

How to solve simultaneous linear equations

There are three ways (in order of increasing difficulty)

METHOD (1): Using the "simultaneous equations" app on your TI calculator.

METHOD (2): Using the matrix inverse operation on your calculator.

METHOD (3): Using Cramer's Method.

Method (1) Your TI calculator must have the simultaneous equation app installed on it. The app prompts for all the coefficients of the standard form and then gives you the solution.

A16

eg Std form

$$\begin{array}{r} 3I_1 + 4I_2 + 5I_3 = 7 \\ I_1 + 2I_2 + 3I_3 = 2 \\ \nearrow 2I_1 - I_2 + 6I_3 = 0 \end{array}$$

You type in these 12 numbers, and the calculator tells you that

$$\begin{array}{l} I_1 = 2.35 \text{ A} \\ I_2 = 0.8 \text{ A} \\ I_3 = -0.65 \text{ A} \end{array}$$

Method (2) Using Matrix inverse function

$$\begin{aligned} \text{Std. form: } 3I_1 + 4I_2 + 5I_3 &= 7 \\ I_1 + 2I_2 + 3I_3 &= 2 \\ 2I_1 - I_2 + 6I_3 &= 0 \end{aligned}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 6 \end{bmatrix}}_A \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_X = \underbrace{\begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}}_B \quad \text{matrix form}$$

$$\Rightarrow A X = B$$

$$\Rightarrow X = A^{-1} B$$

$$X = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} .75 & -1.45 & .1 \\ 0 & .4 & -.2 \\ -.25 & .55 & .1 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} \quad \left(\begin{array}{l} \text{calculated } A^{-1} \\ \text{using calculator} \\ \text{matrix inverse function} \end{array} \right)$$

$$= \begin{bmatrix} 2.35 \\ 0.8 \\ -0.65 \end{bmatrix} \quad \text{So } \begin{cases} I_1 = 2.35 \text{ A} \\ I_2 = 0.8 \text{ A} \\ I_3 = -0.65 \text{ A} \end{cases}$$

Method (3)

Cramer's Rule

see Dr. Sharer's notes on Cramer's Rule.

Std form:

$$\begin{aligned} 3I_1 + 4I_2 + 5I_3 &= 7 \\ I_1 + 2I_2 + 3I_3 &= 2 \\ 2I_1 - I_2 + 6I_3 &= 0 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 6 \end{bmatrix}}_A \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_X = \underbrace{\begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}}_B$$

$$\Delta = |A| = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 6 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 5 & 3 & 4 \\ 1 & 2 & 3 & 1 & 2 \\ 2 & -1 & 6 & 2 & -1 \end{vmatrix}$$

$$= 3(2)6 + 4(3)2 + 5(1)(-1)$$

$$- 5(2)2 - 3(3)(-1) - 4(1)6$$

$$= 36 + 24 - 5 - 20 + 9 - 24 = 20$$

$$\Delta_1 = \begin{vmatrix} 7 & 4 & 5 \\ 2 & 2 & 3 \\ 0 & -1 & 6 \end{vmatrix} = \begin{vmatrix} 7 & 4 & 5 & 7 & 4 \\ 2 & 2 & 3 & 2 & 2 \\ 0 & -1 & 6 & 0 & -1 \end{vmatrix}$$

$$= 7(2)6 + 4(3)0 + 5(2)(-1)$$

$$- 5(2)0 - 7(3)(-1) - 4(2)6$$

$$= 84 + 0 - 10 - 0 + 21 - 48 = 47$$

$$\Delta_2 = \begin{vmatrix} 3 & 7 & 5 \\ 1 & 2 & 3 \\ 2 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 3 & 7 & 5 & 3 & 7 \\ 1 & 2 & 3 & 1 & 2 \\ 2 & 0 & 6 & 2 & 0 \end{vmatrix}$$

$$= 3(2)6 + 7(3)2 + 5(1)0 - 5(2)2 - 3(3)0 - 7(1)6$$

$$= 36 + 42 + 0 - 20 - 0 - 42 = 16$$

$$\Delta_3 = \begin{vmatrix} 3 & 4 & 7 \\ 1 & 2 & 2 \\ 2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 7 & 3 & 4 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 0 & 2 & -1 \end{vmatrix}$$

$$= 3(2)0 + 4(2)2 + 7(1)(-1) - 7(2)2 - 3(2)(-1) - 4(1)0$$

$$= 0 + 16 - 7 - 28 + 6 - 0 = -13$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{47}{20} = 2.35 \text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{16}{20} = 0.8 \text{ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-13}{20} = -0.65 \text{ A}$$