Sample Test 2

This list of sample test questions is subject to updates until we review for the test
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1. Which of the following relations is *not* a function?

   (1) $\{(1,1),(2,1),(3,1)\}$;

   (2) $\{(1,1),(1,2),(1,3)\}$;

   (3) $\{(1,2),(2,1),(3,3)\}$.

2. Find the domain and range of the relation $\{(-1,5)),(0,3),(2,3),(2,4)\}$.

3. Which of the following equations does *not* describe function?

   (1) $x^2+y^2=1$. (This is the equation of a circle.)

   (2) $2x+3y=5$. (This is the equation of a line.)

   (3) $y=x^2$ (This is the equation of a parabola with a vertical axis of symmetry.)

   (On the test I will either provide pictures, or a verbal description as above.)

4. Find the domain of the function $f(x)=\sqrt{x-x^2}$.

5. Find the domain of the function

$$f(x)=\frac{1}{\sqrt{x^2-1}}.$$

6. Find the vertex and axis of symmetry of the parabola $y=-3(x+1)^2-7$. Also answer the question whether the parabola is open up or open down.

7. Find the vertex and axis of symmetry of the parabola $y=2x^2+40x-7$. Also answer the question whether the parabola is open up or open down.

8. A projectile is at $f(t)=500t-10t^2$ meters from ground, t seconds after it is fired. When does the projectile reach its highest altitude, and what is the highest altitude it reaches?

9. By selling x items per month a company can make $P(x)=20,000x-20x^2$ profit. What is the maximum profit the company can make, and how many items should they sell per month?

10. Solve the system of equations

$$\begin{aligned} x - 2y &= 4 \\ 3x + 3y &= 3. \end{aligned}$$

11. Solve the system of equations

$$\begin{aligned} 2x - y &= 3 \\ 4x - 2y &= 6. \end{aligned}$$

12. Solve the system of equations

$$\begin{aligned} 5x &= 1 + y \\ 10x - 2y &= 6. \end{aligned}$$

13. There are x chickens and y rabbits in a yard. The number of their heads is 35 and they have 110 legs altogether. What is the number of chickens and what is the number of rabbits?

14. Subtract the appropriate multiples of the first equation from the second and third equations to eliminate the variable x from the second and third equations.

$$\begin{aligned} x - 2y + 3z &= 2 \\ 3x - y + z &= 6 \\ -2x + y - z &= 1. \end{aligned}$$

15. Find all solutions of the system of equations below. Note the system is in reduced row echelon form.

$$\begin{aligned} x - y + 2z &= 2 \\ y + 5z &= 1 \\ z &= -1. \end{aligned}$$

16. Find all solutions of the system of equations below. Note the system is in reduced row echelon form.

$$\begin{aligned} x - 5y + z &= 10 \\ y - z &= 7 \\ 0 &= -10. \end{aligned}$$

17. Find all solutions of the system of equations below. Note the system is in reduced row echelon form.

$$\begin{aligned} x + 2y - z &= 6 \\ y - 7z &= 1 \\ 0 &= 0. \end{aligned}$$

18. A company is planning to lease 300 vehicles: x vans, y small trucks and z large trucks. Leasing vans costs $20,000$ dollars, leasing small trucks costs $15,000$ dollars, leasing large trucks costs $45,000$ dollars. The company has $4,750,000$ dollars to lease the vehicles. They want the number of large trucks leased to be 3 times the number of vans leased. Write a linear system of equations to solve this problem. Write all equations in the form $ax + by + cz = d$. **Do not solve!**

Solutions:

1. (2), because 1 appears as the first coordinate more than once.

2. The domain is the set of the first coordinates, that is $\{-1, 0, 2\}$, the range is the set of the second coordinates, that is $\{3, 4, 5\}$.

3. (1), a circle fails the vertical line test. A non-vertical line is the graph of a linear function, a parabola with vertical axis of symmetry is the graph of a quadratic function.

4. $[0, 1]$. Under the square root we can only have a nonnegative number, hence the domain is given by the inequality $x - x^2 \leq 0$.

5. $(-\infty, -1) \cup (1, \infty)$. Under the square root we can only have a nonnegative number, and we can not divide by zero. Hence the domain is given by the inequality $x^2 - 1 > 0$.

6. The vertex is $(-1, -7)$, the axis of symmetry is $x = -1$, the parabola is open down.

   We have $y = a(x - h)^2 + k$ with $a = -3$, $h = -1$ and $k = -7$. The vertex is (h, k). The axis of symmetry is vertical and it contains the vertex, so it is $x = -1$. Since $a = -3$ is negative, the parabola is open down.

