# A Self-Adaptive Optimal Fragmentation Protocol for Multi-Channel Cognitive Radio Ad Hoc Networks 

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#### Abstract

In multi-channel cognitive radio ad hoc networks (CRAHNs), packet fragmentation is impacted by new factors besides those in traditional wireless networks due to the unique CR functions. For example, spectrum handoff is the technique for a secondary user ( SU ) to continue its transmission when a primary user ( $\mathbf{P U}$ ) reoccupies its current transmitting channel. Then, a short frame is less likely to be affected by PU activities, which leads to a lower probability of retransmission. However, with the same header size, a long frame can convey more data than a short frame. In addition, the optimal fragmentation in terms of maximizing the throughput is also related to the spectrum handoff delay, node mobility, and the original packet size of the SU. More importantly, all these factors may vary with time and location, which makes this issue extremely challenging. In this paper, by mathematically modeling these impacts and dynamically mining the related parameters, we propose a selfadaptive protocol guiding the SU to derive the up-to-date optimal packet fragmentation. The proposed protocol is based on practical assumptions and taking other necessary CR functions into account such as spectrum sensing, channel hopping, and spectrum handoff. Simulation results validate our probabilistic model and the optimality of the fragmentation we derived. To the best of our knowledge, this is the first practical fragmentation protocol for multi-channel CRAHNs.


## I. Introduction

According to Federal Communications Commission (FCC), almost all the spectrum from 3 KHz to 300 GHz has already been allocated to current wireless services. On the other hand, up to $85 \%$ of the allocated spectrum is underutilized. In order to balance the increase in the spectrum access demand and the inefficiency in the spectrum usage, cognitive radio (CR) [1] emerges as a key technology that enables unlicensed users, or, secondary users (SUs), to opportunistically access the spectrum which are temporarily unused by licensed users, or, primary users (PUs).

In a CR ad-hoc network (CRAHN) [2], SUs are equipped with cognitive radios which can sense the spectrum (spectrum sensing [3]) to seek spectrum holes, or, available channels and dynamically configure their operating parameters to switch to the desired channel (spectrum handoff [4], [5]). These new functions introduce changes to the transmission conditions and methods for SUs as compared to users in traditional wireless networks. These changes greatly affect a fundamental yet important function in the SU MAC layer, packet fragmentation. In the traditional wireless MAC, a long packet is usually fragmented into small frames in order to achieve a satisfactory bit-error-rate (BER) for the PHY layer transmission in a dedicated channel. However, compared with BER, the main factor contributing to the retransmission in

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CRAHNs is the PU's interference, since a SU can always find a channel with a relatively better quality (low BER) but cannot avoid PUs' reoccurrence. Meanwhile, the retransmission delay is also different from the traditional one without spectrum handoffs, which affects the overhead analysis of the frame size design. Thus, packet fragmentation in CRAHNs should be specially designed, which, however, is less investigated.

Nevertheless, there are several remotely related papers studying the optimal packet size in different CR networks with various limitations: i) the optimal SU packet size from the perspective of system design is designed in [6], [7], which is not appropriate to be embedded in a distributed SU MAC layer due to the lack of ample system information; ii) the optimal packet size for physical layer in terms of energy saving and BER control is considered in [8]-[10], but they neither fully adapt with the CR environment (the underlay mode in [9]) nor establish practical channel models (a common control channel in [10] which is difficult to maintain in CRAHNs and overly simplified assumption in [8], [10] where each channel is only associated with one dedicated PU) ; and iii) the optimal frame size for packet fragmentation is proposed in [11]-[13], but also modeled as the underlay mode in a single channel network.

