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Since the tailwater depth of 4 m exceeds the distinguishing condition ($y_3 = 3.12$ m), then submerged flow conditions exist and the flow through the gate is given by Equation 3.219. In this case, Equations 3.220 and 3.221 give

$$\begin{aligned} \lambda &= \frac{y_1}{y_3} = \frac{5}{4} = 1.25 \\ \xi &= \left(\frac{1}{\eta} - 1\right)^2 + 2(\lambda - 1) = \left(\frac{1}{0.134} - 1\right)^2 + 2(1.25 - 1) = 42.3 \end{aligned}$$

and Equation 3.219 gives

$$\begin{aligned} Q &= C_c \frac{\left[\xi - \sqrt{\xi^2 - \left(\frac{1}{\eta^2} - 1\right)^2 \left(1 - \frac{1}{\lambda^2}\right)}\right]^{1/2}}{\frac{1}{\eta} - \eta} b y_g \sqrt{2gy_1} \\ &= 0.61 \frac{\left[42.3 - \sqrt{42.3^2 - \left(\frac{1}{0.134^2} - 1\right)^2 \left(1 - \frac{1}{1.25^2}\right)}\right]^{1/2}}{\frac{1}{0.134} - 0.134} (8)(1.10)\sqrt{2(9.81)(5)} \\ &= 28.7 \text{ m}^3/\text{s} \end{aligned}$$

Therefore, when the tailwater depth is 4 m, the flow through the gate is submerged and equal to $\boxed{28.7 \text{ m}^3/\text{s}}$.

3.93. Since

$$C_c = 1 - 0.75\left(\frac{\theta}{90}\right) + 0.36\left(\frac{\theta}{90}\right)^2$$

To find θ that minimizes C_c,

$$\frac{dC_c}{d\theta} = -\frac{0.75}{90} + 0.72\left(\frac{\theta}{90}\right)\frac{1}{90} = 0$$

which simplifies to

$$-0.00833 + 0.0000889\theta = 0$$

or

$$\theta = 93.7^\circ$$

Since

$$\frac{d^2C_c}{d\theta^2} > 0$$

then $\theta = 93.7^\circ$ is a minimum. Since θ must be between 0° and 90°, then, within this range, the minimum value of C_c occurs at $\boxed{\theta = 90^\circ}$.

3.94. From the given data: $D = 0.5$ m, $n = 0.013$, $k_e = 0.05$, $C_d = 0.95$, $S_o = 0.02$, and $L = 20$ m. The elevation difference, Δh, between the headwater and the top of the culvert at the exit is given by

$$\Delta h = 0.2 + S_o L = 0.2 + (0.02)(20) = 0.6 \text{ m}$$

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Assume that the exit is not submerged, flow is either Type 2 or Type 3. Assume the flow is Type 2:

$$\begin{aligned} Q &= A\sqrt{\frac{2g\Delta h}{2gn^2\frac{L}{R^{4/3}} + k_e + 1}} = \pi(0.25)^2\sqrt{\frac{2(9.81)(0.6)}{2(9.81)(0.013)^2\frac{20}{(0.5/4)^{4/3}} + 0.05 + 1}} \\ &= 0.464 \text{ m}^3/\text{s} \end{aligned}$$

Determine if the culvert flows full. The Manning equation gives

$$Q_{\text{full}} = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S_o} = \frac{1}{0.013}\frac{(\pi 0.25^2)^{5/3}}{(\pi 0.5)^{2/3}}\sqrt{0.02} = 0.534 \text{ m}^3/\text{s}$$

and the culvert flows full when the discharge exceeds 1.07(0.534) = 0.571 m^3/s. Since $Q <$ 0.571 m^3/s, the culvert does not flow full, and the assumption of Type 2 flow is not supported. Assuming Type 3 flow,

$$Q = C_d A\sqrt{2gh} = 0.95\pi(0.25)^2\sqrt{2(9.81)(0.25 + 0.2)} = 0.55 \text{ m}^3/\text{s}$$

Since $Q <$ 0.571 m^3/s, the culvert does not flow full, Type 3 flow is confirmed, and the discharge through the culvert is $\boxed{0.55 \text{ m}^3/\text{s}}$.