7. The vertex is $(10, 593)$, the axis of symmetry is $x = 10$, the parabola is open up.

   We have $y = ax^2 + bx + c$ with $a = 2$, $b = 40$ and $c = -7$. The first coordinate of the vertex (h, k) is given by

   $$h = \frac{-b}{2a} = \frac{-40}{4} = -10.$$

   The axis of symmetry is vertical and it contains the vertex, so it is $x = -10$. The second coordinate k of the vertex is given by

   $$k = f(h) = f(-10) = 2 \cdot 10^2 + 40 \cdot (-10) - 7 = -207.$$

   Since $a = 2$ is positive, the parabola is open up.

8. The graph of $f(t)$ is a parabola, we are looking for its vertex. $f(t) = at^2 + bt + c$ with $a = -10$, $b = 500$ and $c = 0$. The first coordinate of the vertex (h, k) is

$$h = -\frac{b}{2a} = \frac{-500}{-20} = 25.$$

Maximum altitude is reached after $t = 25$ seconds. The maximum altitude is

$$k = f(h) = 500 \cdot 25 - 10 \cdot 25^2 = 6,250 \text{ meters.}$$

9. The graph of $p(x)$ is a parabola, we are looking for its vertex. $P(x) = ax^2 + bx + c$ with $a = -20$, $b = 20,000$ and $c = 0$. The first coordinate of the vertex (h, k) is

$$h = -\frac{b}{2a} = \frac{-20,000}{-40} = 500.$$

Maximum profit is reached by selling $x = 500$ items. The maximum profit is

$$k = f(h) = 20,000 \cdot 500 - 20 \cdot 500^2 = 5,000,000 \text{ dollars.}$$

10. $(2, -1)$. There are two ways to solve such a system: by substitution, or by elimination. Here is the way to solve it by elimination. Multiply the first equation by three and subtract it from the second.

$$\begin{aligned} x - 2y &= 4 \\ 9y &= -9. \end{aligned}$$

From the second equation we get $y = -1$. Substituting this into the first equation we get $x + 2 = 4$, that is, $x = 2$.

11. $(x, 2x-3)$. There are two ways to solve such a system: by substitution, or by elimination. Here is the way to solve it by elimination. Multiply the first equation by two and subtract it from the second.

$$\begin{aligned} 2x - y &= 3 \\ 0 &= 0. \end{aligned}$$

The second equation became $0 = 0$, we have infinitely many solutions. Solving the first equation for y gives $y = 2x - 3$.

12. No solution. Subtract y on both sides in the first equation, to get the familiar form

$$\begin{aligned} 5x - y &= 1 \\ 10x - 2y &= 6. \end{aligned}$$

Multiply the first equation by two and subtract it from the second.

$$\begin{aligned} 5x - y &= 1 \\ 0 &= 4 \end{aligned}$$

The second equation became $0 = 4$, there is no solution.

13. $x = 15$ chickens and $y = 20$ rabbits. We need to solve the system of linear equations

$$\begin{aligned} x + y &= 35 \\ 2x + 4y &= 110. \end{aligned}$$

From the first equation we get $x = 35 - y$. Substituting the value of x into the second equation gives $2(35 - y) + 4y = 110$, that is $70 + 2y = 110$, so $2y = 40$ and $y = 20$. By $x = 35 - y = 35 - 20$ we get $x = 15$.

14. Multiply the first equation by 3 and subtract it from the second:

$$\begin{aligned} x - 2y + 3z &= 2 \\ 5y - 8z &= 0 \\ -2x + y - z &= 1. \end{aligned}$$

Multiply the first equation by 2 and add it to the third:

$$\begin{aligned} x - 2y + 3z &= 2 \\ 5y - 8z &= 0 \\ -3y + 5z &= 5. \end{aligned}$$

15. From the third equation we get $z = -1$. Substituting into the second equation we get $y + 5 \cdot (-1) = 1$, so $y - 5 = 1$ and $y = 6$. Substituting the values of y and z into the first equation we get $x - 6 + 2 \cdot (-1) = 2$, that is, $x - 8 = 2$, and so $x = 10$. The only solution is $(10, 6, -1)$.

16. There is no solution, because the third equation is a contradiction.

17. z is a free variable. From the second equation we get $y = 1 + 7z$. Substituting the value of y into the first equation we get $x + 2(1 + 7z) - z = 6$, that is, $z + 2 + 13z = 6$, and so $x = 4 - 13z$. There are infinitely many solutions, the solution set is given by $(4 - 13z, 1 + 7z, z)$.

18.

$$\begin{aligned} x + y + z &= 300 \\ 20x + 15y + 45z &= 4,750 \\ 3x \qquad\quad - z &= 0. \end{aligned}$$