Furthermore, besides the shortcomings explained above, none of the aforementioned works ( [6]-[13]) can satisfy the following three design guidelines for practical packet fragmentation. First, as mentioned previously, the protocol should capture the unique features of the new CR functions. Currently, the ability of multi-channel handoffs is only considered in [6]. Second, it is impractical to derive a universally optimal frame size and implement it to all packets with different lengths. For example, consider a packet which is 1.5 times longer than the optimal frame size. Is it still optimal to fragment this packet into two frames and only one of them has the optimal size? Only [11] mentioned that each given packet may have its own optimal frame size to fragment. Last, due to the time-varying mobility and activity of both PUs and SUs in CRAHNs, even for the same packet size, the optimal fragmentation changes with the network environment. Only [10] aims to dynamically acquire the optimal packet size.

In this paper, in order to design a practical fragmentation protocol with the above three aspects considered simultaneously, we first mathematically model the retransmission probability of a frame as a function of the given length. Our model takes into account related CR operations such as spectrum sensing, SU-contention, and spectrum handoff. The derivation employs parameters that a SU can either know or learn by itself as much as possible. Then, based on the average retransmission rate counted for different packet lengths in
a short-term transmission history, the unknown (PU-related) parameters can be estimated using our model. Furthermore, the optimal frame size in terms of maximizing the throughput is derived with both known and estimated parameters. Finally, we propose a self-adaptive optimal fragmentation (SAOF) protocol for SUs in CRAHNs. With SAOF, each SU can individually derive the optimal frame size for a given packet based on its latest environment. More specifically, the salient features of SAOF are summarized as follows:

1) SAOF employs a probabilistic retransmission model considering the impact of handoff delay, PU and SU activities, and node mobility. The correctness of the model is validated through extensive simulations.
2) SAOF derives the optimal fragmentation in terms of maximizing the throughput for a given packet, the optimality of which is mathematically proved and validated.
3) SAOF is based on practical primary network assumptions such as the independence between PUs and channels, the unknown time-varying PU traffic, and the unknown time-varying number of neighboring PUs.
4) SAOF can intelligently update its parameters from mining the network environment.
5) SAOF can also cooperate with other frame designs from different perspective such as energy saving to adjust the fragmentation, or enhance the performance of higher layer functions such as congestion control and end-toend delay control.
The rest of this paper is organized as follows. The system model considered in this paper and the problem formulation are introduced in Section II. In Section III, we analyze three factors leading to handoff occurrence. In Section IV, we propose a SAOF protocol with the optimal fragmentation derivation. Simulation results are shown in Section V, followed by the conclusions in Section VI.

## II. System Model and Problem Formulation

## A. System Model

1) Network Environment: The system considered in this paper consists of finite number of PUs and SUs which can operate over $N$ orthogonal channels. In a short term, each SU has $k$ PUs on average randomly distributed within its transmission range $r$. Without loss of generality, suppose $r$ is also the transmission range of a PU. Then, if a SU and any neighboring PU are both active on the same channel, their transmissions interfere with each other. We assume that the PU traffic follows the Poisson distribution in a short term with the average packet arrival rate $\lambda_{P}$ which is homogeneous for all neighboring PUs of a given SU. The PU packet size follows an arbitrary probability distribution with the average length $L_{P}$. Each PU is randomly assigned a channel not occupied by other PUs concurrently like 2G/3G cellular networks. The timevarying parameters such as $k, \lambda_{P}$, and $L_{P}$ differentiate our practical assumptions with the dedicated parameters assumed in most papers. Meanwhile, compared with the common assumption that one PU belongs to one channel, the random-channel-allocation mechanism for PUs complicates our model to better adapt with the realistic scenarios. Other important notations used in the following are summarized in Table I.

TABLE I
List of Symbol Notations

| $N$ | The number of channels in the network |
| :--- | :--- |
| $m$ | The number of busy channels from spectrum sensing |
| $k$ | The number of neighboring PUs of a SU |
| $B$ | The data rate of the channel |
| $t_{d}$ | The spectrum handoff delay |
| $\lambda_{P} / \lambda_{S}$ | Average packet arrival rate of the PU/SU |
| $r$ | The radius of the sensing area of the SU |
| $\bar{v}$ | The average relative velocity of a SU with respect to PUs |
| $L_{P} / L_{S}$ | The length of the PU/SU packet |
| $c$ | The optimal number of SU packet segmentation |
| $l$ | The length of the segmented SU packet |
| $h$ | The length of the header and trailer in the SU frame |
| $q$ | The average channel switching times during handoff |
| $H(l)$ | The handoff occurrence probability in terms of $l$ |
| $T(l)$ | The average service time of a frame in terms of $l$ |
| $X\left(L_{S}\right)$ | The average service time of a packet in terms of $L_{S}$ |
| $\Gamma\left(L_{S}\right)$ | Normalized SU throughput in terms of $\lambda_{S}$ |

