SUBSET: A Joint Design of Channel Selection and Channel Hopping for Fast Blind Rendezvous in Cognitive Radio Ad Hoc Networks

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***Abstract*—Without a common control channel in cognitive radio ad hoc networks (CRAHNs), two secondary users have to first hop on a common available channel before setting up their communication link. Existing papers on this blind rendezvous process mainly focus on the sequence design of channel hopping but do not consider the selection of available channels. Their time to rendezvous (TTR) and operation complexity increase with the number of available channels, which is against the concept that cognitive radios should perform better when there are more unused channels in primary networks. Thus, a new blind rendezvous design that can address this paradoxical issue is desirable. In this paper, we propose a joint design of channel selection and channel hopping for guaranteed blind rendezvous. For the first time, the TTR is significantly reduced to $O(1)$ with a low operation requirement. An analytical model of TTR is also proposed and validated against the simulation. More importantly, under our proposed protocol, TTR decreases with the increasing number of available channels in the network. This is a very attractive feature in spectrum-under-utilized scenarios which has not been achieved by any existing CRAHN rendezvous work.**

I. Introduction

In cognitive radio ad hoc networks (CRAHNs) [1], secondary users (SUs) are allowed to use channels if they do not cause harmful interference to licensed users, or, primary users (PUs). We name these channels available channels for SUs. Due to PUs' distribution and activities in the network, the available channel set (ACS) of a SU may change with its location and time. Thus, unlike traditional wireless ad hoc networks, it is difficult or impractical to find a channel which is commonly available for all SUs as their control channel. This requires SUs to work in a decentralized way to communicate with each other. Under such scenarios, an important issue is how they can find each other on a common available channel without the knowledge of each other's available channels. This challenging process is called *blind rendezvous*.

Channel-hopping (CH) based technique recently emerges as a promising solution for blind rendezvous which can guarantee the rendezvous between any two SUs as long as they have at least one common available channel. In a typical CH process, time is divided into time slots. All channels in the ACS of a SU are ordered in a predefined sequence. Both source SUs (i.e., SUs willing to rendezvous) and listening SUs (i.e., potential receivers) hop on to their available channels one by one according to their predefined sequences. Based on the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) mechanism in IEEE 802.11, a source SU broadcasts a Request-To-Send (RTS) message in each time slot on each channel it hops to check if its destination SU is on the same channel until a Clear-to-Send (CTS) message is received. We define TTR (time to rendezvous) as the number of channels the source SU has hopped on before a successful handshake of an RTS/CTS exchange. Thus, ETTR (expected TTR) and MTTR (maximum TTR) are regarded as two crucial metrics in CH performance.

The drawbacks of current CH-based rendezvous algorithms are threefold. Firstly, these algorithms raise an awkward s-cenario: their TTR increases with the number of available channels. If the total number of channels in the network is N, the state-of-the-art CH design [2]–[4] can achieve ETTR and MTTR at the order of $O(N)$ and $O(N^2)$, respectively. This contradicts the original intention of cognitive radios which should perform better when there are ample unoccupied channels in primary networks. A method to downsize the large ACS of SUs during the rendezvous process is proposed in [5], but the TTR is still proportional to the new size of ACS. Only in [6], instead of N, the TTR is proportional to the number of SUs, but every SU is preassigned a role (source SU or listening SU) in each slot. In addition, the design proposed in neither [5] nor [6] is rendezvous-guaranteed.

Secondly, the TTR in existing effort is not short enough with respect to networking operations at higher layers. To get an acceptable successful handshake rate, it is proved in [7] that the staying time on each hopping channel (i.e., one time slot) should be $4 \sim 6$ times longer than the $(RTS + CTS)$ transmission duration, $t_{RTS+CTS}$. Consequently, the actual rendezvous time is more than $(4 \times t_{RTS+CTS} \times TTR)$. Even if the fastest CH algorithm is adopted, this long rendezvous time can easily result in network congestion even under moderate traffic conditions [8].

Lastly, in existing CH schemes, the energy consumption of a SU is high since it keeps hopping from one channel to another. Even if a SU does not have packets to send, in order to provide the communication chance for potential source SUs, it

This work was supported in part by the US National Science Foundation (NSF) under Grant No. CNS-0953644, CNS-1218751, and CNS-1343355.

978-1-4673-7331-9/15/$31.00 © 2015 IEEE

still needs to keep hopping. Most existing rendezvous schemes ask every SU to follow the same CH algorithm no matter it is an active source node or a passive listening node. In [5], different CH algorithms are designed for the source SU and the listening SU, but the latter still has to hop on different available channels frequently. From this point of view, there is no "idle" user in a secondary network when using current CH methods.

To eliminate the above CH problems, we consider shortening the TTR from the perspective of constructing desirable ACS for the rendezvous-pair. We propose that *the ACS of a listening SU should be a subset of a source SU's ACS*. We use the following simple CH strategy to explain the salient feature of such rendezvous pair no matter how the subset relationship of their ACS is formed. Assume that the listening SU keeps staying on an arbitrary channel in its ACS. The source SU keeps hopping on to every channel one by one in its ACS. Due to the subset relationship, the source SU can always meet the listening SU on one of its available channels eventually and thus rendezvous is guaranteed. As we can see, even in such a rough scheme, the operation of the listening SU is minimized since it only needs to keep listening on one channel and the order of MTTR is $O(N)$, which is already a breakthrough achievement in CH design.

Motivated by the above feature, we investigate the issue of available channel selection in order to realize such ACS pair. In fact, as explained in [9], the concept of available channels is imprecisely mentioned in many CH-based rendezvous papers as the channels that are not occupied by PUs. In this paper, we regard an available channel as the channel that can be used without generating unacceptable interference to PUs. By this definition, whether a channel is available or not for a SU depends on the SU's *interfering range* and the locations of PUs on the same channel. Since the interfering range is variable under different transmission power of a SU [10] and the locations of PUs can be inferred from the sensing period (see details in Section II), the number of available channels for a SU is controllable using appropriate transmission power.

