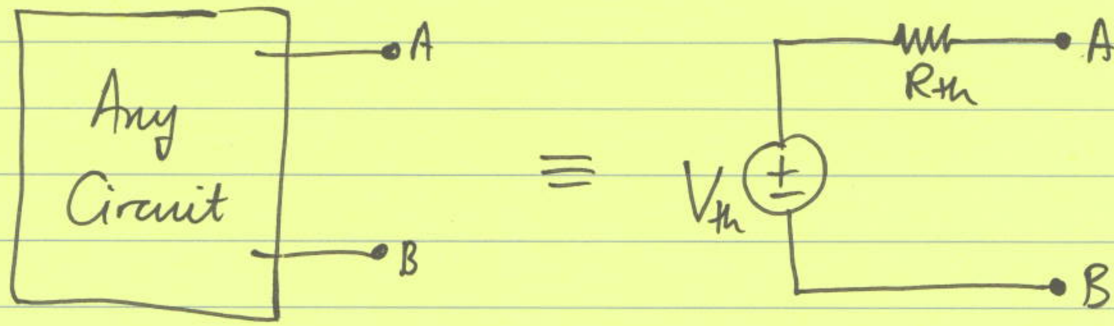
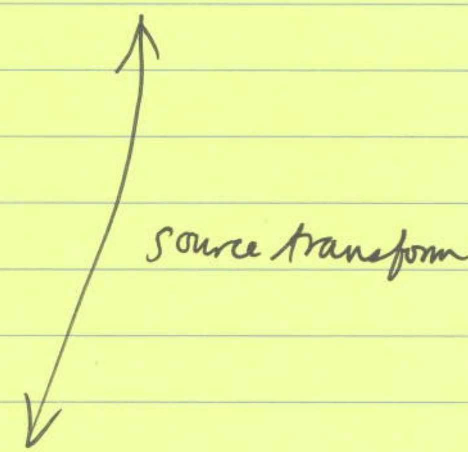
Thevenin Equivalent Circuit

> Any linear circuit with 2 terminals is equivalent to a voltage source in series with a resistance.

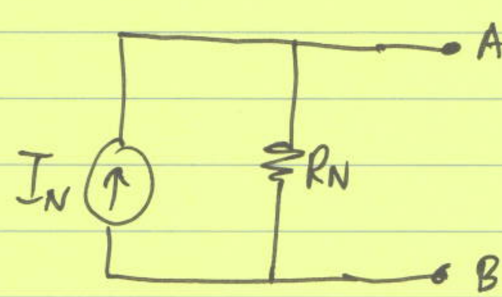


THEVENIN EQUIVALENT

V_{th} = Thevenin Voltage
R_{th} = Thevenin Resistance



$R_N = R_{th}$
$I_N = \dfrac{V_{th}}{R_{th}}$



NORTON EQUIVALENT

V_{Th} = open circuit voltage of circuit = V_{oc}

I_N = short circuit current of circuit = I_{sc}

$$R_{th} = \frac{V_{oc}}{I_{sc}}.$$

How to find the Thevenin Voltage:

V_{th} = the open circuit voltage
So, draw circuit with nothing connected to A, B and then find V_{th} = voltage between A & B.

How to find the Thevenin Resistance:

There are 3 methods:

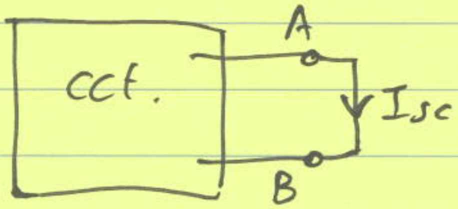
R_{th} method (A): Equivalent resistance method

Suppress all independent sources, then R_T = resistance looking into terminals (A, B).

(This is the easiest method – use when circuit has no dependent sources)

R_{th} method (B) : Short-circuit current method

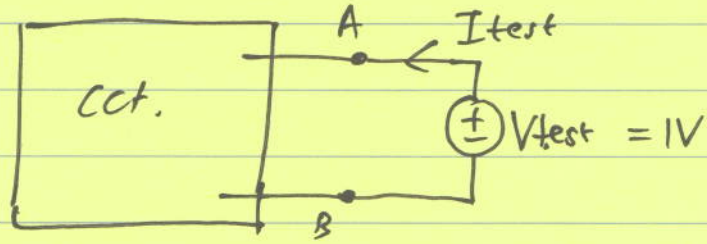
Draw the circuit with a short across A & B and then find Isc.



