Since $E_1 = 3.06$ m, the above relation gives

$$3.06 = \frac{3}{2}\left[\frac{(20/b)^2}{9.81}\right]^{1/3}$$

which yields $b = 2.19$ m. Therefore, the minimum allowable width of the constriction to prevent choking in $\boxed{2.19 \text{ m}}$.

3.36. From the given data: $Q = 36$ m^3/s, $b = 10$ m, $n = 0.030$, $S_o = 0.001$, and $y = 3$ m. Determine the normal depth, using the Manning equation

$$\begin{aligned} Q &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S_o} \\ 36 &= \frac{1}{0.030}\frac{(10y_n)^{5/3}}{(10+2y_n)^{2/3}}\sqrt{0.001} \end{aligned}$$

which gives

$$y_n = 2.45 \text{ m}$$

Determine the critical depth,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3.6^2}{9.81}\right)^{1/3} = 1.09 \text{ m}$$

Since $y > y_n > y_c$, the water surface follows a $\boxed{\text{M1 profile}}$.

Determine S_f using the Manning equation,

$$\begin{aligned} S_f &= \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{nQP^{2/3}}{A^{5/3}}\right]^2 = \left[\frac{nQ(10+2y)^{2/3}}{(10y)^{5/3}}\right]^2 = \left[\frac{(0.030)(36)(10+2\times 3)^{2/3}}{(10\times 3)^{5/3}}\right]^2 \\ &= 0.00056 \end{aligned}$$

The slope of the water surface is given by

$$\frac{dy}{dx} = \frac{S_o - S_f}{1-\text{Fr}^2} = \frac{0.001 - 0.00056}{1-\left(\frac{1.2}{9.81(3)}\right)^2} = \boxed{0.00046}$$

where the velocity is taken as $V = Q/A = 36/(10\times 3) = 1.2$ m/s.

If $y = 2$ m, then $y_n > y > y_c$ and the water surface follows a $\boxed{\text{M2 profile}}$. Therefore the shape of the water surface would be different than when $y = 3$ m.

3.37. From the given data: $Q = 30$ m^3/s, $b = 8$ m, and $n = 0.035$.

$$\begin{aligned} y_c &= \left[\frac{q^2}{g}\right]^{1/3} = \left[\frac{(30/8)^2}{9.81}\right]^{1/3} = 1.13 \text{ m} \\ S_c &= \left[\frac{nQP^{2/3}}{A^{5/3}}\right]^2 = \left[\frac{nQ(8+2y_c)^{2/3}}{(8y_c)^{5/3}}\right]^2 = \left[\frac{(0.035)(30)(8+2\times 1.13)^{2/3}}{(8\times 1.13)^{5/3}}\right]^2 = 0.016 \end{aligned}$$

Hence for $\boxed{\text{Mild Slope: } 0 < S_o < 0.016}$ and for $\boxed{\text{Steep Slope: } S_o > 0.016}$.

3.38. From given data: $b = 6$ m, $m = 2$, $n = 0.045$, $S_o = 0.015$, $Q = 80$ m^3/s, and $y = 5$ m. Calculate normal depth using Manning equation,

$$\begin{aligned} Q &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S_o} \\ 80 &= \frac{1}{0.045}\frac{(by_n + 2y_n^2)^{5/3}}{(b + 2\sqrt{5}y_n)^{2/3}}\sqrt{S_o} = \frac{1}{0.045}\frac{(6y_n + 2y_n^2)^{5/3}}{(6 + 2\sqrt{5}y_n)^{2/3}}\sqrt{0.015} \end{aligned}$$

which gives

$$y_n = 2.21 \text{ m}$$

Calculate the critical depth,

$$\begin{aligned} \frac{Q^2}{g} &= \frac{A_c^3}{T_c} \\ \frac{80^2}{9.81} &= \frac{(by_c + 2y_c^2)^3}{(b + 4y_c)} \\ 652.4 &= \frac{(6y_c + 2y_c^2)^3}{(6 + 4y_c)} \end{aligned}$$

which gives

$$y_c = 2.07 \text{ m}$$

Since $y > y_n > y_c$, the water surface has a $\boxed{\text{M1 profile}}$ and the depth $\boxed{\text{increases}}$ in the downstream direction.

