Assignment 3

Oral questions

1. Let $ABC_\triangle$ be a right triangle with a right angle at C and let C_1 be the orthogonal projection of C on AB. Prove that $|CC_1|$ is the *geometric mean* of $|AC_1|$ and $|C_1B|$, that is $|CC_1| = \sqrt{|AC_1| \cdot |C_1B|}$. Deduce the inequality between the arithmetic and geometric mean: $\sqrt{ab} \leq \frac{a+b}{2}$ for all $a, b \geq 0$.
2. Provide an example showing that (SSA) does not set up a congruence of triangles. Define a smaller class of triangles for which (SSA) does imply congruence. (Make a restriction on the angles considered, you do not need to formally prove your statement.)
3. In class we have shown the following: If $A_1 * A_2 * A_3$ and $A_2 * A_3 * A_4$ then $A_1 * A_3 * A_4$ and $A_1 * A_2 * A_4$. Define the *line segment* AB as the set of all points P satisfying $A * P * B$. Using the cited statement, prove that $A * B * C$ implies that AB is a subset of AC.

Questions to be answered in writing

1. Assume that the distance of the points O_1 and O_2 is d. Draw a circle of radius r_1 around O_1 and a circle of radius r_2 around O_2. Express, in terms of equations and inequalities for r_1, r_2 and d, necessary and sufficient conditions for the two circles to have 0, 1 or 2 points in common. (You do not have to prove your claims, but you have to consider all possibilities, including one circle containing the other one.)
2. Explain how Thales' theorem is a special case of the Star Trek Lemma. Prove Thales' theorem. Prove the Star Trek Lemma in the case when the angle $\angle BOC$ is acute and O is on the line segment AB.