Then $R_{th} = \frac{V_{th}}{I_{sc}}$.

(use when circuit has both dependent and independent sources)

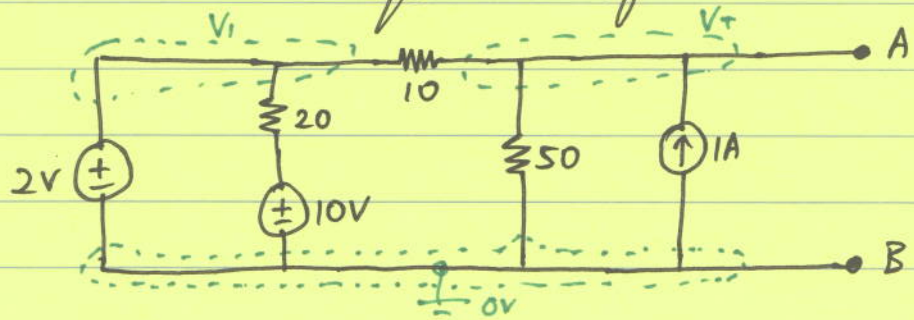
R_{th} method (C) : Test Source Method



Attach a test source to A & B, then $R_{th} = \frac{V_{test}}{I_{test}}$.

(this method works for any circuit, but is much more work than method (A))
(use when all sources are dependent).

Ex Find Thevenin equivalent of:



Vth: $V_1 = 2V$ (KNOWN NODE VOLTAGE special case)

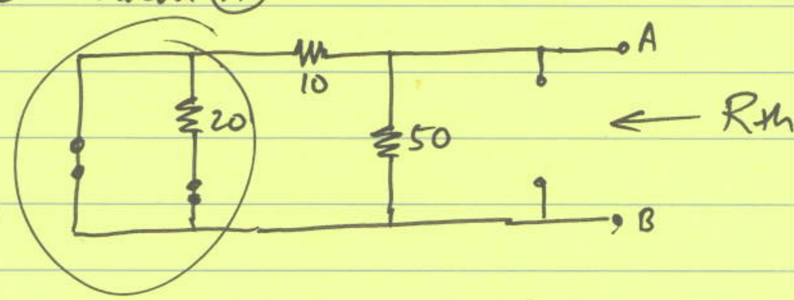
$$\frac{V_T - V_1}{10} + \frac{V_T - 0}{50} + (-1) + 0 = 0$$