2) Communication Steps: Before SUs set up their communications, spectrum sensing and channel hopping [14] are performed by each SU individually. Spectrum sensing helps each SU to obtain its available channels and channel hopping enables any SU pair rendezvous with each other in a finite time as long as they have at least one common available channel [15]-[17]. After rendezvous, the SU pair exchange their channel information to form their common available channel set. If later the communication is interrupted by PUs, they can hop on to their next common available channel and resume the transmission.
3) Packet Fragmentation: Each time when a SU has a string of data (unpacked from higher layer packet) to transmit, the SU MAC splits the packet into $c$ equilong pieces. Then, each piece is added with a header and trailer to form a frame to transmit independently. At the end of a frame, if a SU frame does not collide with a PU packet, the SU transmitter continues to transmit the following frames on the same channel until all frames are successfully transmitted. Therefore, if a SU packet collides with a PU packet, only the collided frame needs to be retransmitted. As for the long PU packet, we assume that it is already fragmented by PU MAC based on the BER control. Since PU packet owns the priority of the assigned channel, its frames (including the retransmitted frames) are seamlessly transmitted in the given channel, which can still be treated as an uncut PU packet in the analysis of SU frame transmission.


Fig. 1. The spectrum handoff process.
4) Handoff Delay: Fig. 1 shows the spectrum handoff process considered in this paper. If a PU starts its packet transmission during a SU's frame transmission in the same channel (say, channel 1), the SU pair will know the failed transmission till the end of the frame (i.e., the receiver cannot decode the collided frame and the transmitter does not receive an acknowledgment (ACK) from the receiver). Then, the SU pair switches to their next common available channel
(say, channel 2) for the retransmission of the previously unsuccessful frame. However, this new channel may already be unavailable due to the expired sensing information: i) at least one of the PU-neighbors of the SU pair reoccupied this channel after the rendezvous; and ii) SU-contention: other SU pairs nearby initiate the transmission on this channel before they arrive. Therefore, the CSMA mechanism is commonly employed to assist the handoff [5]. In other words, only after a successful RTS/CTS handshake on a new channel, can the SU pair finish the handoff process. Denote $q$ as the average number of channel switching before handoff finishing ( $q \geq 1$ ). The total handoff delay, $t_{d}$, can be calculated as:

$$
\begin{equation*}
t_{d}=t_{A C K}+q\left(t_{\text {switch }}+t_{R T S}+t_{C T S}\right) \tag{1}
\end{equation*}
$$

where $t_{\text {switch }}$ is the average operation delay for channel switching.

## B. Problem Formulation

1) Performance Metric: From the perspective of the SU MAC layer design, the main tasks are the throughput increase and the congestion/delay control to support higher layers. For a SU with the average packet arrival rate $\lambda_{S}$ and payload length $L_{S}$, its congestion/busy ratio is $\rho_{S}=\lambda_{S} X\left(L_{S}\right)$ where $X\left(L_{S}\right)$ denotes the average packet service (transmission) time in terms of $L_{S}$. Meanwhile, the average good throughput for such a SU is $\Gamma\left(L_{S}\right)=\frac{L_{S}}{X\left(L_{S}\right)}$. As we can see, for a given $L_{S}$, a lower $X\left(L_{S}\right)$ can increase the throughput and at the same time better serve the congestion control by decreasing the busy rate.

