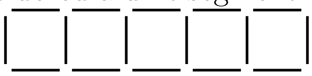
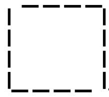
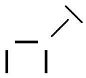
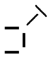
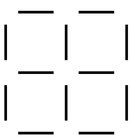
The general topic for this essay is spacial visualization. This set of problems involves the arrangement and rearrangement of toothpicks (aka sticks). In general we try to visualize the final configuration before we start moving sticks around. Each toothpick can be considered a unit segment. Here are a couple of sample problems.[1]

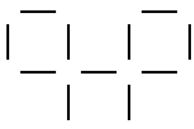
The arrangement  is built from 16 toothpicks. Rearrange 10 of these to build a single square. Here's a solution. .

Here's another example. The five toothpicks are used build a *gnu*:  Move exactly one toothpick to a new location to produce a new gnu.

Solution. Now its pretty clear that we can't move both the neck and the head, so we should try the front or rear legs. The front legs don't work, but look at what we get if we move the rear legs. 

Let's work through one more example before we start on the problems below. Reposition exactly 3 toothpicks to produce a figure with exactly three squares, staring with  .

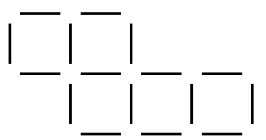
To solve this, realize that you end up with 12 toothpicks that together represent the boundary of three squares. How can this happen if each square has perimeter 4? The answer is that each toothpick is part of the boundary of exactly one square.

Now try to imagine such a figure. 

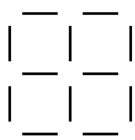
[1]Unauthorized reproduction/photocopying prohibited by law' ©

Have fun with the problems.

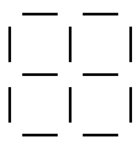
1. Move exactly 2 toothpicks to produce a figure with exactly four unit squares staring with



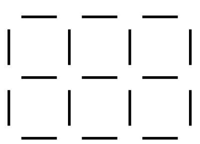
2. Remove exactly 2 toothpicks to produce a figure with exactly two squares,

   staring with  .

3. Remove exactly 4 toothpicks to produce a figure with exactly two squares,

   staring with  .

4. Notice that we can build a 2×3 grid of squares with 17 toothpicks.

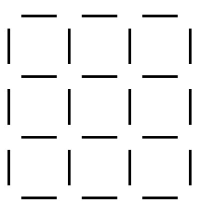


How many toothpicks are required to build a 5×7 grid of squares.

5. How many squares of all sizes are outlined by the toothpicks in the 5×7 grid of squares?

6. In the same way as above, the 2×3 grid has 8 subsquares. Remove exactly four toothpicks to produce a figure with exactly 3 subsquares.

7. Next consider the 3×3 grid shown below.



   (a) Can you remove exactly 8 toothpicks to produce a figure with exactly 2 squares?
   (b) Can you remove exactly 8 toothpicks to produce a figure with exactly 3 squares?
   (c) Can you remove exactly 8 toothpicks to produce a figure with exactly 4 *congruent* squares?
   (d) Remove eight toothpicks leaving five squares.
   (e) Move 12 toothpicks and make two congruent squares.
   (f) Remove four toothpicks leaving one large square and four small squares.
   (g) Remove four toothpicks leaving 5 unit squares.

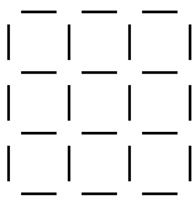
(h) Can you remove exactly 4 toothpicks to produce a figure with exactly 6 squares?

(i) Remove exactly six toothpicks leaving exactly three squares.

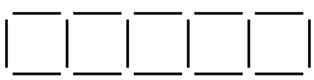
(j) Remove exactly four toothpicks leaving exactly nine squares. These squares need not all be the same size.

(k) Remove six toothpicks leaving two squares and two congruent hexagons.

8. Again consider the 3×3 grid shown.



How many rectangular regions can you find in this grid? Of course you should count squares as special cases of rectangles.

9. The next set of problems all deal with the 1×5 grid: 

(a) Rearrange exactly 8 toothpicks so that the new figure has two congruent squares.

(b) Rearrange exactly 6 toothpicks so that the new figure has exactly three squares.

10. In this problem we are building polygons with given areas using a given number of toothpicks.

(a) Use exactly 14 toothpicks to build a polygon whose area is 12.

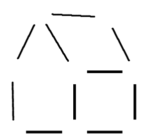
(b) Use exactly 12 toothpicks to build a polygon whose area is 6.

(c) Use exactly 16 toothpicks to build a polygon whose area is 12.

(d) Using exactly 16 toothpicks, how many different areas can you capture with rectangles?

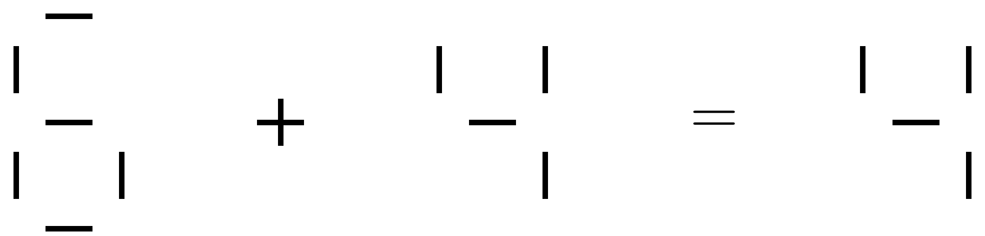
(e) How many rectangles have perimeter 16 and an integral area?

11. The house shown is built with 10 toothpicks. Rotate the house by rearranging just two toothpicks.



12. Show how to make seven equilateral triangles with nine toothpicks of equal length, the sides of the triangles all being the length of one toothpick.

13. Now we're moving a single toothpick to make a correct mathematical statement.



14. Now we're moving single digits to make a correct mathematical statement.
    (a) Move exactly one digit to make

    $$12 + 23 = 25$$

    into a correct statement.
    (b) Try moving one digit to make $102 + 123 = 223$ correct.
    (c) $123 + 312 = 444$.
    (d) $112 - 101 = 113$.

15. Move exactly three toothpicks and the eye to make the fish swim in the other direction.