3.95. From the given data: TW = 10.00 m, D = 380 mm = 0.38 m, and L = 8 m. Since the water depth at the outlet is 10.00 m − 9.50 m = 0.50 m, and the culvert diameter is 0.38 m, then Type 1 flow is expected. Accordingly, the discharge, Q, is given by Equation 3.228 as

$$Q = A\sqrt{\frac{2g\Delta h}{2gn^2 L/R^{\frac{4}{3}} + k_e + 1}} \tag{1}$$

For a corrugated metal pipe, Table 3.13 gives n = 0.028 (conservative value) and Table 3.14 gives k_e = 0.7 for a mitered entrance. Substituting into Equation (1) gives

$$Q = \frac{\pi}{4}(0.38)^2\sqrt{\frac{2(9.81)(HW - 10.00)}{2(9.81)(0.028)^2(8)/(0.38/4)^{\frac{4}{3}} + 0.7 + 1}}$$

which simplifies to the following culvert performance curve

$$\boxed{Q = 0.236\sqrt{(HW - 10)}} \tag{2}$$

If the culvert is required to pass 0.30 m^3/s, then the required headwater elevation satisfies the relation

$$0.3 = 0.236\sqrt{(HW - 10)}$$

which gives $\boxed{HW = 11.62 \text{ m}}$.

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When the tailwater elevation is 9.75 m, the flow through the culvert is Type 2, and the flowrate is based on the crest elevation of the culvert exit, which is equal to 9.50 m + 0.38 m = 9.88 m. Hence the discharge relation is given by

$$Q = 0.236\sqrt{(HW - 9.88)}$$

For $HW = 11.62$ m,

$$Q = 0.236\sqrt{(11.62 - 9.88)} = 0.31 \text{ m}^3/\text{s}$$

Therefore the culvert capacity increases by 0.01 m^3/s or $\boxed{3\%}$.

3.96. From the given data: culvert dimensions = 1.5 m × 1.5 m, $n = 0.013$, $C_d = 0.95$, $k_e = 0.05$, $S_o = 0.007$, and $L = 40$ m.

(a) Free outlet conditions. Assume Type 2 flow, where

$$\Delta h = 0.5 \text{ m} + 0.007(40) = 0.78 \text{ m}$$

and therefore

$$\begin{aligned} Q &= A\sqrt{\frac{2g\Delta h}{2gn^2\frac{L}{R^{4/3}} + k_e + 1}} = (1.5)^2\sqrt{\frac{2(9.81)(0.78)}{2(9.81)(0.013)^2\frac{40}{(1.5^2/6)^{4/3}} + 0.05 + 1}} \\ &= 7.08 \text{ m}^3/\text{s} \end{aligned}$$

Determine if the culvert flows full. Calculate the normal depth, y_n, using the Manning equation,

$$\begin{aligned} Q &= \frac{1}{n}\frac{A_n^{5/3}}{P_n^{2/3}}S_o^{1/2} \\ 7.08 &= \frac{1}{0.013}\frac{(1.5y_n)^{5/3}}{(1.5+2y_n)^{2/3}}(0.007)^{1/2} \\ 1.33 &= \frac{(1.5y_n)^5}{(1.5+2y_n)^2} \\ y_n &= 1.22 \text{ m} \end{aligned}$$

Therefore, the assumption that the culvert flows full (Type 2 flow) is not supported. Assuming Type 3 flow,

$$Q = C_d A\sqrt{2gh} = 0.95(2.25)\sqrt{2(9.81)(0.75 + 0.5)} = 10.6 \text{ m}^3/\text{s}$$

Determine if the culvert flows full. Calculate the normal depth, y_n, using the Manning equation,

$$\begin{aligned} Q &= \frac{1}{n}\frac{A_n^{5/3}}{P_n^{2/3}}S_o^{1/2} \\ 10.6 &= \frac{1}{0.013}\frac{(1.5y_n)^{5/3}}{(1.5+2y_n)^{2/3}}(0.007)^{1/2} \\ 4.468 &= \frac{(1.5y_n)^5}{(1.5+2y_n)^2} \\ y_n &= 1.7 \text{ m} \end{aligned}$$

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Therefore, the assumption that the culvert does not flow full (Type 3 flow) is not supported. The flow is somewhere between Type 2 and Type 3 flow, and the discharge is in the range $\boxed{7.08 \text{ to } 10.6 \text{ m}^3/\text{s}}$.