On the other hand, $X\left(L_{S}\right)$ depends on its number of frames $c$, which is a tradeoff parameter. A higher $c$ can decrease the retransmission rate for each relatively shorter frame but increase the overhead of both the total packet length and the operation times. Thus, one of the design goal is to get the optimal $c$ for different $L_{S}$ in terms of the minimum $X\left(L_{S}\right)$.
2) Derivation of the $X\left(L_{S}\right)$ : Denote $l$ as the length of a frame, then $l=\frac{L_{S}}{c}+h$ where $h$ is the combined header and trailer size. Since $B$ is the data rate of the channel, the length of time needed to transmit one SU frame without retransmission is $\frac{l}{B}$. However, due to the PU activities, the transmission of the SU frame may fail and the frame needs to be retransmitted multiple times before it is successfully received by the receiver, each time with a handoff performed for help. If the total number of transmissions for a SU frame to be successfully received is $i$, the total transmission time is $\frac{l}{B} i+t_{d}(i-1)$, where $t_{d}$ is the handoff delay from (1). In addition, let $H(l)$ be the probability that a handoff performed during a frame transmission with length $l$, which is actually the retransmission probability. Then, the probability that a SU frame is transmitted $i$ times is $H(l)^{i-1}(1-H(l))$. Then, the transmission time of a frame is written as

$$
\begin{align*}
T(l) & \left.=\sum_{i=1}^{\infty}\left[\frac{l}{B} i+t_{d}(i-1)\right)\right] H(l)^{i-1}(1-H(l))  \tag{2}\\
& =\frac{1}{1-H(l)} \frac{l}{B}+\frac{H(l)}{1-H(l)} t_{d} .
\end{align*}
$$

For an original data with size $L_{S}$, its service time is then

$$
\begin{equation*}
X\left(L_{S}\right)=c T(l)=c T\left(\frac{L_{S}}{c}+h\right) \tag{3}
\end{equation*}
$$

## III. Retransmission Analysis

In (2), $H(l)$ is an unknown function, the retransmission probability of a SU frame with length $l$, or, the interference probability during the transmission. As mentioned in Section I, for a given $l, H(l)$ is determined by the PU activities and the mobility of SUs and PUs. We derive the interference probabilistic model considering these factors in the following.

## A. Analysis of One-PU-Neighbor Scenarios

First of all, we assume that there is averagely only one PU-neighbor of a SU. That is, $k=1$. Then, there are two scenarios to be considered. On one hand, the PU is idle before the SU frame transmission. Then, the transmission will be interfered if the PU has packets arrival during $\frac{l}{C p}$ and the channel assigned to the PU is the same channel the SU is using $\left(1 /\binom{N}{1}\right)$. Let $N(t)$ be the number of packet arrivals of the PU in time $t$. Then, the probability of $n$ packet arrivals in time $t$ is $\operatorname{Pr}[N(t)=n]$, which is associated with the distribution of packets arrivals. For example, if the traffic of this PU follows Poisson distribution, $\operatorname{Pr}[N(t)=n]=\frac{\left(\lambda_{P} t\right)^{n}}{n!} e^{-\lambda_{P} t}$. Without loss of generality, suppose the average PU packet size is larger than a SU frame size and all PU frames are consecutively transmitted on its chosen channel. Then, as long as the PU has at least one packet arrival together with above conditions, the SU frame will be interfered. Overall, the interference probability under PU-idle-scenario is:

$$
\begin{equation*}
H_{k=1}^{i d l e}(l)=\left(1-\operatorname{Pr}\left[N\left(\frac{l}{B}\right)=0\right]\right) \frac{1}{N} . \tag{4}
\end{equation*}
$$

On the other hand, if the PU is transmitting a packet before the SU frame transmission on some other channel, then two cases may take place: i) the SU finishes its transmission before the PU does, which is free of interference; and ii) the PU completes its current packet transmission before the SU frame, which need to be further discussed later. To derive the probability of the second case, we denote $t_{1}$ and $t_{2}$ as the transmission starting time of the PU and SU respectively with an illustration in Fig. 2. The probability of the second case equals to the probability that PU finishes earlier cases (case $A: t_{1}+L_{P} \leq t_{2}+l$ ) among all the cases that the SU starts transmission in the middle of the PU's transmission $\left(\right.$ case $\left.B: t_{1} \leq t_{2} \leq t_{1}+L_{P}\right): \operatorname{Pr}[(A, B) \mid B]=\frac{l}{L_{P}}$.