$$\Rightarrow \frac{V_T - 2}{10} + \frac{V_T}{50} - 1 = 0$$

$$\Rightarrow V_T\left(\frac{1}{10} + \frac{1}{50}\right) = 1 + \frac{2}{10}$$

$$\Rightarrow V_T = 1.2/0.12 = 10V$$

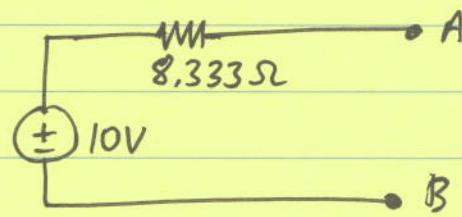
Rth: use method Ⓐ



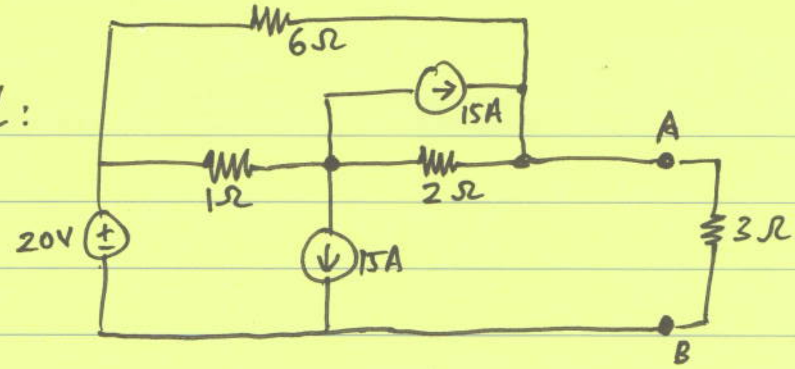
$20 \| 0 = 0\Omega$

$R_{TH} = 50 \| 10 = 8.333\,\Omega$

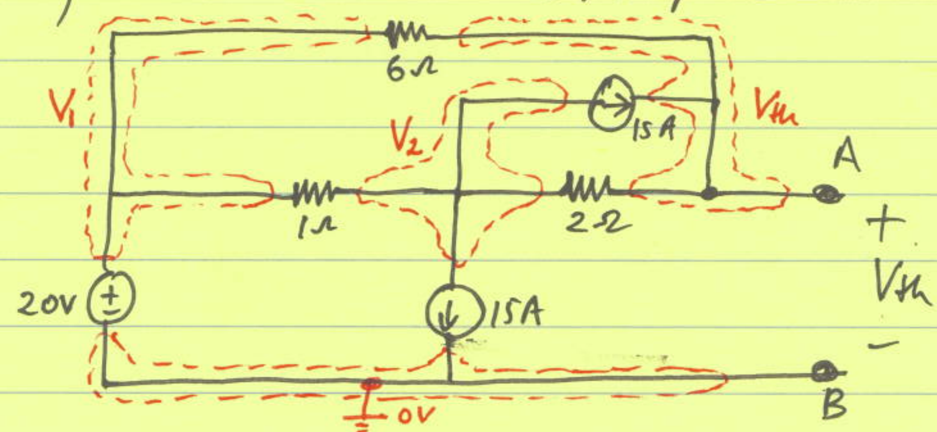
Thev. equiv. is



Ex Find Thev. Equiv. of:



Soln For V_{th}, draw circuit with (A,B) open-circuit:



Special case: Known node voltage $V_1 = 20V$.

Node 1 : $V_1 = 20$

Node 2 : $\frac{V_2 - V_1}{1} + 15 + \frac{V_2 - V_{th}}{2} + 15 = 0$

Node V_{th} : $\frac{V_{th} - V_1}{6} + (-15) + \frac{V_{th} - V_2}{2} + 0 = 0$

Standard form:

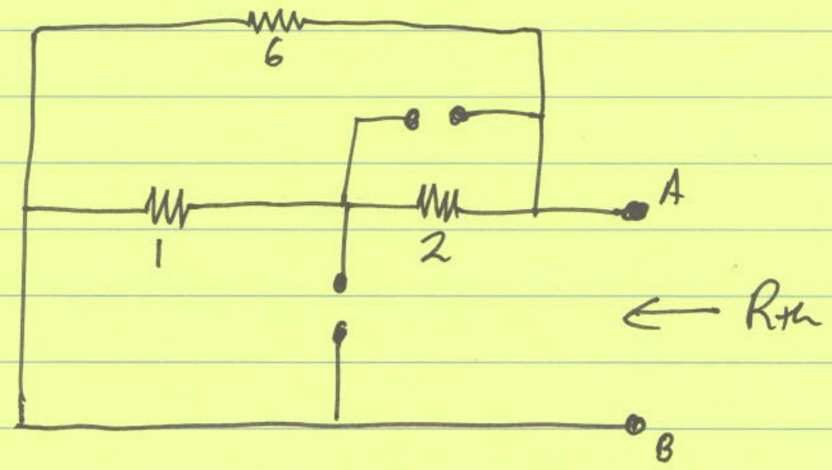
$$\begin{cases} V_1(1) + V_2(0) + V_{th}(0) = 20 \\ V_1(-1) + V_2(1+\frac{1}{2}) + V_{th}(-\frac{1}{2}) = -30 \\ V_1(-\frac{1}{6}) + V_2(-\frac{1}{2}) + V_{th}(\frac{1}{6}+\frac{1}{2}) = +15 \end{cases}$$

$$\Rightarrow \begin{aligned} V_1 &= 20V \\ V_2 &= 3.3333V \\ V_{th} &= 30V \end{aligned}$$

Now get R_{th}:

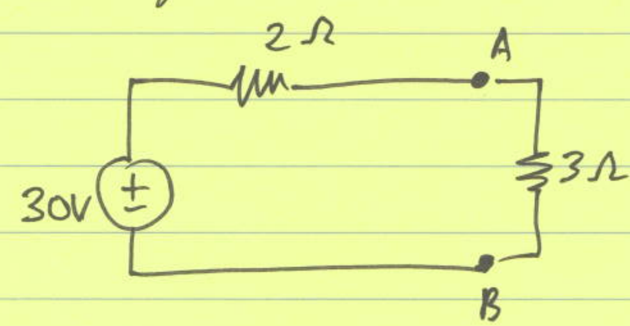
For R_{th} : No dependent sources ⇒ Method A will work.

