Name: **49-er Express/Webwork login name:**

Sample Test II.

The actual test will have less questions. You will have 75 minutes to answer them, without using your notes or communicating with other students. You will have to give the simplest possible answer and show all your work.

1. Using the definition of the derivative, find the derivative of $f(x) = x^2 - x$ at $x = 3$.
   Answer: $\lim_{h\to 0} \frac{(3+h)^2 - (3+h) - 6}{h} = \lim_{h\to 0} \frac{h^2 + 5h}{h} = 5.$
2. Find the derivative of $x \cdot \sqrt{x}$. (Product rule.)
   Answer: $\sqrt{x} + \frac{x}{2 \cdot \sqrt{x}}$.
3. Find the derivative of $\frac{x^2 - x}{2x - 1}$. (Quotient rule.)
   Answer: $1 - \frac{2(x^2 - x)}{(2x-1)^2}$.
4. Find the derivative of $\sqrt{x^2 - 1}$. (Chain rule.)
   Answer: $\frac{x^2 - 1}{x}$.
5. The daily cost to produce x boxes of chocolate is $C(x) = 0.01 \cdot x^2 + x + 10$ dollars. What is the cost incurred at manufacturing the 101st box of chocolate? What is the marginal cost when $x = 100$?
   Answer: Actual cost is $C(101) - C(100) = 213.01 - 210 = 3.01$, marginal cost is $C'(100) = 3.00$.
6. The demand x for a product is given by the formula $x = \sqrt{30 - p}$. Recall that the elasticity of the demand is given by $E(p) = -\frac{pf'(p)}{f(p)}$. Is the demand elastic, inelastic or unitary at the price level $p = 20$?
   Answer: Unitary.
7. Find the second derivative of $x \cdot \sqrt{x-1}$.
   Answer: $\frac{1}{\sqrt{x-1}} - \frac{x}{4(x-1)^{3/2}}$.
8. Find the differential of $y = 2x^2 - x$. Use the differential to find the approximate change of y if x changes from 3 to 2.8. What is the actual change?
   Answer: The differential is $dy = (4x-1)dx$. The approximate change is $(4\cdot 3-1)\cdot(-0.2) = -2.2$. The actual change is $12.88 - 15 = -2.12$.
9. Find the intervals where the function $f(x) = 2x^3 - 15x^2 + 36x$ is increasing.
   Answer: $f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$, positive on $(\infty, 2) \cup (3, \infty)$.

10. Find the minimum and maximum values of $f(x) = x \cdot \sqrt{1-x}$ on the interval $[0.5, 1]$.
    Answer: $f'(x) = \sqrt{1-x} - \dfrac{x}{\sqrt{1-x}} = \dfrac{2-3x}{\sqrt{1-x}}$. Critical points are $x = 2/3$ and $x = 1$. Maximum is $f(2/3) = \dfrac{2\sqrt{3}}{9}$, minimum is $f(1) = 0$.

11. Find the critical points and asymptotes of $f(x) = \dfrac{x^2+x}{x^2-x}$.
    Answer: Horizontal asymptote at $y = 1$, vertical asymptote at $x = 1$, hole at $x = 0$. $f'(x) = \dfrac{x^2-2x}{(x-1)^2}$ (simplified form!), critical points are $x = 0, 1, 2$.

12. Find the local minima and maxima of $f(x) = \dfrac{x^2+x}{x^2-x}$.
    Answer: Critical points are $x = 0, 1$, but neither of them is a zero of the derivative. There is no local minimum.

13. Describe the interval(s) where the graph of $f(x) = x^3 - x^2$ is concave up.
    Answer: Concave up on $(1/3, \infty)$.

14. Describe the interval(s) where the graph of $f(t) = \sqrt{t^2-9}$ is concave up.
    Answer: Nowhere. Concave down on $(-\infty, -3) \cup (3, \infty)$.

15. Find the inflection point(s) of $f(x) = x^3 - 2x^2$.
    Answer: $x = 2/3$.

16. You are given the fact that $x = 5$ is a critical point of $f(x)$ and that $f''(5) = 1.2$. According to the second derivative test, do you have a local minimum or maximum at $x = 5$, or is the test inconclusive?
    Answer: Relative minimum.