Assignment 6

Oral questions

1. Review the proof of Thales' theorem and point out the instance(s) where we use Euclid's fifth postulate, or an equivalent statement. Assume then that Thales' theorem is true. Explain why this implies the existence of a triangle with zero defect.

2. Let O be the center of a circle and A and B two points on the circle. Let M be the midpoint of the line segment AB. Prove in neutral geometry that the line OM is perpendicular to AB. (Hint: Corresponding angles of congruent triangles are congruent.)

3. Given $A*B*C$ on a line and a point D not on the line such that $DC \perp AC$. Prove that $AD > BD > CD$. (Use Lemma 7.9 from our notes.)

Question to be answered in writing

1. Let $ABDC$ be a quadrilateral whose base angles $\angle A$ and $\angle B$ are right angles. Prove that if $AC < BD$ then $\angle D < \angle C$. (Hint: Choose E between B and D on the line BD such that $AC = BE$. Apply Theorem 8.3 (i) and the weak exterior angle theorem. You are allowed to use without proof the fact that E is interior to $\angle ACD$.)

2. Assume that the lines ℓ and ℓ' have a common perpendicular line segment MM'. Prove that MM' is the shortest segment between any point of ℓ and any point of ℓ'. (Hint: Assume $A \in \ell$, $A' \in \ell'$ and compare AA' to MM'. Use the previous written exercise when AA' is perpendicular to ℓ and then use the third oral exercise in the other case.)