i) the case of the earliest possible $t_{2}$


Fig. 2. The cases that the PU is busy before SU frame transmission.
For the second case, there are further two inherent conditions which result different interference probabilities, as illustrated in Fig. 3: i) if the PU still has packets in the queue waiting for the service (the PU congestion/busy ratio $\rho_{P}=\lambda_{P} \frac{L_{P}}{B}$ ), it will immediately starts another transmission. The new selected channel may also be the same channel that the SU is currently using $\left(1 /\binom{N}{1}\right.$ ); and ii) if the PU has zero packet waiting in its buffer $\left(1-\rho_{P}\right)$, it restores to the
first scenario where the PU is idle at the beginning of the left transmission time of the SU frame. Since the average remaining time of the SU frame in such cases is $\frac{l}{2}$ (similar derivation as that of Fig. 3, we can substitute it for $\frac{l}{B}$ in (4) to represent the interference probability under such cases. Finally, the interference probability under PU-busy-scenario can be written as:


Fig. 3. The cases that the PU finishes transmission before SU does.

## B. Analysis of Multi-PU-Neighbor Scenarios

In the $k$ PU-neighbors scenario, suppose $m$ PUs ( $m \leq k$ ) are busy at the beginning of a SU frame transmission. Similar as the analysis in one-PU-neighbor scenario, we can elaborately derive the probability for each case. For example, the probability that $i$ PUs among the $(k-m)$ idle PUs have traffic generated during the SU frame transmission is $\left(P_{0}[l / B]\right)^{k-m-i}\left(1-P_{0}[l / B]\right)^{i}$ where $P_{0}[l / B]$ is the short term of $\operatorname{Pr}[N(l / B)=0]$. In addition, the probability that one of these re-active PUs choose the same channel with the SU is $\binom{N-1}{i-1} /\binom{N}{i}=\frac{i}{N}$. Then, the interference probability of the ( $k-m$ ) idle PUs can be derived as:

$$
\begin{equation*}
H_{k-m}^{i d l e}(l)=\sum_{i=1}^{k-m}\left(P_{0}\left[\frac{l}{B}\right]\right)^{k-m-i}\left(1-P_{0}\left[\frac{l}{B}\right]\right)^{i} \frac{i}{N} \tag{6}
\end{equation*}
$$

We can also derive the interference probability under the $m$ busy PUs in the same way. However, in order to reduce the computational flexibility, the probability expression can be simplified to some extent with negligible difference. Consider the fact that cognitive radio technique is always used under the spectrum not fully utilized environment. That is, $m$ and $i$ are relatively much smaller than $N$. On the other hand, we know in mathematics, when $N$ is much larger than $i, 1-\left(\frac{N-1}{N}\right)^{i} \approx \frac{i}{N}$. Therefore, we replace $\frac{i}{N}$ in (6) with $\left(1-\left(\frac{N-1}{N}\right)^{i}\right)$ and the probability can be derived as a simpler form:

$$
H_{k-m}^{i d l e}(l) \approx 1-\left[1-H_{k=1}^{i d l e}(l)\right]^{k-m}
$$

With the same revision of the probability under busy PUs, the total interference probability under $(k, m)$ can be written as:

$$
\begin{equation*}
H_{k}^{\text {static }}(l) \approx 2-\left[1-H_{k=1}^{i d l e}(l)\right]^{k-m}-\left[1-H_{k=1}^{b u s y}(l)\right]^{m} \tag{7}
\end{equation*}
$$

It is only the interference probability of the static CRNs and the total probability considered the nodes mobility is analyzed in the following.

