Nash Equilibrium Points in a Game of "Seasonal" Stopping.

Isaac Sonin

Dept. of Mathematics, Univ. of North Carolina at Charlotte,

Charlotte, NC 28223, USA

July 31, 2008

Key words: Nash equilibrium point, Markov chain, optimal stopping, Elimination Algorithm; GEL code : C73

The following problem of optimal stopping (OS) of Markov chain (MC) was dubbed previously by an author as OS of "Seasonal" Observations. A player (decision maker) observes a MC (Z_n) with two components, where the first one is an "underlying" finite MC with m states and given transition matrix P, and the other component is one of m independent sequences of i.i.d. r.v. with known distributions. A simple example of such a situation is a case in which there are two dice with four and six equally likely sides and the determination as to which of them is tossed at a given moment is specified by a position of a Markov chain with two states and given stochastic matrix. The goal of a player is to maximize the discounted expected reward over all possible stopping times.

The *Game of Seasonal stopping* is a game where in a similar setting there are N players, $N \leq m$, and a state space partitioned into N nonempty subsets. A player i can stop the game only when the underlying MC is in her "favorable" subset. Correspondingly a policy of each player is defined by her stopping set S_i. The game stops at moment of a first visit of a MC (Z_n) to a union of these stopping sets. Under mild restriction on reward functions, we prove

Theorem 1. A stopping set in every Nash equilibrium point is defined by the threshold values, and at least one such NE point always exists.

Remark 1. Relatively simple examples show that it is possible that there are more than one NE point and they can be ordered by a "greediness index". The "most greedy" NE point is when every player stops immediately when possible, next when all stop only if some threshold is exceeded and so on, with gradual increase in thresholds. An open problem is to describe all such points. Another line of study is the possible cooperation between players in a similar game.

Remark 2. The proof of Theorem 1 and an algorithm of calculations of at least some of the NE points and corresponding value functions is based on the Elimination Algorithm for the Problem of Optimal Stopping developed by an author earlier.