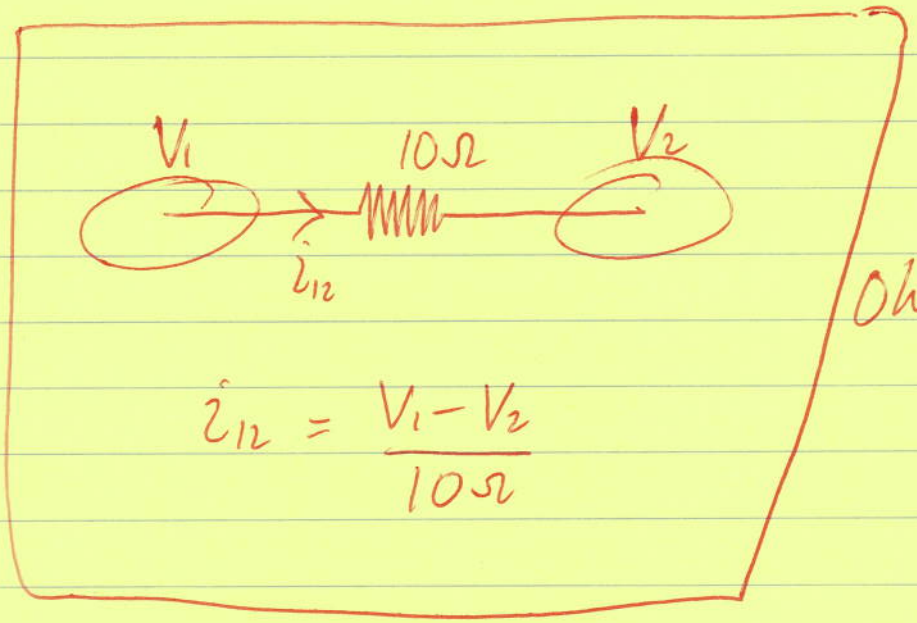
Node = a bunch of wire that joins 2 or more circuit elements together.

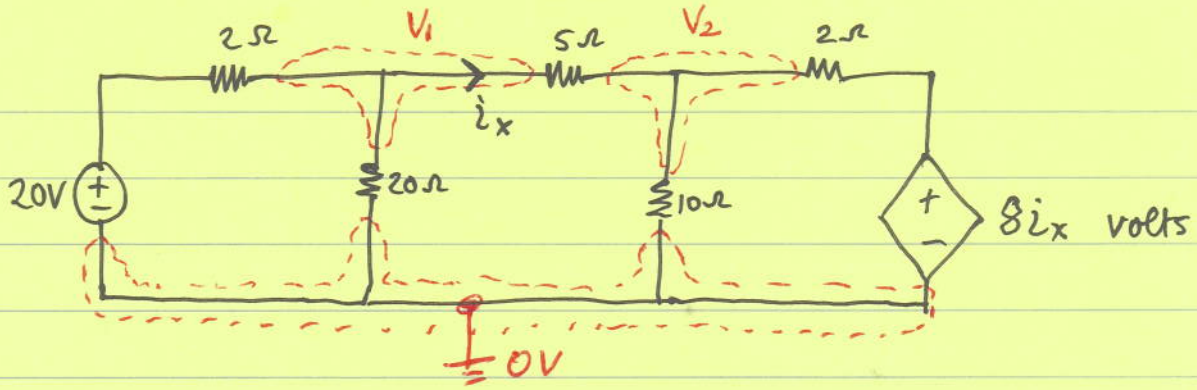
Nonessential node = a node that connects only 2 elements.
Essential node = a node that connects more than 2 elements.

Node Voltage Method

1. Identify all essential nodes.
2. Label one of them as "ground" = 0V.
3. Label the others with node voltages $V_1, V_2, V_3, \ldots$
4. Check for special cases (see later)
5. Write KCL for each node.
6. Solve resulting eqns for $V_1, V_2, V_3, \ldots$



Ex



Node V_1 : $\frac{V_1-20}{2} + \frac{V_1-0}{20} + \frac{V_1-V_2}{5} = 0$ .... ①

Node V_2 : $\frac{V_2-V_1}{5} + \frac{V_2-0}{10} + \frac{V_2-8i_x}{2} = 0$ .... ②

But $i_x = \frac{V_1-V_2}{5}$ (ohm's law). subst. into ②.

