

Assignment 1

Oral questions

1. Prove formulas (1.1) and (1.2) on page 9 of our notes. (You may use trigonometry.)
2. Find an algebraic expression for the n -th triangular number. In other words, find a closed formula for $1 + 2 + \cdots + n$. Then find a geometric argument showing that the sum of two subsequent triangular numbers is a square number.
3. There are two accounts how Thales calculated the height of a pyramid in Egypt. According to Hieronymus, a pupil of Aristotle, Thales compared the shadow of the pyramid to a shadow of a man at the moment, when the man's shadow was equal to his height. In the later version of the story, given by Plutarch, Thales set up a stick, and used similar triangles. Both versions fail to mention the difficulty of obtaining the length of the shadow of the pyramid, measured from the center of its base. Devise a method, based on similar triangles that determines the height of a pyramid from two shadow observations (made at different times of the same day).

Questions to be answered in writing

1. Prove Gregory's formula (1.3) for $\arctan(x)$. (Hint: take the derivative of $\arctan(x)$ and expand it using the generalized binomial theorem. Explain why the constant term in the Taylor series of $\arctan(x)$ must be zero.)
2. Use Heron's formula to find the radius of the inscribed circle of the triangle, whose side lengths are 3, 3, and 5.