Additional Homework problems

This dynamically updated document lists the mandatory and bonus homework problems that were assigned on the board and can not be found in the book.

Last update: Monday, October 4, 2010

B1 Bonus problem, due Wednesday, September 22.

Let $n > 1$ be a fixed positive integer and consider the set $\mathbb{P}$ of positive integers modulo n. For any $a \in \mathbb{P}$ let $[a]$ denote the set $\{a + kn \mid k \in \mathbb{Z}, a + kn > 0\}$, i.e., the set of positive integers congruent to a, modulo n. We have seen in class that $[a] + [b] \subseteq [a + b]$ holds for any pair (a, b) of positive integers. Is it true that $[a] + [b]$ always contains $[a + b]$? If yes, prove your claim, if not, show a counterexample.

B2 Bonus problem, due Wednesday, October 13.

Consider the set of all functions $f : \mathbb{Z} \to \mathbb{Z}$, together with the composition operation. As we saw it in class, this is a semigroup, and the identity element is the function $\imath : x \mapsto x$. Give an example of a function with infinitely many right inverses but no left inverse. A function $g : \mathbb{Z} \to \mathbb{Z}$ is a right inverse of f if $f \circ g = \imath$. Similarly, a function $g : \mathbb{Z} \to \mathbb{Z}$ is a left inverse of f if $g \circ f = \imath$.

B3 Bonus problem, due Wednesday, November 17.

Prove that $\sqrt[5]{2}$ is irrational.

B4 Bonus problem, due Monday, December 6.

Let V be any subset of $\mathbb{C}^n$. Prove that the set $I := \{f(x_1, \ldots, x_n) \in \mathbb{C}[x_1, \ldots, x_n] : f(c_1, \ldots, c_n) = 0 \text{ for all } (c_1, \ldots, c_n) \in V\}$ is an ideal of $\mathbb{C}[x_1, \ldots, x_n]$.