## C. Analysis of Mobility Scenarios

Since PUs are evenly distributed in the system, the average number of PU neighbors ( $k$ ) does not change within the moving duration of a SU. However, a new scenario may contribute to the interference probability compared with the
network with statistic nodes. As illustrated in Fig. 4, when SU moves from location A to location B, it may encounter the new PU who is currently using the same channel with the SU (say, channel 6). The probability of such cases need to be added to the original $H(l)$.


Fig. 4. An interference example under mobile scenarios.
To derive this probability we denote $k^{\prime}$ as the number of new encountered PUs within the transmission range of a SU during its moving. The ratio of $k^{\prime}$ to $k$, is the same as the ratio of the crescent shadow area size $\left(S_{C}\right)$ to the original circular area size $\left(\pi r^{2}\right)$ in the right part of Fig. 4. The circle represents the transmission range of a SU with the radius $r$. We assume that the speed of the SU is $\bar{v}$ which is a relative speed compared to surrounding nodes. The shadow part is the new transmission area during the nodes' moving. Note that the moving time during a frame transmission is $t=l / B$. Then, we derive $S_{C}$ as a function of $l$.

Derivation of $S_{C}(l): S_{C}=2\left(S_{C B D}-S_{C B E}\right)$. Meanwhile, we know $S_{C B E}=S_{C A E}-S_{C A B}$. Suppose $r$ and $\bar{v} t$ are known, we derive $\alpha=\arccos \frac{\bar{v} t}{2 r}$ and $\theta=\pi-\alpha$. Then $S_{C A B}=\frac{\bar{v} t}{2} r \sin \alpha, S_{C A E}=\pi r^{2} \frac{\alpha}{2 \pi}$ and $S_{C B D}=\pi r^{2} \frac{\theta}{2 \pi}$. Then, $S_{C}(l)=(\pi-2 \alpha) r^{2}+\bar{v} t r \sin \alpha$.

After calculating $S_{C}(l), k^{\prime}(l)=\frac{\dot{S}_{C}(l)}{\pi r^{2}} k$. The probability that a new encountered PU is busy on the SU's transmission channels as $\rho_{P} \frac{1}{N}$. Therefore, the interference probability due to nodes' mobility is

$$
\begin{equation*}
H_{k}^{\text {mobile }}(l)=1-\left(1-\rho_{p} \frac{1}{N}\right)^{k^{\prime}(l)} \tag{8}
\end{equation*}
$$

Consequently, the total interference probability with the consideration of mobility is

$$
\begin{equation*}
H_{k}^{t o t a l}(l)=H_{k}^{\text {static }}(l)+H_{k}^{\text {mobile }}(l) \tag{9}
\end{equation*}
$$

## IV. SAOF Protocol

## A. Optimal Fragmentation

Next, we formulate the optimal SU packet fragmentation. From (3), denote $c_{i}$ as the optimal fragmented number for $L_{S}^{i}$. Followed the optimization problem:

$$
\begin{array}{ll}
\underset{c_{i}}{\operatorname{Minimize}} & c_{i} T\left(\frac{L_{S}^{i}}{c_{i}}+h\right) \\
\text { subject to } & (1),(2),(4),(5),(7)-(9),  \tag{10}\\
& c_{i}>0, \text { and } c_{i} \in \mathbb{Z} .
\end{array}
$$

From the later protocol analysis, we know that $X\left(L_{S}^{i}\right)$ is only the function of $c_{i}$ for a given $L_{S}^{i}$. If we assume $c_{i}^{\prime} \in \mathbb{R}$, then the near-optimal $c_{i}{ }^{\prime}$ can be derived mathematically: in the final expression, the items including $c_{i}{ }^{\prime}$ in the power position can use Taylor expansion to approximate. Then the optimal value
of $c_{i}{ }^{\prime}$ in the new expression can be calculated by the method of derivation. Due to the space limitation, the trivial derivational process is not given here. After calculating $c_{i}{ }^{\prime}$, the optimal $c_{i}$ can be determined by comparing the nearest integer in terms of the minimum $X\left(L_{S}^{i}\right)$.