The slope, S_f, of the energy grade line is given by the Manning equation as

$$S_f = \left[\frac{nQP^{2/3}}{A^{5/3}}\right]^2 = \left[\frac{nQ(b + 2\sqrt{5}y)^{2/3}}{(by + 2y^2)^{5/3}}\right]^2 = \left[\frac{(0.045)(80)(6 + 2\sqrt{5} \times 5)^{2/3}}{(6 \times 5 + 2 \times 5^2)^{5/3}}\right]^2 = 0.000508$$

Other hydraulic parameters are

$$\begin{aligned} A &= by + 2y^2 = (6)(5) + 2(5)^2 = 80 \text{ m}^2 \\ V &= \frac{Q}{A} = \frac{80}{80} = 1 \text{ m/s} \\ T &= b + 2my = (6) + 2(2)(5) = 26 \text{ m} \\ D &= \frac{A}{T} = \frac{80}{26} = 3.08 \text{ m} \\ \text{Fr}^2 &= \frac{V^2}{gD} = \frac{1^2}{9.81(3.08)} = 0.0331 \end{aligned}$$

The slope of the water surface is therefore given by

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \text{Fr}^2} = \frac{0.015 - 0.000508}{1 - 0.0331} = \boxed{0.0150}$$

The depth, y_u, 100 m upstream is given by

$$y_u = 5 - 100(0.0150) = \boxed{3.50 \text{ m}}$$

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and the depth, y_d, 100 m downstream is given by

$$y_u = 5 + 100(0.0150) = \boxed{6.50 \text{ m}}$$

3.39. According to the momentum equation

$$\sum F_x = \rho Q(V_2 - V_1)$$

In this case,

$$\begin{aligned} \gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 + \gamma \mathcal{V} \sin\theta &= \rho Q \left(\frac{Q}{A_2} - \frac{Q}{A_1} \right) \\ A_1 \bar{y}_1 - A_2 \bar{y}_2 + \mathcal{V} \sin\theta &= Q \left(\frac{Q}{gA_2} - \frac{Q}{gA_1} \right) \end{aligned}$$

For a rectangular channel of width b,

$$\begin{aligned} A_1 &= by_1 \\ A_2 &= by_2 \\ \bar{y}_1 &= \frac{y_1}{2} \\ \bar{y}_2 &= \frac{y_2}{2} \\ \mathcal{V} &= \frac{y_1 + y_2}{2} 5y_2 b \\ \sin\theta &= S_o \\ Q &= qb \end{aligned}$$

and substituting into the momentum equation gives

$$\begin{aligned} b\frac{y_1^2}{2} - b\frac{y_2^2}{2} + \frac{y_1 + y_2}{2} 5y_2 b S_o &= \frac{q^2 b^2}{gby_2} - \frac{q^2 b^2}{gby_1} \\ \frac{y_1^2}{2} - \frac{y_2^2}{2} + \frac{y_1 + y_2}{2} 5y_2 S_o &= \frac{q^2}{gy_2} - \frac{q^2}{gy_1} \end{aligned}$$

or

$$\boxed{\frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2} - \frac{y_1 + y_2}{2} 5y_2 S_o}$$

3.40. From given data: $Q = 100$ m^3/s, $b = 8$ m, $y_1 = 0.9$ m, and

$$q = \frac{Q}{b} = \frac{100}{8} = 12.5 \text{ m}^2/\text{s}$$

and

$$\begin{aligned} y_2 &= \frac{y_1}{2} \left(-1 + \sqrt{1 + 8\text{Fr}_1^2} \right) \\ &= \frac{y_1}{2} \left(-1 + \sqrt{1 + 8\frac{q^2}{gy_1^3}} \right) = \frac{0.9}{2} \left(-1 + \sqrt{1 + 8\frac{12.5^2}{(9.81)(0.9)^3}} \right) = \boxed{5.52 \text{ m}} \end{aligned}$$