However, it is still very challenging to form the ACS subset relationship for a pair of rendezvous SUs in practical CRAHNs. For example, one challenge lies in the impact of the one-hop distance. In order to include the listening SU's ACS, the source SU should limit its transmission power low enough to interfere less number of PUs and thus make more channels available. However, this may be a problem for the source SU's transmission to reach its destination SU when their distance is large. This issue is addressed in Section III-C. Other practical challenges are discussed in Section II-B and taken into account in our proposed design.

In this paper, we first analyze the relationships of several important parameters such as the transmission power of a SU, the *interfering range* of a SU transmitter, the distance between the rendezvous pair, and the *interference range* of a SU receiver (a different concept from the interfering range of the source SU. See details in Section II-A). Besides, we also consider practical issues including the sensing resolution of SUs, PU location derivation, role-exchange problem, and the interference among SUs. Then, based on these relationships and practical constraints, we propose a mechanism to construct the ACS of a pair of rendezvous SUs. Moreover, two highly efficient CH algorithms are developed based on such ACS pairs to deal with different one-hop distances. Finally, we propose analytical models and conduct simulations to validate the analytical models and evaluate the performance of the proposed design. The salient features of our proposed protocol, SUBSET, are summarized as follows.

1) In Section IV, we prove that both the ETTR and MTTR of the SUBSET protocol are $O(1)$ with a 100% successful rendezvous rate if the two rendezvous SUs have at least one common available channel. To the best of our knowledge, this is the first protocol that can achieve guaranteed blind rendezvous in such a short time period.
2) An **inherent feature** of our CH algorithms is that no matter what the primary network condition is (even in dense high traffic volume networks) and what device parameters PUs and SUs have, the ETTR of our rendezvous scheme will approach 1 (*monotonically decreasing*) as the number of channels in the network increases. Therefore, SUs using our protocol can enjoy faster rendezvous in spectrum highly underutilized networks.
3) Both analytical and simulation results show that the TTR of our SUBSET protocol is only 1 or 2 time slots in most cases. Such a fast rendezvous can significantly reduce the network congestion in practical scenarios, as compared to existing rendezvous algorithms.
4) Our proposed SUBSET protocol does not require idle SUs to keep hopping from a channel to another. Most of the time, they just stay on one channel. Simulation results show that SUBSET requires fewer operations during a long rendezvous time.

The rest of this paper is organized as follows. The system model and parameter analysis are described in Section II. In Section III, both the channel selection and channel hopping issues in SUBSET are addressed with details. In Section IV, TTR analysis is presented. Simulations results of our protocol are shown in Section V, followed by the conclusions in Section VI.

II. System Model and Parameter Analysis

The system considered in this paper consists of finite number of PUs and SUs which can operate over a set of orthogonal channels denoted by $\boldsymbol{C} = \{c_1, c_2, c_3, ..., c_N\}$. Each time when a PU wants to transmit, a channel c_i is randomly selected for the PU. Both the source SUs and listening SUs utilize SUBSET to form appropriate ACSs and follow CH algorithms in SUBSET. This process should be done periodically to avoid channel status change due to PU activities. Note that time-slotted synchronization is *not a necessary requirement*, because the listening SU in SUBSET stays on one channel most of the time.

A. Important Relationships

We first analyze the relationships that parameters must satisfy in our design. In [11], the relationship between a sender's transmission range and a receiver's interference range is studied in 802.11 ad hoc networks. Similar relationship in cooperative sensing in CRAHNs is used in [12]. In this paper, we derive specific relationships of parameters in rendezvous scenarios in CRAHNs.

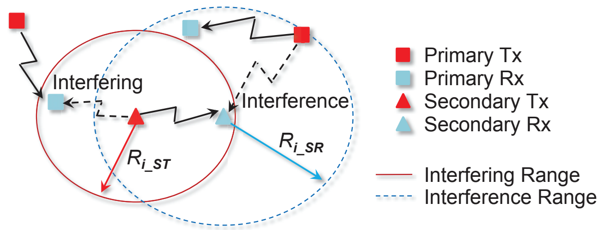


Fig. 1. Two ranges of the rendezvous pair.

As mentioned in Section I, the ACS of a SU depends on its interfering/interference range. In particular, in this paper, we consider secondary transmitter (ST) interfering range and secondary receiver (SR) interference range, as illustrated in Fig. 1.

Interfering Range (R_{i_ST}) is the range centered at a source SU. Any PU receiver (PR) within this range has the possibility of failing to decode a packet sent from its PU transmitter (PT) due to the interference signal from the ST. In other words, for any PR outside this range, its SIR (the ratio of the received signal power from a PT to the received interference power from a ST) should be greater than a certain threshold (Γ_{sir_PU}) which is usually set to be 10dB as in the 802.11b specification. Let P_{r_PT} be the received signal power from the PT and P_{i_ST} be the received interference power from the ST. Then,

$$\frac{P_{r\_PT}}{P_{i\_ST}} \geq \Gamma_{sir\_PU}. \tag{1}$$

According to the path loss model [13] commonly used in wireless networks, when a transmitter propagates a signal to a receiver, the received signal power at the receiver is

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^\alpha},$$

where P_t is the transmission power, G_t and G_r are antenna gains of the transmitter and receiver, respectively, h_t and h_r are the height of the antennas, and d is the transmitter-receiver distance. α is the path loss exponent reflecting the signal attenuation rate which is equal to 4 in the two-ray ground reflection model and is equal to 2 within the Freznel zone.

About fading: since our design (explained in detail in the next section) only requires the existence of different receiving power from different channels, we do not need to establish a very accurate path loss model to estimate the exact location of each PU. Instead, only the closest possible PRs on each channel need to be estimated. Thus, we suppose that each received signal sent by a PU propagates with the largest possible fading, i.e., the received signal power can be represented as

$$P_r = \frac{kP_t}{d^\alpha}, \tag{2}$$

where k is a known constant for the worst possible fading case.

Let P_{t_PT} and P_{t_ST} be the transmission power from the PT and ST, respectively. From (1) and (2), the relationship between $\boldsymbol{P_{t\_ST}}$ and $\boldsymbol{R_{i\_ST}}$ is given by

$$\frac{P_{r\_PT}}{P_{i\_ST}} = \frac{\frac{kP_{t\_PT}}{D^\alpha}}{\frac{kP_{t\_ST}}{R_{i\_ST}^\alpha}} = \frac{P_{t\_PT}}{\boldsymbol{P_{t\_ST}}}\left(\frac{\boldsymbol{R_{i\_ST}}}{D}\right)^\alpha \geq \Gamma_{sir\_PU}, \tag{3}$$

where D is the longest possible PT-PR distance (i.e., the radius of the PU's transmission range).

