Problem 1. The law says *it is a crime for an American to sell weapons to hostile nations*. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

We will prove *West is a criminal*

Knowledge Base

(8) American(x) ^ Weapon(y) ^ Sells(x,y,z) ^ Hostile(z) -> Criminal(x)

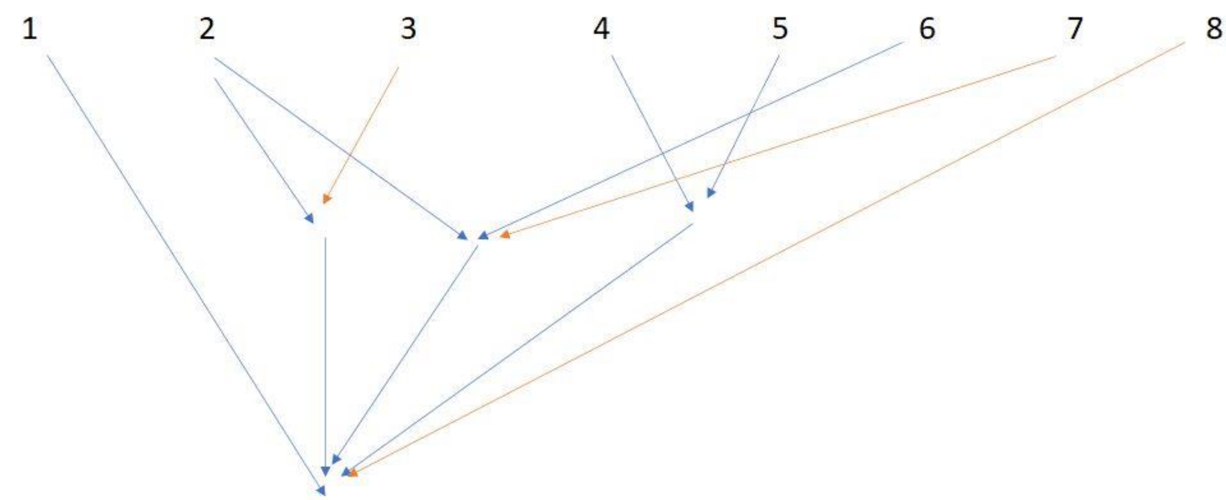
(4) Enemy(Nono, America)   (6) Owns(Nono, M1)   (2) Missile(M1)

(1) American(West)   (7) Missile(x) ^ Owns(Nono,x) -> Sells(West, x, Nono)

(3)Missile(y) -> Weapon(y)   (5) Enemy(z, America) -> Hostile(z)

Criminal(West) ????

Proof: (1), (2)+(3), (4)+(5), (6+2)+(7), *[(1), (2)+(3), (4)+(5), (6+2)+(7)]*+(8), End
MP MP MP MP



Problem 2.

2.1 Show the validity of the formula below by resolution.

(∃x)[[P(x) -> P(A)]^[P(x) -> P(B)]]

Let's take negation of this formula and add to KB.

~(∃x)[[P(x) -> P(A)]^[P(x) -> P(B)]]

$(\forall x)\sim[[\sim P(x) \vee P(A)] \wedge [\sim P(x) \vee P(B)]]$

$[[P(x) \wedge \sim P(A)] \vee [P(x) \wedge \sim P(B)]]$

Clauses in KB:

$P(x)$, $P(x) \vee \sim P(B)$, $\sim P(A) \vee P(x)$, $\sim P(A) \vee \sim P(B)$

Resolution:

$P(x)$, $\sim P(A) \vee \sim P(B)$

$\sim P(B)$

Resolving again with P(x), we get NIL

2.2 Show that statement below is valid following resolution

$(\exists x)(\exists y)\{[P(f(x)) \wedge Q(f(B))] \to [P(f(A)) \wedge P(y) \wedge Q(y)]\}$

Let's take negation of this statement and add it to KB.

$\sim\{\sim[P(f(x)) \wedge Q(f(B))] \vee [P(f(A)) \wedge P(y) \wedge Q(y)]\}$

$[P(f(x)) \wedge Q(f(B))] \wedge \sim[P(f(A)) \wedge P(y) \wedge Q(y)]$

$P(f(x)) \wedge Q(f(B)) \wedge [\sim P(f(A)) \vee \sim P(y) \vee \sim Q(y)]$

Set of clauses:

1. $P(f(x))$
2. $Q(f(B))$
3. $\sim P(f(A)) \vee \sim P(y) \vee \sim Q(y)$

Using resolution:

(4)=(1)+(3): $\sim P(y) \vee \sim Q(y)$ x/f(A)

(5)=(4)+(1): $\sim Q(f(x))$ y/f(x)

(5)+(2): NIL x/B

Problem 3. Convert to the set of clauses:

$(\exists z)[(\exists x)Q(x, z) \vee (\exists x)P(x)] \to \neg\{\neg(\exists x)P(x) \wedge (\forall x)(\exists z)Q(z, x)\}$

$(\exists z)[(\exists x)Q(x, z) \to (\exists x)P(x)] \to \neg\{\neg(\exists x)P(x) \to (\forall x)(\exists z)Q(z, x)\}$

Problem 4.

