1. (a) Find Re $\frac{z}{\bar{z}}$

(b) Find Im $\frac{1}{z}$

(c) Find the polar form of $\frac{1-i}{1+i}$

(d) Find Arg(1+i)

(e) Find all roots of $\sqrt[3]{216}$
2. Are the following functions harmonic? If so, find a corresponding conjugate function $u(x, y)$ for each of them so that $f(z) = u(x, y) + iv(x, y)$ is an analytic function.

(a) $v = x^2 - y^2$

(b) $v = \sin x \cosh y$
3. (a) Find and sketch the image of the region: \( \ln 3 < x < \ln 5 \) under the mapping \( w = e^{2z} \).

(b) Find and sketch the image of the region: \( 1 < y < 2 \) under the mapping \( w = \sin z \).
4. (a) Find the linear fractional transformation that maps $-1, 0, 1$ onto $-1, 0, 3$ respectively.

(b) Find a linear fractional transformation that maps $|z| \leq 1$ onto $|w| \leq 1$ such that $z = 1/3$ is mapped onto $w = 0$.

(c) Show that substituting any linear fractional transformation into a linear fractional transformation gives another linear fractional transformation.
5. Integrate

(a) $\int_C \cos^2 z \, dz$, $C$ is the path from $i$ to 1 along the unit circle.

(b) $\int_C z^2 e^{z^3} \, dz$, $C$ is the path from 2 along the axes to $i$.

(c) Show that $\bar{z} = x - iy$ is not analytic and calculate $\int_C \bar{z} \, dz$, $C$ is the unit circle, counterclockwise.
6. Integrate the following $f(z)$ around the contour $C$ in the counterclockwise sense.

(a) $f(z) = \frac{z^3}{3z-1} + \frac{1 + z}{z-3}$, $C: |z| = 1$

(b) $f(z) = \frac{(z+2)^4 + \cos 2z}{(z+1)^2}$, $C$ is the boundary of square with vertices $\pm \frac{3}{2}$, $\pm \frac{3}{2}i$

(c) $f(z) = \frac{\ln(z-2)}{(2z-1)^3}$, $C: |z-i| = 2$