Interference Range (R_{i_SR}) is the range centered at a SR. Any PT within this range may cause the SR failing to receive a packet correctly due to the PT's radio interference. Let Γ_{sir_SU} be the SIR threshold of a SU, P_{r_ST} and P_{i_PT} be the received signal power from the ST and PT, respectively, and d is the ST-SR distance. The relationship between $\boldsymbol{P_{t\_ST}}$, $\boldsymbol{R_{i\_SR}}$, and $\boldsymbol{d}$ is:

$$\frac{P_{r\_ST}}{P_{i\_PT}} = \frac{\frac{kP_{t\_ST}}{d^\alpha}}{\frac{kP_{t\_PT}}{R_{i\_SR}^\alpha}} = \frac{\boldsymbol{P_{t\_ST}}}{P_{t\_PT}}\left(\frac{\boldsymbol{R_{i\_SR}}}{\boldsymbol{d}}\right)^\alpha \geq \Gamma_{sir\_SU}. \tag{4}$$

B. Practical Constraints

In a CRAHN, the design of blind rendezvous should also consider the following practical issues.

Sensing resolution: Due to the limitation of SU's antenna sensitivity, there is usually a sensing range (R) of a SU within which any PT can trigger SU's carrier sense detection. The detection threshold (P_{d_SU}) is thus given by

$$P_{d\_SU} = \frac{kP_{t\_PT}}{R^\alpha}. \tag{5}$$

Since the activities of PUs outside this sensing range cannot be detected by a SU, $R_{i\_ST} \leq R$ is required to avoid interfering possible PUs outside the sensing range. On the other hand, for a successful rendezvous, $P_{r\_ST} \geq P_{d\_SU}$ is necessary, which indicates

$$P_{t\_ST} \geq P_{t\_PT}\left(\frac{d}{R}\right)^\alpha. \tag{6}$$

PR location derivation: When a source SU determines its interfering range R_{i_ST} on a channel, R_{i_ST} should exclude the closest possible PR in order not to cause interference to any nearby PUs.

However, a PR cannot be sensed immediately by the ST since it does not generate any power while receiving, which is the hidden primary receiver problem. Nevertheless, the ST can still recognize a PR indirectly. For example, if the primary network is in the half-duplex mode, a PU will send an acknowledge (ACK) back to its communication user after each reception. Hence, a hidden PR's location can be derived from a long-term sensing. In CRNs, the long-term sensing can be substituted by mining a SU's own sensing history or cooperative sensing [12], [14]. If the primary network is in the full-duplex mode, the locations of the communication PU pair can also be inferred from their combined signals [15], [16]. In this paper, since we only aim to estimate the location of the closest possible PR to a ST, a simpler yet effective method is proposed.

Let the **aggregated** received power at the ST from all PUs on c_i be $P_{r\_c_i}$ for the full-duplex mode. In the half-duplex mode, the maximum $P_{r\_c_i}$ from the ST's recent sensing history is chosen. For easy explanation, we use $P_{r\_c_i}$ uniformly for both modes. Then,

$$P_{r\_c_i} = \sum_{j=1}^{n} \frac{kP_{t\_PT}}{D_j^{\alpha}}, \quad (7)$$

where n is the number of PUs on c_i and D_j is their distance to the ST. The interfering range on this channel, $R_{i\_ST}^{c_i}$, should satisfy

$$R_{i\_ST}^{c_i} = Minimum\{D_1, D_2, ..., D_n\}, \quad n \geq 1.$$

Thus,

$$P_{r\_c_i} = \sum_{j=1}^{n} \frac{kP_{t\_PT}}{D_j^{\alpha}} \leq \frac{nkP_{t\_PT}}{(R_{i\_ST}^{c_i})^{\alpha}}$$

$$\implies (R_{i\_ST}^{c_i})^{\alpha} \leq \frac{nkP_{t\_PT}}{P_{r\_c_i}}.$$

Consider the closest PR case: when $n = 1$, the relationship between $R_{i\_ST}^{c_i}$ and $P_{r\_c_i}$ is:

$$R_{i\_ST}^{c_i} = \left(\frac{kP_{t\_PT}}{P_{r\_c_i}}\right)^{\frac{1}{\alpha}}. \quad (8)$$

Role-exchange problem: After the listening SU receives a RTS or a data packet on the rendezvous channel, a CTS or ACK is expected to send. Consequently, the listening SU also needs to consider the transmission interfering issue.

Let P'_{t_ST} be the transmission power of the listening SU who just received a correct RTS or data packet. Now, the interference range of this SU becomes its interfering range since the closest PU may be just outside this range. To avoid interfering this potential PU when sending CTS/ACK, by (3),

$$\frac{P_{t\_PT}}{P'_{t\_ST}} \left(\frac{R_{i\_SR}}{D}\right)^{\alpha} \geq \Gamma_{sir\_PU}. \quad (9)$$

Interference among SUs: The interference among SUs can be avoided by limiting SU's transmission power. For example, if a SU's maximum transmission power is higher than a PU's transmission power (i.e., $P_{t\_ST}^{max} > P_{t\_PT}$), replace P_{t_PT} in (4) with $P_{t\_ST}^{max}$ and the interference range may not be able to prevent the interference from an outside ST. Hence, we should set $P_{t\_ST} \leq P_{t\_PT}$.

On SR's side, if its minimum received signal power is lower than that of a PU (i.e., $P_{r\_ST}^{min} < P_{r\_PT}$), replace P_{r_PT} in (3) with $P_{r\_ST}^{min}$ and the interfering range may still affect an outside SR's reception. Therefore, $\frac{kP_{t\_PT}}{D^{\alpha}} \leq \frac{kP_{t\_ST}}{d^{\alpha}}$ is required.

Combining the above two requirements together, the transmission power of a SU should have the following constraint to avoid interference among SUs themselves,

$$\left(\frac{d}{D}\right)^{\alpha} \leq \frac{P_{t\_ST}}{P_{t\_PT}} \leq 1, \quad (10)$$

which is a practical setting in CRAHNs.

