Sample Test 3

This list of sample test questions is subject to updates until we review for the test
Last update: November 12, 2020

1. Find the sum of the matrices $A = \begin{bmatrix} 2 & 3 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

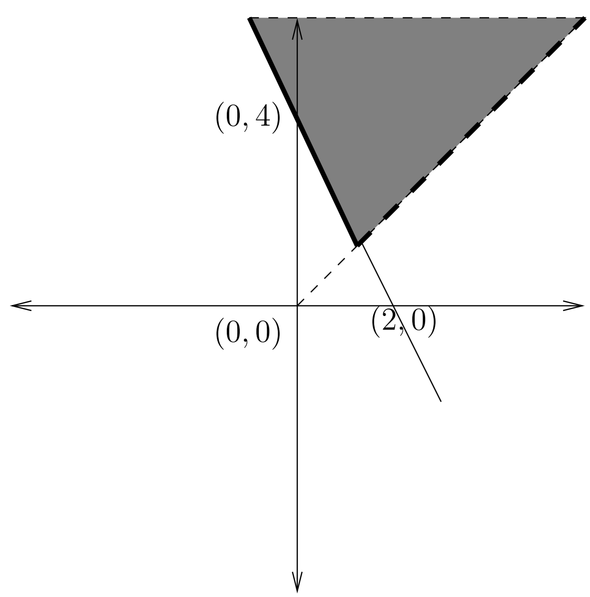
2. Find the sum of the matrices $A = \begin{bmatrix} 1 & 3 & 4 \\ -3 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 1 & 0 \\ 0 & 8 \end{bmatrix}$.

3. Find the dimensions of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

4. Multiply the matrix $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$ with the scalar (-5).

5. Find
$$2 \cdot \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} - 3 \cdot \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}.$$

6. Write a system of inequalities describing the shaded infinite region bounded by the line $y = x$ and the line passing through the points $(0, 4)$ and $(2, 0)$.



7. Write a system of inequalities describing the set of points on or above the line $2x - y = 3$ and on or to the left of the line $x = 3$.

8. List the corners of the feasible region of the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$2x + 3y \leq 12$$

Also describe the shape of the region: is it a triangle, a quadrilateral, or is it unbounded.

9. List the corners of the feasible region of the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 7$$

Also describe the shape of the region: is it a triangle, a quadrilateral, or is it unbounded.

10. List the corners of the feasible region of the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$y \leq 10 - x$$
$$x \leq 5$$

Also describe the shape of the region: is it a triangle, a quadrilateral, or is it unbounded.

11. Find the maximum value of the objective function $5x + y$ on the bounded feasible region whose corners are $(0, 0)$, $(5, 0)$, $(5, 3)$, $(1, 7)$ and $(0, 7)$. Also describe the set of points where the maximum is achieved.

12. Find the maximum value of the objective function $x + y$ on the bounded feasible region whose corners are $(0, 0)$, $(5, 0)$, $(5, 3)$, $(1, 7)$ and $(0, 7)$. Also describe the set of points where the maximum is achieved.

13. You make x ham and y veggie sandwiches for sale. The ham sandwich takes 2 slices of ham, the veggie needs none. Both sandwiches take one slice of bread each. The veggie sandwich needs 3 lettuce leaves the ham sandwich needs one. Only the veggie sandwich

needs one portobello mushroom. You have 30 slices of ham, 50 slices of bread, 20 lettuce leaves and 15 portobello mushrooms. Write down the inequalities describing the feasible region. What objective function would you maximize to find the maximum income if the ham sandwich costs 4 dollars and the veggie sandwich costs 5 dollars. *Do not solve!*

14. How many subsets does the set $\{a, b, c, d\}$ have?

15. Find $A \cup B$ and $A \cap B$ for $A = \{x \mid x \text{ is an integer and } 2 \leq x \leq 6\}$ and $B = \{1, 3, 5, 7, 9\}$.

16. Find the complement of $A = \{3, 4, 6, 8\}$ with respect to the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

17. Which of the following is true for $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6\}$ and $C = \{1, 2, 3, 4\}$?

    (a) $C \subseteq B \subseteq A$

    (b) $C \subseteq A \subseteq B$

    (c) $A \subseteq C \subseteq B$

    (d) $A \subseteq B \subseteq C$

Solutions:

1.
$$A + B = \begin{bmatrix} 7 & 5 & 1 \\ -2 & 1 & 2 \end{bmatrix}.$$

2. The sum does not exist, the matrices are of different sizes.

3. It is a 2×3 matrix.

4.
$$(-5) \cdot A = \begin{bmatrix} -5 & 15 \\ 10 & -30 \end{bmatrix}.$$

5.
$$\begin{bmatrix} -2 & 0 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -9 \\ 0 & 2 \end{bmatrix}.$$

6. The equation of the line passing through $(0, 4)$ and $(2, 0)$ has slope -2 and y-intercept 4. The region has points on or above this line and strictly above the line $y = x$. Hence the system is
$$y \geq -2x + 4$$
$$y > x.$$

7.
$$y \geq 2x - 3$$
$$x \leq 3.$$

8. It is a triangle, the corners are $(0, 0)$, $(6, 0)$ and $(0, 4)$.

9. It is an unbounded region, its corners are $(7, 0)$ and $(0, 7)$.

10. It is a quadrilateral, its corners are $(0,0)$, $(5,0)$, $(5,5)$ and $(0,10)$.

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11.

x	y	$5x+y$
0	0	0
5	0	25
5	3	28
1	7	12
0	7	7

The maximum is 28, and it is only achieved at the corner $(5,3)$.

12.

x	y	$x+y$
0	0	0
5	0	5
5	3	8
1	7	8
0	7	7

The maximum is 8, and it is achieved on the line segment connecting the corners $(5,3)$ and $(1,7)$.

13. The feasible region is given by

$$\begin{aligned} 2x &\leq 30 \text{ (ham)} \\ x+3y &\leq 20 \text{ (lettuce)} \\ x+y &\leq 50 \text{ (bread)} \\ y &\leq 15 \text{ (portobello mushroom)} \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

The objective function you want to maximize is $4x+5y$.

14. The set has $n=4$ elements, the answer is $2^4=16$.

15. $A \cup B = \{1,2,3,4,5,6,7,9\}$ and $A \cap B = \{3,5\}$.

16. $A' = \{1, 2, 5, 7\}$.

17. (c).