1. For each of the following equations, determine whether the power series method works near \( x = 0 \) and whether the Frobenius method works (Justify your answer).

(a) \( x^2y'' - (x + 6)y' - 3y = 0; \)

(b) \( (x + 1)y'' - (x + 2)y' - (x + 3)y = 0; \)

(c) \( x^2y'' - xy' - (x^2 + 3)y = 0. \)
2. Find one solution of the following equation using the Frobenius method (suppose $x_0 = 0$):

$$xy'' + 2y' + 4xy = 0.$$
3. Show orthogonality on the given interval and determine the corresponding orthonormal set of functions

\[1, \cos 8nx, \sin 8nx, \quad n = 1, 2, \ldots, \quad (0 \leq x \leq \pi/4).\]
4. Find the Laplace transform $\mathcal{L}\{f\}$ of the given functions:

(a) $f(t) = 2t + e^{3t} \cos 4t + e^{5t} \sinh 6t$;

(b) $f(t) = \sin(2t + 3) + e^{2+3t}$;

(c) $f(t) = t^2 u(t - 3) + \delta(t - 6)$. 
5. Find $f(t)$ if $F(s) = \mathcal{L}\{f\}$ equals

(a) $F(s) = \frac{1}{s^2 + 4s + 13} + \frac{1}{s^2}$;

(b) $F(s) = \frac{e^{-s}}{s^2 - 9}$;

(c) $F(s) = \frac{1}{s^2(s - 1)}$. 
6. Solve the given initial value problem

\[ y_1' = 5y_1 + y_2, \quad y_2' = y_1 + 5y_2, \quad y_1(0) = -3, \quad y_2(0) = 7. \]