III. Proposed SUBSET Design

In this section, we establish the design of SUBSET from channel selection to channel hopping. The main goal of SUBSET is to achieve blind rendezvous in CRAHNs as quickly as possible.

Motivation: A short TTR of current CH algorithms is based on two requirements: more common available channels and less uncommon available channels in the ACSs of the rendezvous pair. Motivated by this observation, a fundamental step in our design is to build desirable ACSs for the CH algorithm, i.e., *the ACS of a listening SU is the largest possible subset of the ACS of a source SU*.

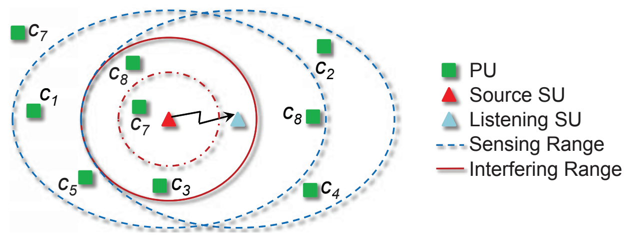


Fig. 2. An illustration of desirable ACSs for the rendezvous pair.

Example: We illustrate the subset relationship in Fig. 2. In existing CH-based rendezvous papers, both the source SU and the listening SU only use the channels not occupied by PUs for constructing the CH sequence. In practice, these are the channels not being used in their sensing range. Thus, the ACSs of the source SU and the listening SU in Fig. 2 are $\{c_2, c_4, c_6\}$ and $\{c_1, c_5, c_6\}$, respectively. They only have one common available channel c_6 and four uncommon available channels, c_2, c_4, c_1, and c_5.

However, in SUBSET, whether a channel is available depends on the interfering/interference range. In Fig. 2, assume that the interference range is equal to the SU's sensing area. Then, the ACS of the listening SU is still $\{c_1, c_5, c_6\}$. In order to have this set be the source SU's largest possible subset, the interfering range of the source SU should be the red solid circle in Fig. 2. Now, the ACS of the source SU is $\{c_1, c_2, c_4, c_5, c_6\}$, which makes all channels in the ACS of the listening SU, c_1, c_5, and c_6, common available channels. At the same time, this set has the least number of uncommon available channels, c_2 and c_4. A smaller interfering range such as the dash-dot circle may harm the rendezvous by increasing the number of uncommon available channels and decreasing the reception power at the listening SU.

A. ACS Construction

Next, we show how to construct the desirable ACS with the subset relationship step by step.

1) ACS for the listening SU: For a listening SU, in order to generate minimum interference to all potential source SUs' signal, from (4), we prefer a listening SU to stay on the channel with the largest interference range ($R_{i\_SR}^{max}$). Based on the analysis in the role-exchange problem in Section II, after a packet is correctly received, the interference range will become the listening SU's interfering range which is R. Thus, we have $R_{i\_SR}^{max} = R$. Those sensed-idle channels thus can be chosen into its ACS:

$$ACS_{listening} = \{c_i \mid P_{r\_c_i} \leq P_{d\_SU}, i = 1, 2, ..., N\}. \quad (11)$$

2) Transmission power of the listening SU: A listening SU sends a CTS with the maximum transmission power it can use, because the location of the source SU is unknown. Since the interfering range is R, based on (3), this transmission power (P'_{t_ST}) should be:

$$P'_{t\_ST} = \frac{P_{t\_PT}}{\Gamma_{sir\_PU}} \left(\frac{R}{D}\right)^{\alpha}, \quad (12)$$

which is the upper bound of the constraint (9).

3) ACS for the source SU: To ensure that the ACS of the listening SU is the largest possible subset of the ACS of the source SU, the source SU's minimum interfering range of the selected channel should be included in the interference range of the listening SU as shown in Fig. 2.

Suppose that the distance between a source SU and its destination SU is $\boldsymbol{d}$. Then, the source SU's maximum interfering range Γ should satisfy

$$\Gamma + \boldsymbol{d} = R. \quad (13)$$

Any idle channel or channel occupied by PUs located farther than this range Γ can be selected to the source SU's ACS. From the relationship in (8), this means that any channel with received power lower than P_γ can be selected, where

$$P_\gamma = \frac{kP_{t\_PT}}{(\Gamma)^{\alpha}}. \quad (14)$$

Then, the ACS for the source SU can be formed by:

$$ACS_{source} = \{c_i \mid P_{r\_ci} \leq P_\gamma, i = 1, 2, ..., N\}. \quad (15)$$

4) Transmission power of the source SU: For those channels with $R_{i\_ST}^{c_i} \geq \Gamma$, from (3), the maximum transmission power of the source SU on such channels should be:

$$P_{t\_ST}^{c_i} = \frac{P_{t\_PT}}{\Gamma_{sir\_PU}} \left(\frac{R_{i\_ST}^{c_i}}{D}\right)^{\alpha}. \quad (16)$$

However, since there is an interfering range limit to form the subset relationship (13), the source SU should only use the limit transmission power:

$$P_{t\_ST} = \frac{P_{t\_PT}}{\Gamma_{sir\_PU}} \left(\frac{\Gamma}{D}\right)^{\alpha}. \quad (17)$$

5) Derivation of the maximum d: So far, for a given $\boldsymbol{d}$, the source SU can form a desirable ACS from (15). However, when $\boldsymbol{d}$ becomes large, the transmission power from (17) may not be enough for the distant listening SU to decode. To guarantee the rendezvous, we require an upper bound of $\boldsymbol{d}$ that can satisfy the relationship for decoding in (4) as well as other necessary constraints in Section II. We list them as follows:

$$\begin{cases} P_{t\_ST} = \frac{P_{t\_PT}}{\Gamma_{sir\_PU}} \left(\frac{R - \boldsymbol{d}}{D}\right)^{\alpha} \\ \frac{P_{t\_ST}^{min}}{\boldsymbol{d}^{\alpha}} \geq \Gamma_{sir\_SU} \frac{P_{t\_PT}}{R^{\alpha}} \\ P_{t\_ST}^{min} \geq P_{t\_PT} \left(\frac{\boldsymbol{d}}{D}\right)^{\alpha} \end{cases}. \quad (18)$$