Problem Statement: 1. Ravi likes all kind of food. 2. Apples and chicken are food 3. Anything anyone eats and is not killed is food 4. Ajay eats peanuts and is still alive 5. Rita eats everything that Ajay eats.

Prove by resolution that *Ravi likes peanuts* using resolution.

i. (∀x)[food(x) → likes(Ravi, x)]
ii. food(Apple) ^ food(chicken)
iii. (∀a)(∀b)[eats(a, b) ^ ~killed(a) → food(b)]
iv. eats(Ajay, Peanuts) -> alive(Ajay)
v. (∀c)[eats(Ajay, c) → eats (Rita, c)]
vi. (∀d)[alive(d) → ~killed(d)]
vii. (∀e)[~killed(e) → alive(e)]
Conclusion: likes (Ravi, Peanuts)

Convert into CNF
i. ~food(x) v likes(Ravi, x)   x,a,b,c,d,e – Skolem variables
ii. food(apple)
iii. food(chicken)
iv. ~ eats(a, b) v killed(a) v food(b)
v. eats(Ajay, peanuts)
vi. alive(Ajay)
vii. ~eats(Ajay, c) v eats(Rita, c)
viii. ~alive(d) v ~ killed(d)
ix. killed(e) v alive(e)
Conclusion: likes(Ravi, Peanuts)
Negate the conclusion: x. ~ likes(Ravi, peanuts)

(1)=(xi)+(i): ~food(peanuts)
(2)=(iv)+(1)+(v): killed(Ajay)
(3)=(vi)+(viii): ~killed(Ajay)
(2)+(3): NUL

Problem 5.
Knowledge Base
Whoever can read is literate: (∀x)[R(x) -> L(x)]
Dolphins are not literate: (∀x)[D(x) -> ~L(x)]
Some dolphins are intelligent: (∃x)[D(x) ^ I(x)]
Prove that:
Some who are intelligent cannot read: (∃x)[I(x) ^ ~R(x)] ???
Clauses: Breath First Strategy

~R(x) v L(x)   ~D(x)v ~L(x)   D(A)   I(A)   ~I(x) v R(x)

~R(x)v ~D(x)   ~L(A)   R(A)

~R(A)

NIL

Problem 4 (we built 5 levels graph)
Every child loves Santa. Everyone who loves Santa loves any reindeer. Rudolph is a reindeer, and Rudolph has a red nose. Anything which has a red nose is weird or is a clown. No reindeer is a clown. Scrooge does not love anything which is weird
Prove that: Scrooge is not a child.   /following breadth first/

LEVEL I
~Child(x) v L(x,Santa),   ~L(x,Santa) v ~Reind(y) v L(x,y),   Reind(Rudolph), Red(Rudolph),   ~Red(x) v Weird(x) v Clown(x),   ~Reind(x) v ~ Clown(x), ~Weird(x) v ~L(Scrooge, x),   Child(x),
LEVEL 2

LEVEL3

Problem 1o
Consider the following facts:
(1) Every child loves anyone who gives the child any present.
(2) Every child will be given some present by Santa if Santa can travel on Christmas eve.
(3) It is foggy on Christmas eve.
(4) Anytime it is foggy, anyone can travel if he has some source of light.
(5) Any reindeer with a red nose is a source of light.
Prove that: If Santa has some reindeer with a red nose, then every child loves Santa.
Alphabet: child(x) – x is a child, loves(x,y) – x loves y, gives (x,y) – x gives y a present, travel(x) – x can travel on Christmas, Foggy – foggy on Christmas Eve, has(x, y)- x has y,

SL-source of light, RRN- reindeer with a red nose.

Knowledge Base:

[gives(x,y) ^ child(y)] -> loves(y,x)
[travel(Santa) ^ child(y)] -> gives(Santa, y)
Foggy
Foggy ^ has(x,SL) -> travel(x)
has(x, RRN) -> has(x, SL)
Prove: has(Santa, RRN) ^ child(y) -> loves(y, Santa) ????

Clauses

~gives(x,y) v ~child(y) v loves(y,x)
~travel(Santa) v ~child(y) v gives(Santa, y)
Foggy
~Foggy v ~has(x,SL) v travel(x)
~has(x, RRN) v has(x, SL)
Prove: ~has(Santa, RRN) v ~child(y) v loves(y, Santa)

Clauses

1) ~gives(x,y) v ~child(y) v loves(y,x)
2) ~travel(Santa) v ~child(y) v gives(Santa, y)
3) Foggy
4) ~Foggy v ~has(x,SL) v travel(x)
5) ~has(x, RRN) v has(x, SL)
6) has(Santa, RRN)
7) child(y)
8) ~loves(y, Santa)

Resolution

(9)=(3)+(4): ~has(x,SL) v travel(x)
(10)=(1)+(7)+(8): ~gives(Santa,y)
(11)=(10)+(2)+(7): ~travel(Santa)
(12)=(5)+(6): has(Santa, SL)
(9)+(12)+(11): NIL