② $\Rightarrow \frac{V_2-V_1}{5} + \frac{V_2-0}{10} + \frac{V_2-8\left(\frac{V_1-V_2}{5}\right)}{2} = 0$ ... ②'

Standard form $\begin{cases} V_1\left(\frac{1}{2}+\frac{1}{20}+\frac{1}{5}\right) + V_2\left(-\frac{1}{5}\right) = +\frac{20}{2} \\ V_1\left(-\frac{1}{5}-\frac{8}{2\times5}\right) + V_2\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{2}+\frac{8}{2\times5}\right) = 0 \end{cases}$

Solve using calculator or Cramer's Rule

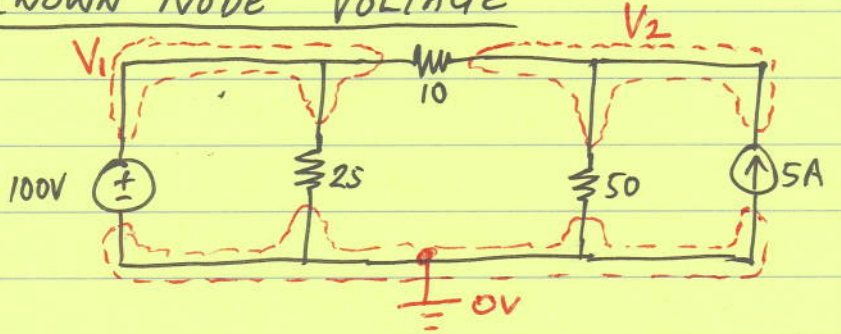
$\Rightarrow \begin{cases} V_1 = 16V \\ V_2 = 10V \end{cases}$

Node Voltage Special Cases

Case (1): Voltage source directly from ground to another node
$\Rightarrow$ Known Node Voltage

Case (2): Voltage source directly from one non-ground node to another
$\Rightarrow$ Supernode

CASE (1) KNOWN NODE VOLTAGE



10V source directly from ground to V_1 $\Rightarrow$ Known node voltage V_1.

(K) node 1 : $V_1 = 100V$

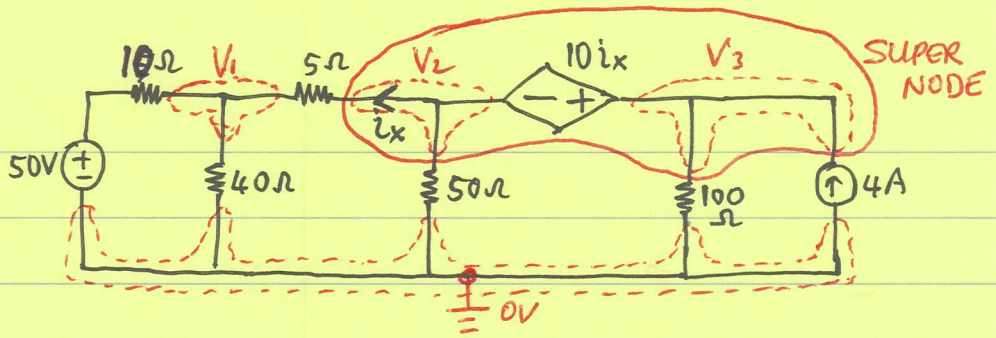
node 2 : $\frac{V_2 - V_1}{10} + \frac{V_2 - 0}{50} + (-5) = 0$

Std. form: $V_1(1) + V_2(0) = 100$

$V_1\left(\frac{-1}{10}\right) + V_2\left(\frac{1}{10} + \frac{1}{50}\right) = +5$

$\Rightarrow \begin{cases} V_1 = 100V \\ V_2 = 125V \end{cases}$

CASE ② SUPERNODE



$10i_x$ voltage source is connected directly from node ② to node ③.
Form a SUPERNODE consisting of nodes ②&③ and the voltage source.

Node ①: $\frac{V_1-50}{10} + \frac{V_1-0}{40} + \frac{V_1-V_2}{5} = 0$ ——— ①

Supernode:
$$\begin{cases} \frac{V_2-V_1}{5} + \frac{V_2-0}{50} + \frac{V_3-0}{100} + (-4) = 0 \quad \text{——— ②} \\ V_3 - V_2 = 10 i_x \quad \text{——— ③} \end{cases}$$

But $i_x = \frac{V_2-V_1}{5}$

$$\Rightarrow V_3 - V_2 = 10\left(\frac{V_2-V_1}{5}\right)$$
$$\Rightarrow V_3 - V_2 = 2V_2 - 2V_1$$
$$\Rightarrow V_3 - 3V_2 + 2V_1 = 0 \quad \text{——— ③'}$$

Std. form:
$$\begin{cases} V_1\left(\frac{1}{10}+\frac{1}{40}+\frac{1}{5}\right) + V_2\left(-\frac{1}{5}\right) + V_3(0) = +\frac{50}{10} \\ V_1\left(-\frac{1}{5}\right) + V_2\left(\frac{1}{5}+\frac{1}{50}\right) + V_3\left(\frac{1}{100}\right) = +4 \\ V_1(2) + V_2(-3) + V_3(1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} V_1 = 55.03\,V \\ V_2 = 64.43\,V \\ V_3 = 83.22\,V \end{cases}$$