

Exercises

1. Convert to clausal form: $(\forall x)\{[(\exists y)(\forall z)P(x,y,z)] \rightarrow (\forall u)Q(x,u)\}$
2. Consider the following facts:

All hounds howl at night.

Anyone who has any cats will not have any mice.

Light sleepers do not have anything which howls at night.

John has either a cat or a hound.

Prove: *If John is a light sleeper, then John does not have any mice.*

Language predicates: HOUND(x), HOWL(x), HAVE(x,y), CAT(x), MOUSE (x), **Constant:** John

$(\forall x)(\text{HOUND}(x) \rightarrow \text{HOWL}(x))$

$(\forall x)(\forall y)[\text{HAVE}(x,y) \wedge \text{CAT}(y) \rightarrow \neg (\exists z)(\text{HAVE}(x,z) \wedge \text{MOUSE}(z))]$

$(\forall x)[\text{LS}(x) \rightarrow \neg (\exists y) (\text{HAVE}(x,y) \wedge \text{HOWL}(y))]$

$(\exists x)[\text{HAVE}(\text{John},x) \wedge (\text{CAT}(x) \vee \text{HOUND}(x))]$

$\text{LS}(\text{John}) \rightarrow \neg (\exists z)[\text{HAVE}(\text{John},z) \wedge \text{MOUSE}(z)]$ - this we need to prove

$\neg \text{HOUND}(x) \vee \text{HOWL}(x)$

$\neg \text{HAVE}(x,y) \vee \neg \text{CAT}(y) \vee \neg \text{HAVE}(x,z) \vee \neg \text{MOUSE}(z)$

$\neg \text{LS}(x) \vee \neg \text{HAVE}(x,y) \vee \neg \text{HOWL}(y)$

$\text{HAVE}(\text{John},a) \wedge (\text{CAT}(a) \vee \text{HOUND}(a))$

$\text{LS}(\text{John}) \wedge \text{HAVE}(\text{John},b) \wedge \text{MOUSE}(b)$ - negation of what we need to prove

3. Construct language to talk about queries in data tables having schema $T(A,B,C,D)$, where A, B, C, D are attributes. You should define the syntax of queries and their semantics (what is the interpretation domain?)

4. Consider the following facts:

Every child loves Santa.

Everyone who loves Santa loves any reindeer.

Rudolph is a reindeer, and Rudolph has a red nose.

Anything which has a red nose is weird or is a clown.

No reindeer is a clown.

Scrooge does not love anything which is weird

Prove that: *Scrooge is not a child.*

Note: 'has a red nose' can be a single predicate

5. Consider the following facts:

Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.

Every dog chases some rabbit.

Mary buys carrots by the bushel.

Anyone who owns a rabbit hates anything that chases any rabbit.

John owns a dog.

Someone who hates something owned by another person will not date that person.

Prove that: *If Mary does not own a grocery store, she will not date John.*

Note: Represent these clauses in predicate calculus, using only those predicates which are necessary. For example, you need not represent 'person', and phrases such as 'who buys carrots by the bushel' may be represented by a single predicate.

6. Consider the following facts:

Every Austinite who is not conservative loves some armadillo.
Anyone who wears maroon-and-white shirts is an Aggie.
Every Aggie loves every dog.
Nobody who loves every dog loves any armadillo.
Clem is an Austinite, and Clem wears maroon-and-white shirts.
Answer the question: *Is there a conservative Austinite?*

7. Consider the following facts:

Anyone whom Mary loves is a football star.
Any student who does not pass does not play.
John is a student.
Any student who does not study does not pass.
Anyone who does not play is not a football star.
Prove that: *If John does not study, then Mary does not love John.*

8. Consider the following facts:

Every coyote chases some roadrunner.
Every roadrunner who says "beep-beep" is smart.
No coyote catches any smart roadrunner.
Any coyote who chases some roadrunner but does not catch it is frustrated.
Prove that: *If all roadrunners say "beep-beep", then all coyotes are frustrated.*

9. Consider the following facts:

Anyone who rides any Harley is a rough character.
Every biker rides [something that is] either a Harley or a BMW.
Anyone who rides any BMW is a yuppie.
Every yuppie is a lawyer.
Any nice girl does not date anyone who is a rough character.
Mary is a nice girl, and John is a biker.
Prove that: *If John is not a lawyer, then Mary does not date John.*

10. Consider the following facts:

Every child loves anyone who gives the child any present.
Every child will be given some present by Santa if Santa can travel on Christmas eve.
It is foggy on Christmas eve.
Anytime it is foggy, anyone can travel if he has some source of light.
Any reindeer with a red nose is a source of light.
Prove that: *If Santa has some reindeer with a red nose, then every child loves Santa.*