(b) For a submerged outlet with tailwater 0.5 m above the crown of the culvert at the exit,

$$\Delta h = S_o L = 0.007(40) = 0.28 \text{ m}$$

and

$$\begin{aligned} Q &= A\sqrt{\frac{2g\Delta h}{2gn^2\frac{L}{R^{4/3}} + k_e + 1}} = (1.5)^2\sqrt{\frac{2(9.81)(0.28)}{2(9.81)(0.013)^2\frac{40}{(1.5^2/6)^{4/3}} + 0.05 + 1}} \\ &= \boxed{4.25 \text{ m}^3/\text{s}} \end{aligned}$$

Find Δh for $Q = 10.6$ m^2/s, where

$$\Delta h = \left[\frac{2gn^2L}{R^{4/3}} + k_e + 1\right]\frac{Q^2}{2gA^2} = \left[\frac{2(9.81)(0.013)^2(40)}{(2.25/6)^{4/3}} + 0.05 + 1\right]\frac{10.6^2}{2(9.81)(2.25)^2} = 1.74 \text{ m}$$

So the headwater must be $1.5 + 0.5 + 1.74 - 0.28 = 3.46$ m above the inflow-channel invert. For $Q = 7.08$ m^3/s, $\Delta h = 0.78$ m. Therefore the range of headwater elevations is from $\boxed{2.50 \text{ to } 3.46 \text{ m}}$ above the channel invert.

3.97. From the given data: $Q = 0.80$ m^3/s, $n = 0.013$, $H = 1$ m, $S_o = 2\% = 0.02$, $L = 10$ m, $k_e = 0.5$, and $C_d = 1$. The data indicate that the difference between the headwater elevation and the crown of the culvert exit is given by

$$\Delta h = LS_o + H - D = (10)(0.02) + 1 - D = 1.2 - D$$

where D is the culvert diameter. Assume the flow is either Type 2 or Type 3 flow. For Type 2 flow, Equation 3.228 gives

$$0.80 = \frac{\pi}{4}D^2\sqrt{\frac{2g(1.2 - D)}{2(9.81)(0.013)^2(10)/(D/4)^{4/3} + 0.5 + 1}}$$

which simplifies to

$$1.02 = D^2\sqrt{\frac{19.62D^{4/3}(1.2 - D)}{0.211 + 1.5D^{4/3}}}$$

Solving iteratively for D gives

$$D = 0.65 \text{ m}$$

Since $H > 1.2D$, inlet submergence verified. It must also be verified that the normal depth of flow is greater than the culvert diameter. When the culvert just flows full,

$$Q = \frac{1}{n}AR^{2/3}S_o^{1/2}$$

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where $n = 0.013$, $A = \pi(0.65)^2/4 = 0.332$ m^2, $R = D/4 = 0.65/4 = 0.163$ m, $S_o = 0.02$, and

$$Q_{\text{full}} = \frac{1}{0.013}(0.332)(0.163)^{2/3}(0.02)^{1/2} = 1.08 \text{ m}^3/\text{s}$$

Therefore, when the flow is 0.80 m^3/s and the diameter is 0.65 m, the culvert does not flow full. This implies Type 3 flow, where

$$Q = C_d A\sqrt{2gh}$$

where $C_d = 1$ and

$$0.80 = (1)\frac{\pi}{4}D^2\sqrt{2(9.81)(1 - D/2)}$$

which yields

$$D = 0.52 \text{ m}$$

Since $A = \pi(0.52)^2/4 = 0.212$ m^2, $R = D/4 = 0.52/4 = 0.13$ m, then the full-flow discharge is given by the Manning equation as

$$Q_{\text{full}} = \frac{1}{0.013}(0.212)(0.13)^{2/3}(0.02)^{1/2} = 0.59 \text{ m}^3/\text{s}$$

and the maximum flowrate in the culvert is $1.07(0.59) = 0.63$ m^3/s. Therefore the culvert flows full and the flow is not Type 3. Since the flow is between Type 2 and Type 3, select the larger culvert diameter of $\boxed{\text{65 cm}}$.

3.98. From the given data: $H = 2$ m, TW $= 1$ m, $Q = 1$ m^3/s, $L = 15$ m, and $S_o = 0.015$. Assuming Type 1 flow, then Equation 3.228 gives

$$Q = A\sqrt{\frac{2g\Delta h}{2gn^2L/R^{4/3} + k_e + 1}} \tag{1}$$

where $A = \pi D^2/4 = 0.785D^2$, $\Delta h = H - \text{TW} + S_oL = 2 - 1 + (15)(0.015) = 1.225$ m, $n = 0.013$, $R = D/4 = 0.25D$, and taking $k_e = 0.50$, Equation 1 gives

$$\begin{aligned} 1 &= 0.785D^2\sqrt{\frac{2(9.81)(1.225)}{2(9.81)(0.013)^2(15)/(0.25D)^{4/3} + 0.50 + 1}} \\ 1 &= 3.85D^2\sqrt{\frac{1}{0.316D^{-4/3} + 1.50}} \end{aligned}$$

Solving gives $D = 0.61$ m. This indicates that the exit is submerged, confirming Type 1 flow. Use a culvert with $\boxed{D = 61 \text{ cm}}$.