Then, we have $l_{i}=L_{S}^{i} / c_{i}$ where $l_{i}$ is the corresponding frame size for $L_{S}^{i}$. Note that a global optimal frame size $l$ does not exist. To prove it, we suppose there is a global optimal frame size $l$. Then the arrival/generated data length of a SU must satisfy $L_{i}=c_{i} l$. However, since $c_{i}$ is an integer, $L_{i}$ contradicts with the arbitrary-size assumption. Therefore, $l_{i}$ is a local optimal size depending on the given $L_{S}^{i}$.

## B. Protocol Details



Fig. 5. The block diagram of the proposed protocol.
Fig. 5 is the complete block diagram of the proposed protocol. Each time when a data string with an arbitrary size $L_{S}^{i}$ need to be transmitted (say, $L_{S}^{1}$ ), SAOF intelligently fragments it into equilong smaller frames with size $l_{i}$ to get the maximum throughput. $l_{i}$ is calculated by (10) with the parameter set $\left\{N, r, B, \bar{v}, q, m, L_{P}, k, H\left(l_{i}\right), \lambda_{P}\right\}$. Among these parameters: i) $N, r, B$ and $\bar{v}$ is the known information from the system and the SU itself; ii) $q$ is hard to mathematically derived but fortunately it is a independent parameter which can be counted and concluded through each frame transmission process; iii) A timely $m$ can be directly obtained from the spectrum sensing since it equals to the number of unavailable channels; iv) $L_{P}$ is straightforward to obtain from the sensing statistics since we only need the average values of the PU traffic information. In addition, if $L_{P}$ also changes with time, the updated $L_{P}$ can also be calculated from the short-term sensing history; v) Similarly, an updated $H\left(l_{i}\right)$ can also be elaborately counted through the frame-transmission short-term history; and vi) The two primary network parameters $k$ and $\lambda_{P}$ cannot be obtained as easy as $L_{P}$. For $k$, the number of idle PUs cannot be detected through spectrum sensing. On the other hand, since neither $k$ nor the channel selected by each PU on each transmission is known to the $\mathrm{SU}, \lambda_{P}$ cannot be inferred from the sensing. However, these two parameters can be learned by regression calculation from (9) since all the parameters required in (9) except $k$ and $\lambda_{P}$ are known or can be obtained as we claimed above. In fact, there should be enough simultaneous equations originated from (9) to derive $k$ and $\lambda_{P}$ by recording different $H\left(l_{i}\right)$ values for different $l_{i}$.

Meanwhile, the calculated $k$ and $\lambda_{P}$ can on the other hand assist the SU estimate the minimum service time $\left(X\left(L_{S}^{i}\right)\right)$ of possible $L_{S}^{i}$ arrived in the near future $\left(L_{S}^{2}, L_{S}^{3}\right.$, etc). Such information complements the SU to finish another task: congestion control of the packet arrival rate $\lambda_{S}$. At last, the protocol works in a dynamical way to keep mining and updating these time-varying parameters in order to better serve the SU transmission under the changing network environment.

## V. Performance Evaluation

In this section, we evaluate the throughput performance in SAOF. Firstly, we validate the proposed analytical models via extensive simulations. Then, we compare the network performance under SAOF with the scenario under various SU packet fragmentation. Finally, we compare the optimal fragmentation results under different SU packet size with varying primary environment. The default simulation parameters are summarized in Table II which mainly adopted from 802.11. In our system, one time slot equals to the transmission time of an ACK packet.

