Sample Final Exam Questions (mandatory part)

The actual final exam will have a mandatory and an optional section. The optional questions will be similar to the ones on the previous (sample) tests, and need to be answered only if you do not want me to re-use your average test score. The questions below are supposed to help you prepare for the mandatory part of the final.

1. Find a minimum cut and a maximum flow for the network shown in Figure 1. **Show all your work!**

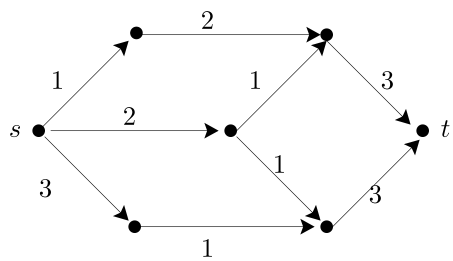


Figure 1: Network with source s and sink t

2. Suppose the vertices in Figure 1 are cities, and the capacities indicate how many wagons can travel along that edge each day. Suppose wagons can leave each city once a day and each edge takes 2 days to travel. Explain how would you find the number of wagons that can be sent from s to t in 12 days. (Only describe how would you set up the problem.)

3. Give an example of a maximal flow that is not maximum. Explain how finding an augmenting path in the slack picture can help correct mistakes.

4. State the Ford-Fulkerson theorem for network flows. Explain how the network flow algorithm may be used to prove it for integer capacities. Indicate what problem you may encounter if you allowed non-integer flows and capacities, and what theorem in analysis makes the Ford-Fulkerson theorem still valid.

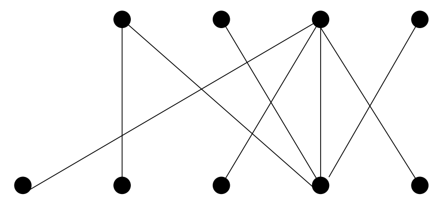
5. State Menger's theorem (edge version) and explain how network flows may be used to prove them. (See your notes and "messenger problems" in the book). You only need to state, how would you set up the network associated to the graph.

6. Explain how the question of finding a maximum size matching and a minimum size edge cover in a bipartite graph may be translated into the question of finding a maximum flow and a minimum cut in a network flow. Prove that the maximum size of a matching is the same as the minimum size of a cover in a bipartite graph.

7. Find a maximum size matching and a minimum size cover in the bipartite graph below by stating and solving a related network flow problem. *Show all your work!*

8. State and prove Hall's Theorem.

9. State and prove Birkhoff's Theorem.



10. Write the following doubly stochastic matrix as a convex combination of permutation matrices:

$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

11. Solve a transportation problem, just like in the following exercises in our textbook: 4.5/1,3,5,7. *Show all your work!*

12. Prove that a transportation problem has always an optimal solution S, for which the associated set of edges $E(S)$ contains no circuit.

13. Explain while repeated use of the northwest corner rule yields a spanning tree solution to the transportation problem.

14. State and prove the formula expressing the cost of a spanning tree solution to the transportation problem in terms of the supply and demand numbers and the price values associated to the vertices. Explain how these prices are calculated, and why they are uniquely defined.

15. State the dual problem to the transportation problem. Explain why a common solution value to both problems is the value of the optimum solution. Explain also why the "solutions" found to the dual problem in the algorithm solving the transportation problem are not valid, except for the one reached at the end.

16. Suppose that while solving the transportation problem the cost does not decrease in a step. Are you sure you can stop there? Why not?

17. Explain why you may need edges with a zero flow on them in a spanning tree solution of the transportation problem. How could the use of the northwest corner rule force the presence of such an edge even in the initial solution?

18. Suppose you find an edge that could be added to a spanning tree solution in the algorithm finding an optimal solution. Explain how you would rearrange the flow and what would you do if you end up with several edges in the circuit created having zero flow on them.

19. Use the branch and bound method to solve the traveling salesperson problem indicated in 3.3/1.

20. State a condition for the cost function which, if satisfied, allows to use a polynomial time algorithm finding a sub-optimal solution to the traveling salesperson problem whose cost is at most twice the minimum cost?

Good Luck. Gábor Hetyei