Finally, we derive the upper bound of $\boldsymbol{d}$ as

$$d_r = \frac{R^2}{R + D(\Gamma_{sir\_SU}\Gamma_{sir\_PU})^{\frac{1}{\alpha}}} \quad (19)$$

with the condition

$$\frac{\Gamma_{sir\_SU}}{R^{\alpha}} \geq \frac{1}{D^{\alpha}}. \quad (20)$$

Or

$$d_r = \frac{R}{1 + (\Gamma_{sir\_PU})^{\frac{1}{\alpha}}} \quad (21)$$

with the condition

$$\frac{\Gamma_{sir\_SU}}{R^{\alpha}} \leq \frac{1}{D^{\alpha}}. \quad (22)$$

We denote this upper bound as the rendezvous range (d_r) for SUBSET. Since the source SU does not know how far away the destination SU is during rendezvous, it should limit its interfering range for the worst case $d = d_r$ in order to form the subset relationship with any listening SU that is located within d_r distance. Thus,

$$\Gamma = R - d_r; \quad (23)$$

Since homogeneous antennas are considered in this paper, we assume that the sensing range (R) of SUs and PUs is the same and their SIR thresholds are also the same, i.e., $\Gamma_{sir\_PU} = \Gamma_{sir\_SU} = 10$. In 802.11 design, $R > D$ is required [17], [18]. Particularly, we adopt the default setting $R = 2.2D$ in ns-2 [19]. Thus, the left-hand side of both (20) and (22) is $\frac{10}{(2.2D)^{\alpha}}$. If $\alpha = 2$, (20) holds and $d_r = 0.4D$. Similarly, if $\alpha = 4$, (22) holds and $d_r = 0.8D$.

B. CH Algorithm

After forming such an ACS pair, we propose a specific CH algorithm which can further reduce the TTR. Algorithm 1 and Algorithm 2 give the pseudo code for the rendezvous pair. For a source SU, it orders the channels in its ACS by their indexes (low to high) and hops on to them one by one in a cyclic way until rendezvous. For a receiver, it also arranges its ACS by the indexes of channels (low to high) and keeps staying on the first ordered channel until a correct RTS is received. If the first ordered channel becomes busy (i.e., a power above the detection threshold is sensed), it hops on to its next ordered channel and stays there.

Algorithm 1: The CH algorithm for the source SU

Input: ACS and P_{t_ST};
1: Order channels by their index: $\{c_{k1}, c_{k2}, ..., c_{km} | k1 < k2 < ... < km\}$;
2: $i = 1$;
3: Hop on to c_{ki} and send RTS with P_{t_ST};
4: **while** *not rendezvous* **do**
  $i = i + 1$;
  $j = ((i - 1) \bmod m) + 1$; /* next ordered channel */
  Hop on to c_{kj} and send RTS with P_{t_ST};

Based on our CH design, the listening SU stays on the first ordered channel most of the time, which saves the energy consumption of SUs.

Algorithm 2: The CH algorithm for the listening SU

Input: ACS and P'_{t_ST};
1: Order channels by their index: $\{c_{k1}, c_{k2}, ..., c_{km} | k1 < k2 < ... < km\}$;
2: $i = 1$; $j = i$;
3: Stay on c_{ki};
4: **while** *no correct RTS is received* **do**
  if *not idle* **then**
    $i = i + 1$;
    $j = ((i - 1) \bmod m) + 1$;
    Stay on c_{kj};
5: Send CTS on c_{kj} with P'_{t_ST};

C. Long-Distance SUBSET

Motivation: We consider a special issue namely *long-distance rendezvous*. In this scenario, the transmission range of a SU (D_{ST}) is longer than the required rendezvous range in SUBSET. Then, the distance (d) between the rendezvous pair may exceed the rendezvous guaranteed range, $d_r < d < D_{ST}$. Note that to have a successful transmission, a minimum interfering range Γ is generated given a transmission distance d. From (4) and (5), $R_{i\_ST} \geq \frac{d}{R} \sqrt[\alpha]{\Gamma_{sir\_SU} \Gamma_{sir\_PU}} D$, which implies $\Gamma = 4.54d$ when $\alpha = 2$ or $1.44d$ when $\alpha = 4$ in our setting. Therefore, when $d > 0.4D$, $d + \Gamma > R$. The same relationship holds when $d > 0.8D$. Thus, our subset relationship no longer exists.

Example: As illustrated in Fig. 3, if the rendezvous pair still follows Algorithm 1 and 2, the rendezvous is unsuccessful: the listening SU keeps staying on c_1 and the source SU keeps hopping on c_2 and c_3.

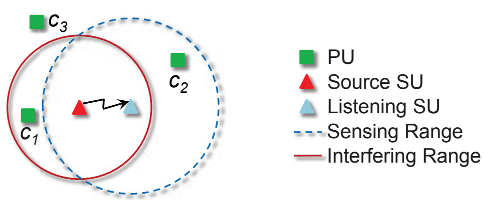


Fig. 3. An illustration of long-distance rendezvous.

Method: To solve this problem, we propose to modify Algorithm 1 for the source SU. Instead of only hopping on to the selected channels with fixed transmission power, the source SU sends an RTS with the maximum allowable transmission power on each channel in the network. Thus, when it hops on to the channel where the listening SU stays, the transmission power required on this channel may not be enough for the listening SU to decode the RTS signal. However, the received power may be more than the detection threshold and trigger the listening SU'carrier sense. Then, according to Algorithm 2, the listening SU will choose the next ordered channel in its ACS to stay. As the process continues, the listening SU has to keep changing channels until the channel it stays on can receive the correct RTS. In this way, the rendezvous can be guaranteed as long as the rendezvous pair has at least one common available channel. For example, in Fig. 3, by running the new algorithm, the source SU starts to hop from c_1. The listening SU detects that c_1 is not idle and chooses c_3 to stay. Finally, they will rendezvous on c_3.

