## Bonus Quiz 1.

The deadline to return this bonus quiz is March 27. The points will be added to your homework score.

1. (5 points) What is the coefficient of $x^{6}$ in $\left(1+x^{2}-x^{3}\right)^{3}$ ?
2. (5 points) Write the polynomial $x^{5}-x^{3}+x$ as a linear combination of the falling factorials $(x)_{n}$.
3. (10 points) Solve exercise $3.1 / 20$ from the textbook.
4. ( $15+10$ points) A set of integers is sparse if it contains no two consecutive integers. For example $\{1,3,7\}$ is sparse but $\{1,3,6,7,11\}$ is not because 6 and 7 are consecutive integers. Express in terms of $n$ the number of sparse subsets of $\{1,2, \ldots, n\}$. You get the 10 additional points on top of the 15 , if you are able to prove your formula.
5. $\left(10+10+10\right.$ points) The sequence $a_{0}, a_{1}, \ldots$ is given by $a_{0}=a_{1}=1$ and by the recursion formula $a_{n+1}=-a_{n}-a_{n-1}$. Find a closed formula for $a_{n}$. You get only 10 points if you find the formula. You will get 10 additional points if you prove the formula by induction. You will get 30 points if you find and prove the formula using generating functions (and complex numbers).

Good luck.

Gábor Hetyei

