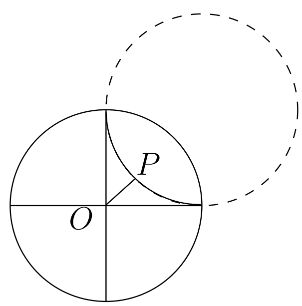
Assignment 10

Oral questions

1. Prove that the distance function $d(A, B) = |\log(AB, PQ)|$ of the Poincaré disk model is additive: if $A * C * B$ on a Poincaré line then $d(AC) + d(CB) = d(AB)$. Fix a Poincaré line with ideal points P and Q and a point A on it. Move another point B along the Poincaré line from P to Q. Show that $d(A, B)$ changes from ∞ to 0 and then back to ∞.

2. Let O be the center of the circle of inversion, P' the inverse of P and Q' the inverse of Q. Assume that O, P, and Q form a triangle. Show that $OPQ_\triangle$ is similar to $OQ'P'_\triangle$. Use this result to show that inversion preserves the cross-ratio: if A, B, P, and Q are four points distinct from the center O of the circle of inversion and A', B', P', and Q' are their inverses then $(AB, PQ) = (A'B', P'Q')$.

3. Schweikart's constant is the distance d for which the angle of parallelism is $\Pi(d) = 45^o$. Prove that for the length function of the Poincaré disk model, Schweikart's constant equals $\log(1+\sqrt{2})$. Do not use Lobachevski's Theorem (Theorem 9.4) but the formula given in Theorem 9.1, and the picture below. (Explain why d is the length of the line segment OP.)



Question to be answered in writing

1. Identify the points of the Euclidean plane with complex numbers and choose a circle of inversion centered at zero, with radius r. Consider another circle centered at the real number r_1, of radius r_1. (This circle contains the origin.) Use complex numbers to prove that the inverse of this circle is a vertical line.