New rendezvous range: We derive the new rendezvous guaranteed range d'_r. The worst case is that a PR is located at the edge of the listening SU's sensing range and meanwhile, it is the closest PU to the source SU. Under this circumstance, the minimum allowable interfering range of the source SU, $R_{i\_ST} = R - d$. Then, the corresponding transmission power should satisfy (3):

$$P_{t\_ST} \leq \frac{P_{t\_PT}}{10} \left(\frac{R-d}{D} \right)^{\alpha}. \tag{24}$$

On the other hand, the maximum allowable transmission power should be able to trigger the listening SU's carrier sense. From (6) and (24),

$$\frac{P_{t\_PT}}{10} \left(\frac{R-d}{D} \right)^{\alpha} \geq P_{t\_PT} \left(\frac{d}{R} \right)^{\alpha}. \tag{25}$$

Using the same setting $R = 2.2D$ in (25),

$$d \leq \frac{4.84D}{2.2 + 10^{\frac{1}{\alpha}}}.$$

$$\implies d'_r = \begin{cases} 0.9D, & \alpha = 2 \\ 1.2D, & \alpha = 4 \end{cases}. \tag{26}$$

Rendezvous guaranteed: Note that when $d \geq 0.48D$ in the $\alpha = 2$ scenario, the interfering range exceeds the sensing range, $\Gamma = 4.54d \geq 4.54 \times 0.48D = R$. Hence the transmission range cannot be set over $0.48D$ when $\alpha = 2$. On the other hand, $d < D$ is required in (10). Therefore, the new design can satisfy long-distance rendezvous under both $\alpha = 2$ and $\alpha = 4$.

D. Protocol Details

Algorithm 3: The SUBSET protocol for SU

Input: k, P_{t_PT}, D, R, D_{ST} and Γ_{sir};
1: **if** (*condition (20) holds*) Calculate d_r using (19);
2: **if** (*condition (22) holds*) Calculate d_r using (21);
3: Sense all channels and obtain P_{r_ci} $(i = 1, 2, ..., N)$;
4: **if** *source SU* **then**
  Calculate P_{t_ST} using (17) and (23);
  Calculate P_{γ} using (14) and (23);
  if $D_{ST} \leq d_r$ **then**
    Calculate ACS using (15);
    Run Algorithm 1;
  if $D_{ST} \geq d_r$ **then** /* long-distance */
    $ACS = \{c_i, i = 1, 2, ..., N\}$; $i = 1$;
    Calculate $P^{ci}_{t\_ST}$ using (8) and (16);
    Hop on to c_i and send RTS with $P^{ci}_{t\_ST}$;
    while *not rendezvous* **do**
      $i = i + 1$; $j = ((i - 1) \bmod m) + 1$;
      Hop on to c_j and send RTS with $P^{cj}_{t\_ST}$;
5: **if** *listening SU* **then**
  Calculate P_{d_SU} using (5);
  Calculate ACS using (11) and P'_{t_ST} using (12);
  Run Algorithm 2;

Algorithm 3 gives the entire protocol showing our joint design of channel selection and channel hopping. As we can see, our proposed SUBSET is very easy for implementation, yet very efficient (based on the analysis in Section IV).

IV. TTR Analysis

In this section, we first propose two analytical models for calculating ETTR and MTTR of our CH algorithm with ACSs of ***any*** subset relationship. Then, we derive the ETTR and MTTR of our SUBSET protocol using these models.

A. Analytical Models

Let n be the number of channels in the source SU's ACS and m be the listening SU's. Since they share the subset relationship, $n \geq m$. Fig. 4 illustrates a possible distribution of the paired ACSs. Assume that channels are already ordered by their indexes from low to high. The ACS of the listening SU (ACS_2) is a subset of m channels randomly chosen from the ACS of the source SU (ACS_1).

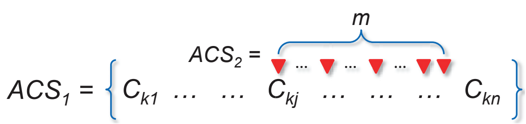


Fig. 4. An illustration of subset distribution.

MTTR: The largest possible indexed first-channel in ACS_2 is the *(n-m+1)th* channel in ACS_1, i.e., the m channels in ACS_2 are exactly the last m channels in ACS_1. Therefore, in SUBSET,

$$MTTR(n, m) = n - m + 1 \tag{27}$$

which is *only related to* n *and* m.

ETTR: The total number of the distribution cases of ACS_1 and ACS_2 is $\binom{n}{m}$. If the source SU rendezvous on the *jth* channel in its ACS, then $TTR = j$. In this case, the first channel in ACS_2 must be the same channel and other channels can be chosen from the channels after it. As shown in Fig. 4, the number of possible cases becomes to $\binom{n-j}{m-1}$. Let P_j be the probability of $TTR = j$. Then, $P_j = \binom{n-j}{m-1}/\binom{n}{m}$. The ETTR in SUBSET can be derived by

$$ETTR = \frac{\binom{n-1}{m-1} + 2\binom{n-2}{m-1} + \cdots + (n-m+1)\binom{m-1}{m-1}}{\binom{n}{m}}. \tag{28}$$

We first analyze the expression

$$f(k) = \binom{n-k}{m-1} + \cdots + \binom{m}{m-1} + \binom{m-1}{m-1}.$$

Using $\binom{m-1}{m-1} = \binom{m}{m}$ and the combinatorial law $\binom{p}{q} = \binom{p-1}{q-1} + \binom{p-1}{q}$ to the last two items, $\binom{m}{m-1} + \binom{m-1}{m-1} = \binom{m}{m-1} + \binom{m}{m} = \binom{m+1}{m}$. Then, keep applying the law to the last two items of the new formed equation iteratively, $f(k) = \binom{n-k}{m-1} + \cdots + \binom{m+1}{m-1} + \binom{m+1}{m} = \binom{n-k}{m-1} + \cdots + \binom{m+2}{m} = \cdots = \binom{n-k+1}{m}$. Thus, the numerator of (28) (NUM) can be rewritten as

$$\begin{aligned} NUM &= f(1) + f(2) + \cdots + f(n-m+1) \\ &= \binom{n}{m} + \binom{n-1}{m} + \cdots + \binom{m+1}{m} + \binom{m}{m}, \end{aligned}$$

which has the same pattern as $f(k)$. Using the same method, we can derive $NUM = \binom{n+1}{m+1}$. Therefore,

$$ETTR(n, m) = \binom{n+1}{m+1} / \binom{n}{m} = \frac{n+1}{m+1}, \tag{29}$$

which also *only depends on two variables* n *and* m.

