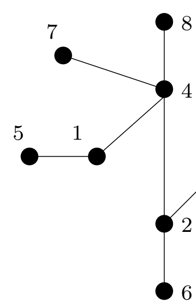
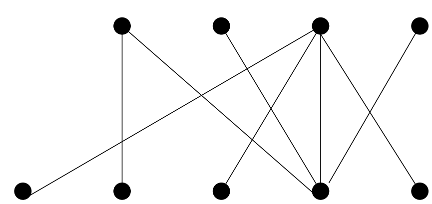
Sample Final Exam Questions (mandatory part)

The actual final exam will have a mandatory and an optional section. The optional questions will be similar to the ones on the previous (sample) tests, and need to be answered only if you do not want me to re-use your average test score. The questions below are supposed to help you prepare for the mandatory part of the final.

1. Draw the tree whose Prüfer code is 222. *Show all your work!*
2. Find the Prüfer code of the tree shown below, *Show all your work!*



3. Explain how the question of finding a maximum size matching and a minimum size edge cover in a bipartite graph may be translated into the question of finding a maximum flow and a minimum cut in a network flow. Prove that the maximum size of a matching is the same as the minimum size of a cover in a bipartite graph.
4. Find a maximum size matching and a minimum size cover in the bipartite graph below by stating and solving a related network flow problem. *Show all your work!*



5. State and prove Hall's Theorem.
6. State and prove Birkhoff's Theorem.
7. Write the following doubly stochastic matrix as a convex combination of permutation matrices:

$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

8. Solve a transportation problem, just like in the following exercises in our textbook: 4.5/1,3,5,7. *Show all your work!*
9. Prove that a transportation problem has always an optimal solution S, for which the associated set of edges $E(S)$ contains no circuit.

10. Explain how a system of prices can be associated to a spanning tree solution and express the transportation cost of the solution in terms of the supply and demand prices. Prove your formula.

11. Use bubble sort to put the following list of numbers into increasing order: $4, 1, 3, 2$. List all steps.

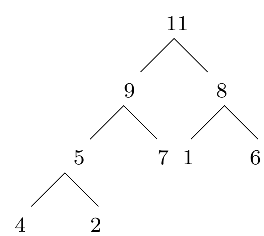
12. What is the number of comparisons in the bubble sort algorithm, if it is used to sort a list of n-entries? Justify your formula.

13. Use merge sort to put the following list of numbers into increasing order: $4, 1, 5, 3, 2$. Indicate all steps in your illustration.

14. What is the maximum number of comparisons used to sort a list of n items using merge sort? Prove your formula.

15. Define what is a "heap" used in the heap sort algorithm.

16. Use heap sort to sort the entries at the nodes of the heap below. Show all phases.



17. Use the branch and bound method to solve the traveling salesperson problem indicated in 3.3/1.

18. What properties must a travel cost matrix have to make the approximate algorithm applicable? How does the algorithm work? What can you guarantee: how good is the approximate solution compared to the optimal solution? Prove your claim!

19. Define a progressively finite game.

20. Define the Grundy function (Grundy numbering) on the progressively finite game. Prove that the second player has a winning strategy if an only if the games starts at a position whose Grundy number is not zero.

21. Using Grundy numbering, describe the winning strategy for the following game: there are 30 sticks two players take turns removing up to five sticks. The winner is the player who removes the last stick. Which player has a winning strategy?

Good Luck. Gábor Hetyei