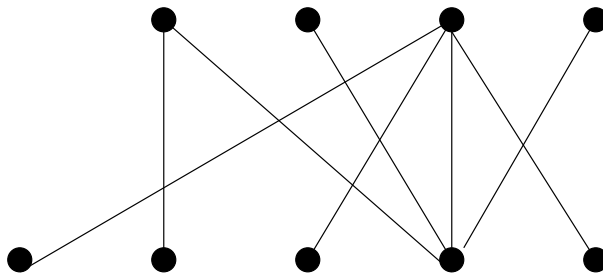# Sample Test 1
The actual test will have only 5 *questions.*

1. Draw the line graph of the graph whose incidence matrix is

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

   (Here the rows are associated to edges.)

2. Show that every 5-regular graph has an even number of vertices.

3. Assume a graph G has at least one cycle. Estimate the girth of the graph in terms of the diameter and give an example where your bound is sharp.

4. A forest has 10 vertices and 5 edges. What is the number of its connected components?

5. Prove that a graph is bipartite if and only if it contains no odd cycle. Indicate the main idea that allows to replace the term "cycle" with "induced cycle" in the statement.

6. For which values of d has the d-cube an Eulerian tour?

7. Prove that the cutspace of a graph is a vector space, that is, it is closed under taking symmetric differences.

8. Write up a basis for for the cycle space of the 3-cube. Draw a picture and indicate what spanning tree are you using. What is the dimension of the cutspace of the 3-cube?

9. Describe the symmetric difference of two matchings.

10. Find a maximum size matching and a minimum size cover in the graph below.

Good Luck.   Gábor Hetyei