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# Study Guide for the Midterm Exam

## Definitions, notions and axioms to remember

1. Axioms: Euclid's postulates (page 38-39, I may ask you also about Birkhoff's and Hilbert's axioms, or to use the SMSG postulates, but if I do so, I will provide a copy of those), Playfair's postulate (page 48), Elliptic Parallel Postulate (page 78).
2. Definitions and notions: incidence, betweenness, rays, line segments, similarity, congruence (see Section 3.2), defect of a triangle, and the following triangle centers: centroid, orthocenter, incenter, circumcenter, Fermat point; sensed ratio, parallelism. You should also be able to use inner products and complex numbers.

## Statements you should remember with their proof

1. From our textbook: Incidence theorem 1 in section 1.4 (pages 29-30), the sum of the angles of a triangle in Euclidean geometry is  $180^\circ$  (section 2.2), two distinct lines can not intersect in more than one point (section 2.4), Isocles Triangle Theorem (Theorem 3.2.7), Perpendicular Bisector Theorem (Theorem 3.2.8), Exterior Angle Theorem (Theorem 3.2.9), triangle congruence conditions (Theorems 3.3.1, 3.3.2, 3.3.3 and 3.3.5), Triangle Inequality (Theorem 3.3.7), Saccheri-Legendre Theorem (Theorem 3.5.1, also the proofs of the lemmas used), base of a Saccheri quadrilateral is not longer than the summit (Theorem 3.6.6), shortest distance between the base and the summit of a Saccheri quadrilateral is the segment connecting the midpoints (Theorem 3.6.9), if a rectangle exist, then there is a rectangle with two arbitrarily large sides (Theorems 3.6.11 and 3.6.12), if one triangle has angle sum  $180^\circ$  then the geometry is Euclidean (Theorems 3.6.13 through 3.6.18), sum of all angles in a triangle is  $180^\circ$  (Theorem 4.2.2), Euclidean exterior angle theorem (Corollary 4.2.3), opposite sides of a parallelogram are congruent (Theorem 4.2.4), parallel transversals theorem (Theorem 4.2.5), Median Concurrence Theorem (Theorem 4.2.7), area formulas (Theorems 4.3.2 through 4.3.6).
2. From lecture and handouts: Symmetries of the Saccheri quadrilateral (Theorems 3.6.1, 3.6.2, 3.6.4), summit angles of a Saccheri quadrilateral can not be obtuse (Theorem 3.6.3), least distance between the base and summit of a Saccheri quadrilateral is at the common perpendicular (Theorem 3.6.9), theorems on the existence of a rectangle (Theorems 3.6.12 and 3.6.13), Ceva's theorem (see also Theorem 4.7.4), existence of the incenter (neutral!).
3. From homework: sum of the interior angles of a triangle from Euclid's fifth postulate, Theorem 3.3.6, SSS congruence (Theorem 3.3.9), fourth angle of a Lambert quadrilateral is not obtuse (Theorem 3.6.7), sides between the two right angles are not longer than the opposite sides in a Lambert quadrilateral (Theorem 3.6.8) distance formula, midpoint formula, Median Concurrence Theorem (see also Theorem 4.2.7), existence of the Euler line.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

## Statements you should know (without proof)

1. From our textbook: first 6 consequences of negating Euclid's fifth postulate on page 76, defect measures area (sums up the remaining consequences on page 76), elementary facts about congruence (Theorems 3.2.1 through 3.2.4) Pasch Axiom (Theorem 3.2.5), Crossbar Theorem (Theorem 3.2.6), Hinge Theorem (Theorem 3.3.8), equivalent forms of Euclid's fifth postulate (theorems in section 3.4), if there is one rectangle then all triangles have angle sum  $180^\circ$  (Theorem 3.6.15), the Euclidean parallel postulate is equivalent to every triangle having angle sum  $180^\circ$ .  
Pythagoras' theorem (+converse), equivalence of Playfair's postulate to Euclid's fifth postulate, Heron's formula, Pasch theorem and crossbar theorem.
2. From lecture: description of Pythagorean triplets.

## What to expect

The exam will be *closed book*. You will have 80 minutes. Some questions may ask you to state and prove a theorem from the list I gave, others may be exercises similar to your homework assignments. There may be questions about examples, whether they have certain properties.