B. Normal SUBSET

To analyze the performance of SUBSET, we only need to know the average n and m in a network. Assume that PUs are evenly distributed. Denote K as the number of PUs in a unit area. If the active rate of a PU is ρ, then the average number of channels occupied by PUs in a listening SU's sensing range is $K\rho\pi R^2$. Then, the average number of available channels for the listening SU is $m = N - K\rho\pi R^2$. Using the same way, we can derive that $n = N - K\rho\pi\Gamma^2$. Therefore, the estimations of ETTR and MTTR in the normal case are:

$$\begin{cases} ETTR = \dfrac{N+1-K\rho\pi(R-d_r)^2}{N+1-K\rho\pi R^2} \\ MTTR = K\rho\pi d_r(2R-d_r)+1 \end{cases}, \tag{30}$$

both of which are $O(1)$.

Note that $\lim_{N\to\infty} ETTR = 1$, which means that ETTR approaches 1 when there are more channels in the network *no matter what the primary network condition is and what the PU and SU devices are* (K, ρ, R, and d_r).

C. Long-Distance SUBSET

In the long-distance design, the listening SU will eventually stay on a common available channel of the source SU and the listening SU. Therefore, the TTR is the index number of their first common available channel since the source SU hops on every channel one by one. Thus, the problem is similar to the normal case shown in Fig. 4, where ACS_2 is the common available channel set and ACS_1 is the set of all channels.

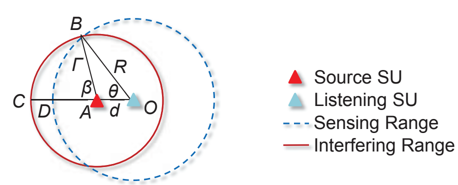


Fig. 5. Changing interfering range with a known d.

In this way, we have $n = N$. Let m' be the number of channels in the common available channel set. Note that when $d \leq d_r$, $\Gamma + d \leq R$, which means that the rendezvous pair still shares the subset relationship and thus $m' = m1 = N - \pi R^2$. When $d > d_r$, they do not own the subset relationship and $m' = m2 = N - K\rho S_U$, where S_U represents the size of the union area of the rendezvous pair. Using Fig. 5, we derive S_U as a function of d.

Derivation of S_U***:*** To derive S_U, we first need to know the size of the intersection area S_I in Fig. 5. By observation, S_I is equal to the interfering area of the source SU minus the crescent area S_C: $S_I = \pi\Gamma^2 - S_C$, where $S_C = 2(S_{BAC} - S_{BAD})$. Meanwhile, we know $S_{BAD} = S_{BOD} - S_{BOA}$. Since R, d, and Γ are known ($\Gamma = 1.44d$), we derive $\theta = cos^{-1}\frac{R^2+d^2-\Gamma^2}{2Rd}$, $\beta = \pi - cos^{-1}\frac{\Gamma^2+d^2-R^2}{2\Gamma d}$, and $S_{BOA} = \frac{1}{2}Rd\sin\theta$. Then $S_{BOD} = \pi R^2\frac{\theta}{2\pi}$ and $S_{BAC} = \pi\Gamma^2\frac{\beta}{2\pi}$. Then, $S_U = \pi\Gamma^2 + \pi R^2 - S_I$.

After calculating m', the ETTR and MTTR for the long-distance SUBSET are

$$\begin{cases} ETTR_l = \int_0^{d_r} \frac{N+1}{m1+1} + \int_{d_r}^{D_{ST}} \frac{N+1}{m2+1} \\ MTTR_l = MTTR(N, m') = N - m' + 1 \end{cases}. \quad (31)$$

To get the maximum value of $MTTR_l$, m' should be as small as possible. The extreme value can be derived when the union area is the largest, i.e., when $d = D_{ST}$. Due to the space limit, the final expression is not shown here. Notice that both $ETTR_l$ and $MTTR_l$ in (31) have the same forms as in the normal case. Therefore, *the ETTR and MTTR in long-distance SUBSET are also* $O(1)$.

V. Performance Evaluation

In our simulation, 1) PUs and SUs are evenly distributed in the simulation area; 2) Each PU is randomly assigned a channel when a new packet needs to be transmitted; 3) Packet arrivals follow the Poisson distribution; and 4) Each SU randomly chooses a SU within its transmission range as its destination SU when it has a new packet to transmit and becomes a source SU. The parameters used in our simulation are listed in Table I.

TABLE I
Simulation Parameters

The antenna related constant	−25.54 dB
The minimum required SIR for PU/SU	10 dB
The path-loss factor α	2
The transmission power of a PU	2w
Side length of the simulation area L	500 m
Channel data rate	2 Mbps
PU/SU packet size	50 slots
The size of (RTS+CTS)	160 + 112 bits (802.11 b/g)
Simulation time	10000 slots

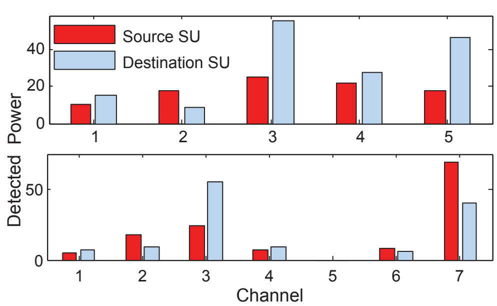


Fig. 6. Detected power on each channel.

Fig. 6 illustrates the spectrum condition detected by a potential rendezvous pair ($d = 80$m) in a moment during simulation. If there are only five channels in a primary network, as shown in the top figure, under the SUBSET protocol, the listening SU will stay on channel 2 and the hopping sequence for the source SU is $\{c_1, c_2, c_5, c_4, c_3\}$. Then, TTR is 2. When the number of channels increases, the number of common available channels also increases, which expedite the rendezvous process and both SUs hop on channel 5 with 1 time slot.

A. Analysis Validation

Fig. 7 shows the analytical and simulation results of the ETTR under different number of channels. The analytical results are calculated using our analytical model proposed in Section IV where $K = 1$, $\rho = 0.375$.

