PAlmetto Number Theory Series

September 21-22, 2019

Abstracts

Scott Ahlgren (Illinois)
Schedule: Sunday, Fretwell 113, 9:00 AM
Title: Kloosterman sums, Maass forms, and partitions
Abstract: I will discuss recent work on uniform bounds for sums of Kloosterman sums and coefficients of Maass forms in half-integral weight. This work is motivated by applications to classical problems involving partitions and an old conjecture of Andrews about the coefficients of the "third-order" mock theta function.

Shabnam Akhtari (Oregon)
Schedule: Saturday, Fretwell 113, 5:00 PM
Title: Representation of integers by binary forms
Abstract: Let $F(x, y)$ be an irreducible binary form with integer coefficients and of degree at least 3. Let m be a nonzero integer. By a well-known result of Thue, the equation $F(x, y) = m$ has only finitely many solutions in integers x and y. I will discuss some old and new quantitative results on the number of solutions to Thue equations. I will also talk about a joint work with Manjul Bhargava, where we use explicit upper bounds for the number of solutions of Thue equations to show that many such equations have no solutions.

Angelica Babei (Vanderbilt)
Schedule: Saturday, Fretwell 113, 3:30 PM
Title: Metacommutation of primes in Eichler orders
Abstract: In this talk, we present the metacommutation problem in locally Eichler orders. From this problem arises a permutation of the set of locally principal left ideals of a given prime reduced norm. Previous results on the cycle structure were determined for locally maximal orders. As we extend these results, we present an alternative, combinatorial description of the metacommutation permutation as an action on the Bruhat-Tits tree. Joint work with Sara Chari.

Kubra Benli (Georgia)
Schedule: Saturday, Fretwell 121, 10:45 AM
Title: Changes in digits of primes
Abstract: The infinitude of primes that change into a composite number after any change in a single digit is proved by Erdős. We will talk about quantitative improvements once we restrict to the case when the resulting composite numbers have as many distinct prime factors as expected from a 'normal' integer.

Stevo Bozinovski (SC State)
Schedule: Sunday, Fretwell 113, 10:00 AM
Title: Brain rhythms and Number Theory: An observation
Abstract: Brain rhythms in a human EEG are defined by their frequency ranges. The ranges are empirically obtained and various authors define them differently. A theory is needed about an underlying process that defines the ranges of brain rhythms. This work

shows that a process in number theory can be used to define ranges of the brain rhythms. This is a joint work with Adrijan Bozinovski.

Fatma Cicek (Rochester)
Schedule: Saturday, Fretwell 113, 4:30 PM
Title: Selberg's Central Limit Theorem and Its Analogues
Abstract: One of the most influential probabilistic results in analytic number theory is Selberg's central limit theorem. Roughly, it states that the logarithm of the Riemann zeta-function on the critical line has an approximate two-dimensional normal distribution. We assume RH and write $\rho = \frac{1}{2} + i\gamma$ to consider the distribution of the following sequences

$$\log \zeta(\rho + z) \quad \text{and} \quad \log |\zeta'(\rho)|$$

for $0 < \gamma \leq T$ where z is a sufficiently small nonzero complex number and T is large. Our results show that on the further assumption of a certain zero-spacing hypothesis, analogues of Selberg's central limit theorem hold for these sequences.

Anup Dixit (Queen's University)
Schedule: Sunday, Fretwell 128, 10:30 AM
Title: On the classification problem for L-functions
Abstract: L-functions play a vital role in number theory. These functions arise from arithmetic and geometric objects and their value distribution encodes significant information about the underlying structure. The degree and the conductor are certain invariants associated to L-functions. A classical converse question is to determine how many L-functions are there with a given degree and conductor. In this talk, we discuss this question in the context of general Dirichlet series and highlight its application to the study of families of number fields.

Alex Dunn (Illinois)
Schedule: Saturday, Fretwell 121, 10:15 AM
Title: The twisted second moment of modular half integral weight L-functions
Abstract: Given a half-integral weight holomorphic Kohnen newform f on $\Gamma_0(4)$, we prove an asymptotic formula for the second moment of the L-function attached to $f \otimes \chi$, over all primitive χ modulo a prime p. Our result is unconditional, it does not rely on the Ramanujan—Petersson conjecture for the form f. This gives a very sharp Lindelöf on average result for L-series attached to Hecke eigenforms without an Euler product. The Lindelöf hypothesis for such series was originally conjectured by Hoffstein. (joint work with A.Zaharescu).

Duc Huynh (Georgia Southern)
Schedule: Sunday, Fretwell 113, 10:30 AM
Title: Nice elliptic curves
Abstract: Recall the 6 to 1 ramified Galois cover $\lambda : X(2) \to X(1)$ of modular curves, where λ is the modular lambda function. We will show that if $\lambda^2 - \lambda + 1 \in \mathbb{Q}^2 \setminus \{0, 1\}$, then the associated elliptic curves possess much more explicit arithmetic data. In particular, the torsion subgroup can be explicitly computed, and a lower bound for the ranks of a subclass can be given.

