1. (a) Is the series $\sum_{n=1}^{\infty} \frac{e^{in}}{n^2 + 2}$ convergent or divergent? (Please give explanations for your answer.)

(b) Is the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ convergent or divergent? (Please give explanations for your answer.)

(c) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(1+i)^n} (z-i)^n$;

(d) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^n} (z+1)^n$. 
2. Find the Maclaurin series of the following functions and determine the precise regions of convergence:

(a) \( \frac{1}{1 + z^2} \).

(b) \( \sin^2 z \).

(c) \( \frac{1}{(z + i)^2} \).

(d) \( \int_0^z \cos t^2 dt \).
3. Find all the Taylor and Laurent series of the function \( f(z) = \frac{1}{z+1} \)
(assume the center is 1).

4. Evaluate the following integral (counterclockwise)

\[ \oint_C \frac{e^z + z}{z^3 - z} \, dz; \quad C : |z| = 1/2 \]
5. Evaluate the following integral

\[
\int_0^{2\pi} \frac{1}{2 - \cos \theta} d\theta.
\]

6. Evaluate the improper integral

\[
\int_{-\infty}^{\infty} \frac{\sin 2x}{x^2 + x + 1} dx
\]
7. Find the potential $\Phi(r, \theta)$ in the unit disk $r < 1$, having the given boundary values $\Phi(1, \theta) = 1 + 2 \cos 2 \theta + 3 \cos^2 2 \theta$.

8. Find the electrostatic potential between the two portions of a circular plate $C_1 : |z| = 1$ and $\theta \in (0, \pi/2)$ and (potential $U_1 = 0$ volts) and $C_2 : |z| = 1$ and $\theta \in (\pi/2, 2\pi)$ ($U_2 = 110$ volts).
9. Find the temperature field $T$ in the first quadrant of the $z$-plane if the temperature at the $x$-axis and the $y$-axis is given:

\[
T = \begin{cases} 
10^\circ C, & \text{if } 0 \leq x < 1 \text{ and } y = 0, \\
0, & \text{if } 1 < x \text{ and } y = 0
\end{cases}
\]

and

\[
T = \begin{cases} 
10^\circ C, & \text{if } 0 \leq y < 2 \text{ and } x = 0, \\
0, & \text{if } 2 < y \text{ and } x = 0.
\end{cases}
\]