

## Assignment 4

### Oral questions

1. Given  $A * B * C$  on a line and a point  $D$  not on the line such that  $DC \perp AC$ . Prove that  $AD > BD > CD$ . (Use Theorem 3.3.6 stating that opposite to larger sides you have larger angles.) (Neutral geometry.)
2. Represent points in the plane using their Cartesian coordinates. Explain how Pythagoras' theorem implies the *distance formula* stating that the distance between the points  $A = (a_1, a_2)$  and  $B = (b_1, b_2)$  is  $|AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ . Use this formula to verify that the midpoint of the line segment  $AB$  is  $M = (\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2})$ . (Don't forget to check that  $M$  is on the line  $AB$ .) (Euclidean geometry.)

### Questions to be answered in writing

1. Use the midpoint formula shown in the previous exercise to prove the existence of the centroid as follows. Consider the triangle  $ABC_{\Delta}$  where  $A = (a_1, a_2)$ ,  $B = (b_1, b_2)$  and  $C = (c_1, c_2)$ . Prove that the point  $G = (\frac{a_1+b_1+c_1}{3}, \frac{a_2+b_2+c_2}{3})$  belongs to all three medians. Thus not only all three medians intersect in a common point but we have a formula to express this point in terms of the coordinates of the vertices. Introducing  $A_1$  for the midpoint of the line segment  $BC$ , prove that the distance  $|A_1G|$  is the half of the distance  $|GA|$ . (Euclidean geometry.)
2. Consider a triangle  $ABC_{\Delta}$  and choose a Cartesian coordinate system that has its origin at the circumcenter  $O$  of  $ABC_{\Delta}$ . Using the notation  $\underline{a} := \overrightarrow{OA}$ ,  $\underline{b} := \overrightarrow{OB}$ ,  $\underline{c} := \overrightarrow{OC}$ , and inner products, show that the point  $H$  defined by  $\overrightarrow{OH} = \underline{a} + \underline{b} + \underline{c}$  has the following property: the line  $AH$  is orthogonal to the line  $BC$  (and, similarly  $BH \perp AC$  and  $CH \perp AB$ ). Thus the orthocenter must exist. Using the formulas from the oral exercises, explain how this formula also implies the theorem about the Euler line (not only its existence, but also the stated distance proportions!) (Euclidean geometry.)