Steven Jin (Maryland)
Schedule: Sunday, Fretwell 128, 10:00 AM

Title: Linnik's large sieve and the L^1 norm of exponential sums
Abstract: The method of proof of Balog and Ruzsa and the large sieve of Linnik are used to investigate the behaviour of the L^1 norm of a wide class of exponential sums over the square-free integers and the primes. Further, a new proof of the lower bound due to Vaughan for the L^1 norm of an exponential sum with the von Mangoldt Λ function over the primes is furnished. Ramanujan's sum arises naturally in the proof, which also employs Linnik's large sieve. This is joint work with Emily Eckles, Brian Tobin, and Andrew Ledoan.

Alex Kontorovich (Rutgers)
Schedule: Saturday, Fretwell 113, 9:00 AM
Title: Sphere Packings and Arithmetic
Abstract: We will discuss recent work on "crystallographic" sphere packings (defined in work with Nakamura), and the subclass of "superintegral" such. (A quintessential example is the classical Apollonian Circle Packing.) These exist in finitely many dimensions, and in fact in finitely many commensurability classes in each dimension. This is a consequence of the Arithmeticity Theorem, that such packings come from arithmetic hyperbolic reflection groups.

Debanjana Kundu (Toronto)
Schedule: Sunday, Fretwell 113, 11:45 AM
Title: Structure of Fine Selmer Groups
Abstract: In this talk I will discuss the relationship between the classical Iwasawa theory and the the theory of elliptic curves. Some of the work I will talk about is joint work with R. Sujatha.

Eun Hye Lee (Stonybrook)
Schedule: Saturday, Fretwell 121, 11:15 AM
Title: On Certain Multiple Dirichlet Series
Abstract: In this talk, I will construct a Multiple Dirichlet series associated to the prehomogeneous vector space of binary cubic forms and prove some of analytic properties. This is joint with Ramin Takloo-Bighash.

Shenhui Liu (Maine)
Schedule: Saturday, Fretwell 121, 3:00 PM
Title: A GL_3 analog of Selberg's result on $S(t)$
Abstract: In 1940's, Selberg studied the average behavior of $S(t) = 1/\pi \arg \zeta(1/2 + it)$. And later Selberg worked on $S(t, \chi)$ for Dirichlet L-functions for primitive characters χ of prime moduli. About fifty years later, Hejhal and Luo studied the spectral analog of $S(t)$ in the context of L-functions for Hecke–Maass cusp forms for $\mathrm{SL}_2(\mathbb{Z})$. In this talk, we will discuss the average behavior of $S(t, F) = 1/\pi \arg L(1/2 + it, F)$ for Hecke–Maass forms F for $\mathrm{SL}_3(\mathbb{Z})$. This is joint work with Sheng-Chi Liu.

Amita Malik (Rutgers)
Schedule: Saturday, Fretwell 121, 3:30 PM
Title: Zeros of derivatives of completed Riemann zeta function
Abstract: In this talk, we discuss the distribution of zeros of derivatives of completed Riemann zeta function. Along the way, we prove a zero density result and an explicit formula. This is joint work with Arindam Roy.

Hayan Nam (Iowa State)
Schedule: Saturday, Fretwell 121, 11:45 AM
Title: Counting numerical semigroups using polytopes
Abstract: A numerical semigroup is an additive monoid that has a finite complement in the set of non-negative integers. For a numerical semigroup S, the genus of S is the number of elements in $\mathbb{N} \setminus S$ and the multiplicity is the smallest nonzero element in S. In 2008, Bras-Amorós conjectured that the number of numerical semigroups with genus g is increasing as g increases. Later, Kaplan posed a conjecture that implies Bras-Amorós conjecture. In the second half, we prove Kaplan's conjecture when the multiplicity is 4 or 6 by counting the number of integer points in a polytope. Moreover, we find a formula for the number of numerical semigroups with multiplicity 4 and genus g.

Siddhi Pathak (Penn State)
Schedule: Sunday, Fretwell 128, 11:00 AM
Title: On the transcendence of certain infinite series
Abstract: In 1737, Euler solved the Basel problem by evaluating the values of the Riemann zeta-function at even positive integers. He showed that $\zeta(2k)$ is a rational multiple of π^{2k}. Since then, there have been several generalizations of Euler's result. One such question is to evaluate and determine the arithmetic nature of the more general series, $\sum_{n=1}^{\infty} A(n)/B(n)$, where A(X) and B(X) are suitable polynomials. Although it is possible to express these sums in terms of the polygamma functions, their arithmetic nature still remains a mystery. In this talk, we will discuss analogs of this problem in two distinct scenarios.

Sarah Peluse (Oxford)
Schedule: Saturday, Fretwell 113, 2:00 PM
Title: Bounds in the polynomial Szemerédi theorem
Abstract: Let $P_1, \ldots, P_m$ be polynomials with integer coefficients and zero constant term. Bergelson and Leibman's polynomial generalization of Szemerédi's theorem states that any subset A of $\{1, ..., N\}$ that contains no nontrivial progressions $x, x + P_1(y), ..., x + P_m(y)$ must satisfy $|A| = o(N)$. In contrast to Szemerédi's theorem, quantitative bounds for Bergelson and Leibman's theorem (i.e., explicit bounds for this o(N) term) are not known except in very few special cases. In this talk, I will discuss recent progress on this problem.