Suppress all independent sources :

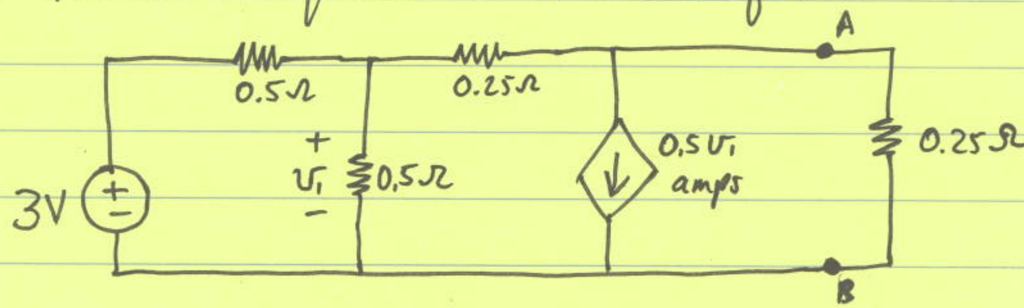


$$R_{th} = (1+2) \| 6 = 3 \| 6 = \frac{3 \times 6}{3+6} = 2\,\Omega$$

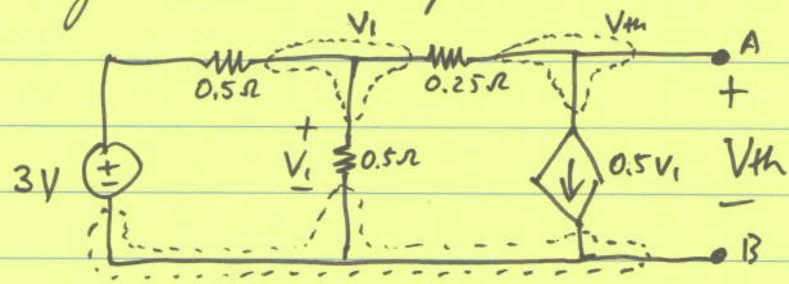
So, Thev. equiv. circuit is :



Ex Find Thevenin equivalent circuit of:



Soln: First get V_{th} = open-circuit voltage:



Node V_1: $\frac{V_1 - 3}{0.5} + \frac{V_1 - 0}{0.5} + \frac{V_1 - V_{th}}{0.25} = 0$

Node V_{Th}: $\frac{V_{th} - V_1}{0.25} + (0.5V_1) + 0 = 0$

Standard form:
$$\begin{cases} V_1\left(\frac{1}{.5} + \frac{1}{.5} + \frac{1}{.25}\right) + V_{th}\left(-\frac{1}{.25}\right) = \frac{3}{.5} \\ V_1\left(-\frac{1}{.25} + 0.5\right) + V_{th}\left(\frac{1}{.25}\right) = 0 \end{cases}$$

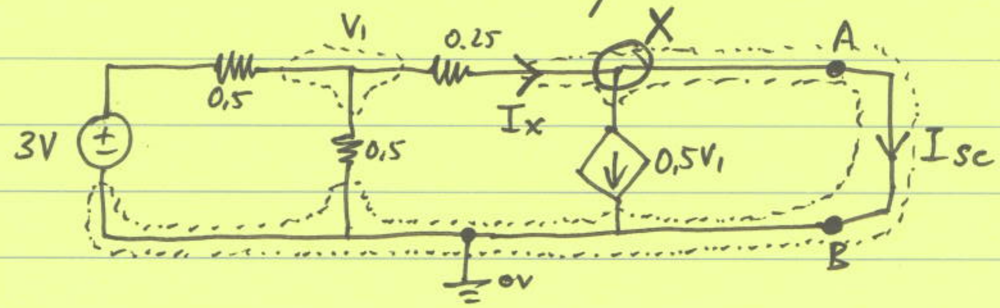
$$\Rightarrow \begin{cases} V_1(8) + V_{th}(-4) = 6 \\ V_1(-3.5) + V_{th}(4) = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_{th} \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -3.5 & 4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.3333\,V \\ 1.1667\,V \end{bmatrix}$$

$V_{th} = 1.16667\,V$.

