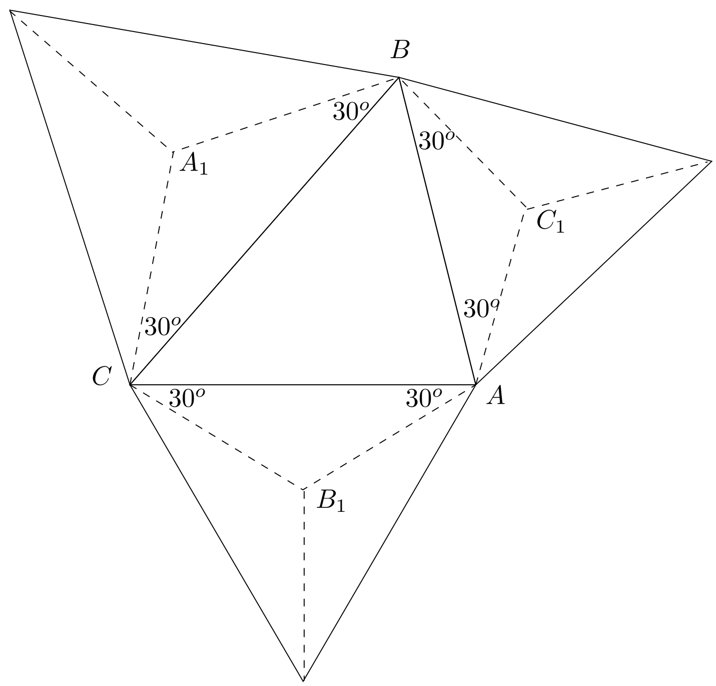
Assignment 8

Oral questions

1. 4.7/13 (Nagle point)
2. For a triangle $\triangle ABC$ let A', B', and C', respectively, be the points where the incircle is tangent to the sides BC, AC, and AB, respectively. Prove that the lines AA', BB' and CC' are concurrent. (The common intersection is the *Gergonne point*.)

Questions to be answered in writing

1. Use Ceva's theorem to prove that the orthocenter exists.
2. Prove Napoleon's theorem: Given an arbitrary triangle $ABC_{\triangle}$, the centers of the equilateral triangles exterior to $ABC_{\triangle}$ form an equilateral triangle. (Illustration and hints on next page.)



Hints: Represent the points A, B, C, A_1, B_1, C_1 with complex numbers a, b, c, a_1, b_1, c_1. Observe that multiplying with

$$\rho := \frac{1}{\sqrt{3}}\left(\cos(30^o) + i \cdot \sin(30^o)\right)$$

rotates the vector $\overrightarrow{BA} = a - b$ into $\overrightarrow{BC_1} = c_1 - b$. Use this observation to express c_1 in terms of a, b and ρ. Express then a_1 and c_1 similarly in terms of a, b, c and ρ. Show that $c_1 - a_1$ is obtained by multiplying $b_1 - a_1$ with

$$\frac{\rho}{1-\rho} = \frac{2\rho - 1}{\rho} = \frac{\rho - 1}{2\rho - 1}.$$

It is probably easier to do so if you find the quadratic equation whose roots are ρ and its conjugate. Finally show that

$$\frac{\rho}{1-\rho} = \cos(60^o) + i \cdot \sin(60^o)$$

meaning that $\overrightarrow{A_1C_1}$ is obtained from $\overrightarrow{A_1B_1}$ by a 60^o rotation.