TABLE II
Static Simulation Parameters

| The radius of SU transmission range | 10 m |
| :--- | :---: |
| The size of MAC (header+trailer) | $30+4$ Bytes |
| The average channel switching delay | $100 \mu \mathrm{~s}$ |
| The size of (RTS+CTS) | $20+14$ Bytes |
| The size of a MAC ACK | 14 Bytes |
| Channel data rate | 2 Mbps |
| The average length of PU packets | 100 slots |



Fig. 6. $H(l)$ under different intra- and outer- conditions
Fig. 6 illustrates the impact of different parameters ( $l, \lambda_{P}$, $N$, and $\bar{v}$ respectively) on SU frame transmission. From the results we can see that: i) the simulation and analytical results coincide very well, i.e., the simulation results validate the correctness of our retransmission/interference model $H(l)$; ii) a larger $k$ always dramatically increases $H(l)$ under various conditions; iii) as shown in Fig. 6(a), the $H(l)$ increases with the frame length which motivates the design for the optimal fragmentation; iv) it is observed in Fig. 6(b) that
$\lambda_{P}$ is also a key factor together with $k$ that can largely affect $H(l)$. Therefore, our protocol is highly required for mining the changing $k$ and $\lambda_{P}$; and $v$ ) from Fig. 6(c) and (d), fragmentation definitely needs to be performed when there are less channels and high $\frac{\bar{v}}{r}$ in the network due to the high $H(l)$.


Fig. 7. Average SU packet throughput under different fragmentation
Next, Fig. 7 demonstrates the simulation results of the average SU throughput for a given packet under various fragmentation with different $k$ and $\lambda_{P}$. It is illustrated that, for a given $L_{S}$, when the number of frames increases, the good throughput of the SU (as claimed in Section II-B1, $\left.\Gamma\left(L_{S}\right)=\frac{L_{S}}{X\left(L_{S}\right)}\right)$ first increases and then decreases. Thus, there always exists an optimal fragmentation that maximizes $\Gamma\left(L_{S}\right)$ for a given $L_{S}$. Besides, it is observed that: i) under the same primary network, if each SU packet with whatever $L_{S}$ takes its own optimal fragmentation, they can always achieve almost the same throughput ( $86.3 \%$ in Fig. 7(b) and $78.3 \%$ in Fig. 7(a)). Such equilong service rate of packets can further help to decrease the queuing delay for SU packets [18]; ii) for packet with a smaller payload ( $L_{S}=50$ ), more fragments beyond the optimal number can largely decrease the throughput due to the high ratio of the overhead (the header and the handoff delay) to the frame size; iii) except the fragmentation, the throughput is heavily influenced by the environment of the primary network. When SUs under a high-traffic dense-node network (Fig. 7(a)), the throughput is degraded from that under a relatively sparse network (Fig. 7 (b)); and iv) the throughput under the optimal $c$ is much higher than that under no fragmentation $(c=1)$. Particularly, for packets with higher payload $\left(L_{S}=150\right)$, the prior throughput is almost 2.7 times than the latter one in the dense primary network.

TABLE III
THE OPTIMAL PACKET FRAGMENTATION

| $L_{S}$ (slot) | 50 |  | 100 |  | 150 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Environment | sparse | dense | sparse | dense | sparse | dense |
| Optimal $c$ | 2 | 3 | 3 | 5 | 5 | 8 |
| $l$ (slot) | 25 | 17 | 33 | 20 | 30 | 19 |

Finally, the optimal SU packet size for above scenarios calculated by our proposed protocol is given in Table III. It is shown that: i) the optimal $c$ derived by SAOF coincides with that observed in Fig. 7, which validates the optimality of SAOF; ii) there is no universal optimal frame size for all $L_{S}$, which identifies SAOF as a practical protocol; and iii) even for a given $L_{S}$, its optimal fragmentation is different depends on the network environment, which enhances the necessity of SAOF's self-adaptive feature.

## VI. Conclusion

In this paper, the SU packet fragmentation issue in CRAHNs is investigated for the first time. Compared with traditional wireless networks, we regard PU interference and handoff delay as the unique fragmentation factors for the desired network. Probabilistic retransmission model is established based on these novel factors. In addition, we associated our model with the time-varying PU activities in a practical way in order to learn the related parameters dynamically. Then, we proposed SAOF for the optimal fragmentation in terms of maximizing the throughput. Simulation results have verified both the correctness of the proposed mathematical model and the optimality of the fragmentation. The up-to-date parameters estimated by SAOF can provide vital information for the design in higher layer functions.

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