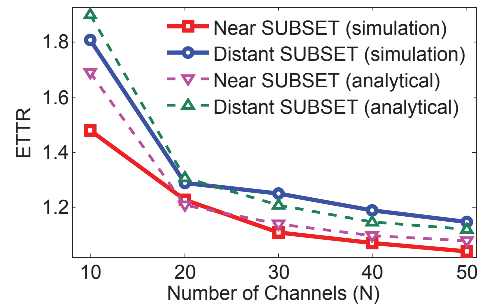
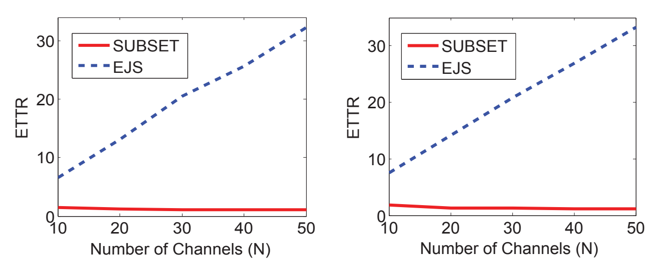


Fig. 7. ETTR vs. N.

From Fig. 7 we summarize: *i)* The ETTR of both SUBSET designs approaches 1 as the number of channels in the network increases. This feature truly accords with the goals of cognitive radios to perform better in spectrum-under-utilized scenarios; *ii)* The difference between the simulation and analytical results is 4.3% with 0.07 standard derivation, which validates our analytical models; *iii)* Near-SUBSET performs better than the long-distance SUBSET because the source SU in the design has to hop on every channel before rendezvous; and *iv)* The average number of unavailable channels is about $K\rho\pi R^2 = 5.7$. When $N = 10$, the network is in spectrum scarcity. Even under this scenario, our proposed SUBSET protocols can still achieve rendezvous within 2 slots.

B. ETTR and MTTR

The ETTR and MTTR of both the near- and distant-SUBSETs are compared with the state-of-the-art CH protocol Enhanced Jump-Stay (EJS) [3]. All these protocols can achieve a 100% successful rendezvous rate. The near-SUBSET protocol is used when ($D_{ST} \le d_r$) and distant-SUBSET is applied when ($D_{ST} > d_r$).

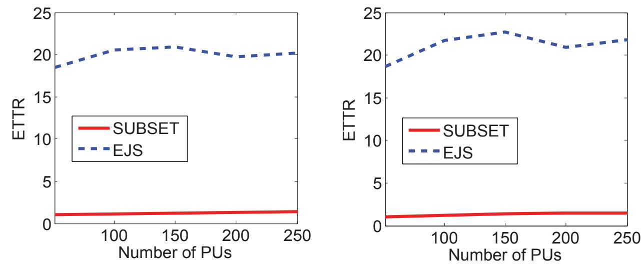


(a) $D_{ST} \le 0.8D$. (b) $D_{ST} \le 1.2D$.

Fig. 8. ETTR vs. N in different protocols ($K = 1$ and $\rho = 0.37$)

From Fig 8, when the number of channels in the network increases, the ETTR of EJS increases with $O(N)$, while our SUBSET protocol maintains the same performance due to our $O(1)$ design.

From Fig 9, when the number of PUs increases, the ETTR performance of all the protocols does not change much. However, the ETTR of SUBSET is still less than 2 time slots even in high-density high-traffic-volume primary networks.



(a) $D_{ST} \leq 0.8D$. (b) $D_{ST} \leq 1.2D$.

Fig. 9. ETTR vs. K in different protocols ($N = 30$ and $\rho = 0.52$)

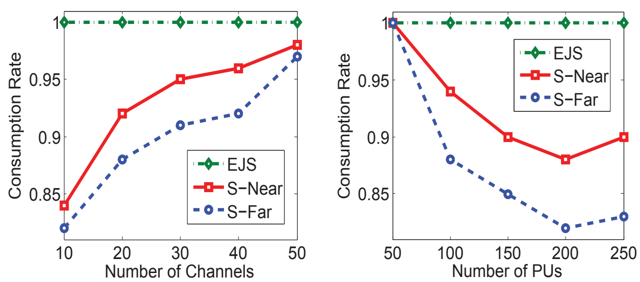
The performance of MTTR is shown in Table II. The results well reflect our $O(1)$ advantage of MTTR, which is significantly lower in SUBSET.

TABLE II
MTTR vs. Number of Channels

Number of channels		10	20	30	40	50
Near Rendezvous	SUBSET	4	4	3	3	2
	EJS	275	296	377	361	463
Distant Rendezvous	SUBSET	9	4	4	3	2
	EJS	269	283	284	379	468

A high MTTR can easily cause network congestion. In a recent study [8], a rendezvous threshold is derived for avoiding network congestion under similar parameters. This threshold is around 10. The MTTR of SUBSET shown in the table is lower than this threshold, which indicates that SUBSET can support a congestion free network.

C. Energy Consumption of Idle SUs



(a) C vs. N ($K = 1$). (b) C vs. K ($N = 30$).

Fig. 10. Idle SU consumption rate under different conditions in CRAHNs.

We define a metric C to evaluate the energy consumption of an idle SU in CRAHNs. Let $n(l)$ be the number of channels a listening SU hopped in a CH process. Then, $C = \frac{n(l)}{TTR}$ which represents the consumption rate of a listening SU during the rendezvous time. Fig. 10(a) shows the impact of spectrum scarcity on C and Fig. 10(b) illustrates C in different PU distributions. It is obvious that SUs with SUBSET can enjoy a longer battery life due to less activities during rendezvous, especially when the rendezvous may take a longer time (SUBSET-far), a worse spectrum condition (smaller N as in (a)), or a worse network condition (higher K as in (b)).

VI. Conclusion

In this paper, an efficient and adaptive protocol, SUBSET, is proposed by joint design of channel selection and channel hopping. We also proposed analytical models for calculating ETTR and MTTR of our CH algorithm with ACSs of any subset relationship. From both analytical and simulation results, SUBSET can achieve fast blind rendezvous with $O(1)$ ETTR and MTTR. To the best of our knowledge, this is the first work that can reduce TTR to $O(1)$ in guaranteed blind rendezvous design. In addition, under SUBSET, for the first time, TTR decreases with the increasing number of available channels in the network. Simulation results also show that SUBSET can achieve fast rendezvous in different spectrum and network environments with less operation requirements.

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