Now get R_{th}:

For R_{th}: Circuit has a dependent source so use method Ⓑ:



Node V_1:
$$\frac{V_1 - 3}{0.5} + \frac{V_1 - 0}{0.5} + \frac{V_1 - 0}{0.25} = 0$$

$$\Rightarrow V_1\left(\frac{1}{.5} + \frac{1}{.5} + \frac{1}{.25}\right) = \frac{3}{.5}$$

$$\Rightarrow V_1(2+2+4) = 6$$

$$\Rightarrow V_1 = 6/8 = 0.75\text{ V}$$

KCL Ⓧ:
$$(-I_x) + (0.5V_1) + I_{sc} = 0$$

But $I_x = \frac{V_1 - 0}{0.25} = \frac{0.75}{0.25} = 3\text{A}$ (ohms law)

and $0.5V_1 = 0.5(0.75) = 0.375\text{A}$

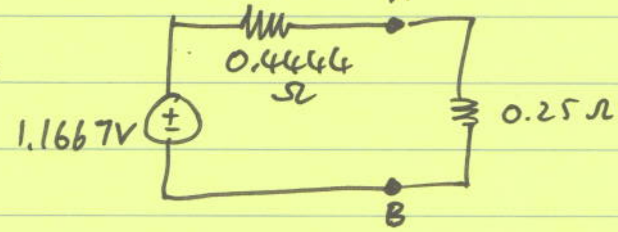
$$\Rightarrow (-3) + (0.375) + I_{sc} = 0$$

$$\Rightarrow I_{sc} = 3 - 0.375 = 2.625\text{ A}$$

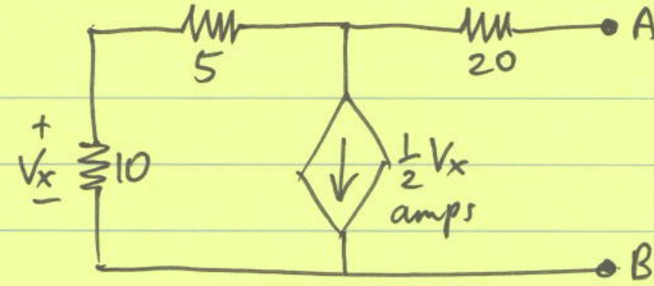
$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{0.75\text{V}}{2.625\text{A}} = 0.4444\,\Omega$$

Correction: should be 1.16667V not 0.75V

Thev equiv is:

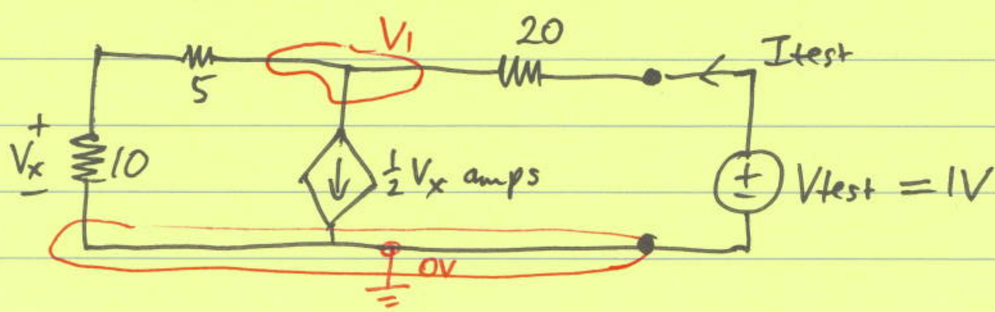


Ex Find Thev. Equivalent of



Soln: V_{th} = open circuit voltage = 0V since there is no independent voltage or current source.

R_{th} : Will have to use Method (B) :



Let $V_{test} = 1V$.

KCL node ①: $\frac{V_1 - 0}{15} + \left(\frac{1}{2}V_x\right) + \frac{V_1 - 1V}{20} = 0$

But $V_x = V_1 \cdot \frac{10}{10+5} = \frac{2}{3}V_1$ (voltage divider)

$\Rightarrow \frac{V_1}{15} + \frac{1}{2}\left(\frac{2}{3}V_1\right) + \frac{V_1 - 1}{20} = 0$

$\Rightarrow V_1\left(\frac{1}{15} + \frac{1}{3} + \frac{1}{20}\right) = \frac{1}{20}$

$\Rightarrow V_1 = \frac{1}{20} \Big/ \left(\frac{1}{15} + \frac{1}{3} + \frac{1}{20}\right) = 0.111111\,V$

$\Rightarrow I_{test} = \frac{1V - 0.111111V}{20\Omega} = 0.04444\,A$

$\Rightarrow R_{th} = \frac{V_{test}}{I_{test}} = \frac{1V}{0.04444A} = 22.5\Omega$

Thev. equiv cct =