Larry Rolen (Vanderbilt)
Schedule: Saturday, Fretwell 113, 3:00 PM
Title: Mock modular Eisenstein series with Nebentypus
Abstract: By the theory of Eisenstein series, generating functions of various divisor functions arise as modular forms. It is natural to ask whether further divisor functions arise systematically in the theory of mock modular forms. In joint work with Mertens and Ono, we establish, using the method of Zagier and Zwegers on holomorphic projection, that this is indeed the case for certain (twisted) "small divisors" summatory functions, which correspond to the classical theta functions of Shimura. These include generating functions for combinatorial objects such as the Andrews spt-function and the "consecutive parts" partition function. Finally, in analogy with Serre's result that the weight 2 Eisenstein series is a p-adic modular form, we show that these forms possess canonical congruences with modular forms.

Kristen Scheckelhoff (Greensboro)
Schedule: Saturday, Fretwell 113, 10:15 AM
Title: Tessellations of Hyperbolic 3-Space
Abstract: The space of positive definite binary Hermitian forms is a 4-dimensional cone. Homothety equivalent forms in this cone can be identified with hyperbolic 3-space $\mathbb{H}^3$. A generalization of work of Voronoi shows that configurations of minimal vectors of perfect Hermitian forms give rise to tessellations of $\mathbb{H}^3$ by three-dimensional ideal polytopes that depend on a choice of imaginary quadratic number field. In this joint work with Kalani Thalagoda and Dan Yasaki, we study the types of polytopes that arise in this decomposition, and prove that the number of vertices of such a polytope is bounded above by 12.

Kalani Thalagoda (Greensboro)
Schedule: Saturday, Fretwell 113, 10:45 AM
Title: Group presentation of $GL_2(\mathcal{O}_F)$
Abstract: In this talk, we report on work in progress on computing explicit group presentations for $GL_2(\mathcal{O}_F)$, where F is an imaginary quadratic number field. The method uses a 2-dimensional CW-complex discovered by Ash and McConnell called the well-rounded retract. We adapt work of Brown and Braun–Coulangeon–Nebe–Schönnenbeck to this case. This is joint work with Kristen Scheckelhoff and Dan Yasaki.

Zack Tripp (Vanderbilt)
Schedule: Saturday, Fretwell 113, 11:45 AM
Title: Bounds on multiplicities of zeros of a family of zeta functions
Abstract: In "The Pair Correlation of Zeros of the Zeta Function", Montgomery finds the asymptotics of the pair correlation function in order to give a lower bound on the proportion of zeros that are simple (assuming the Riemann Hypothesis). We will discuss some of the necessary tools to extend his proof to pair correlation for zeros of Dedekind zeta functions of abelian extensions, and as in the Riemann zeta case, we can then use this to obtain results on multiplicities of zeros for these zeta functions. However, we also are able to relate the counts of multiplicities to Cohn-Elkies sphere-packing type bounds, allowing us to use semi-definite programming techniques to obtain better results in lower degree extensions than could be found from a direct analysis. In particular, we are able to conclude that more than 45% of the zeros are distinct for Dedekind zeta functions of quadratic number fields. This is based on joint work with M. Alsharif, D. de Laat, M. Gibson, M. Milinovich, L. Rolen, and I. Wagner.

Ian Wagner (Vanderbilt)
Schedule: Saturday, Fretwell 113, 4:30 PM
Title: Partitions and a conjecture of John Thompson
Abstract: For a finite group G, let $K(G)$ denote the field generated over $\mathbb{Q}$ by its character values. For alternating groups, G. R. Robinson and J. G. Thompson determined $K(A_n)$ as an explicit multiquadratic field. Confirming a speculation of Thompson, we show that arbitrary suitable multiquadratic fields are similarly generated by the values of A_n-characters restricted to elements whose orders are only divisible by ramified primes. We also extend this result to suitable linear groups and show that cyclotomic fields and subfields are generated by the values of characters restricted to elements with prime power order. This is a joint work with Madeline Locus Dawsey and Ken Ono.

Tom Wright (Wofford College)
Schedule: Sunday, Fretwell 113, 11:00 AM
Title: A Conditional Density for Carmichael Numbers
Abstract: Currently, there is a large gap between the best known lower bound for the number of Carmichael numbers up to x and the conjectured density of those same numbers. In this talk, we show that the assumption of a nice conjecture about primes in arithmetic progressions would essentially get rid of this gap, improving the lower bound almost all the way up to the conjectured density.

Dan Yasaki (Greensboro)
Schedule: Saturday, Fretwell 113, 11:15 AM
Title: Steinberg Homology and Real Quadratic Fields
Abstract: The Steinberg module is one of the most important representations of the general linear group of degree n. When n = 2, the Steinberg module may be thought of as the module of universal modular symbols. In this joint project with Avner Ash, we investigate this case comparing the homology of congruence subgroups with coefficients in Steinberg modules arising from real quadratic number fields.