3.99. From the given data: $b = 2$ m, $Q = 4$ m^3/s, $S_o = 0.1$, $L = 25$ m, $k_e = 0.1$, and for a concrete culvert $n = 0.013$. Find the normal depth of flow in the culvert using the Manning equation,

$$\begin{aligned} Q &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2} \\ 4 &= \frac{1}{0.013}\frac{(2y_n)^{5/3}}{(2 + 2y_n)^{2/3}}(0.1)^{1/2} \end{aligned}$$

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which gives

$$y_n = 0.24 \text{ m}$$

Find the critical depth, y_c, where

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$
$$\frac{4^2}{9.81} = \frac{(2y_c)^3}{2}$$

which gives

$$y_c = 0.74 \text{ m}$$

Therefore the culvert slope is *steep*. Assume Type 5 flow. Applying Equation 3.235 (neglecting the upstream velocity) gives

$$Q = A_c\sqrt{2g(\Delta h + V_1^2/2g - h_i)}$$
$$4 = (0.74 \times 2)\sqrt{2(9.81)\left(\Delta h - 0.1 \times \frac{4^2}{2(9.81)(2 \times 0.74)^2)}\right)}$$

which gives

$$\Delta h = 0.41 \text{ m}$$

Since y_c occurs at about $1.4y_c$ downstream of the culvert entrance, the headwater depth is $y_c + \Delta h - S_o(1.4y_c) = 0.74 + 0.41 - 0.1(1.4 \times 0.74) = \boxed{1.05 \text{ m}}$. Since the entrance is unsubmerged, Type 5 flow is confirmed.

3.100. The relationship between the depth of flow, y_1, at the culvert entrance and the depth of flow, y_2 at the culvert exit can be estimated using the standard-step method, where

$$L = \frac{\left[y + \frac{Q^2}{2gA^2}\right]_2^1}{\bar{S}_f - S_o}$$

which can be put in the form

$$\frac{L}{2}(S_{f1} + S_{f2} - 2S_o) = y_1 + \frac{Q^2}{2gA_1^2} - y_2 - \frac{Q^2}{2gA_2^2} \qquad (1)$$

where

$$S_{f1} = \left[\frac{nQ}{A_1R_1^{2/3}}\right]^2 \qquad (2)$$
$$S_{f2} = \left[\frac{nQ}{A_2R_2^{2/3}}\right]^2 \qquad (3)$$

In this case,

$$A_1R_1^{2/3} = \frac{A_1^{5/3}}{P_1^{2/3}} = \frac{(2y_1)^{5/3}}{(2+2y_1)^{2/3}} \qquad (4)$$
$$A_2R_2^{2/3} = \frac{A_2^{5/3}}{P_2^{2/3}} = \frac{(2y_2)^{5/3}}{(2+2y_2)^{2/3}} \qquad (5)$$

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Combining Equations (2) and (4), and taking $n = 0.013$ and $Q = 10$ m^3/s gives

$$S_{f1} = \left[(0.013)(10)\frac{(2+2y_1)^{2/3}}{(2y_1)^{5/3}}\right]^2 = 0.00168\frac{(2+2y_1)^{4/3}}{y_1^{10/3}} \tag{6}$$

and similarly

$$S_{f2} = 0.00168\frac{(2+2y_2)^{4/3}}{y_2^{10/3}} \tag{7}$$

The culvert tailwater depth, y_2, satisfies the Manning equation in the downstream channel, hence

$$Q = \frac{1}{n}AR^{2/3}S_o^{1/2} = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2} \tag{8}$$

where $Q = 10$ m^3/s, $n = 0.022$, $S_o = 0.005$, $b = 5$ m, $m = 2$, and

$$\begin{aligned} A &= by_2 + my_2^2 = 5y_2 + 2y_2^2 \\ P &= b + 2\sqrt{1+m^2}y_2 = 5 + 2\sqrt{1+2^2}y_2 = 5 + 4.47y_2 \end{aligned}$$

Substituting into Equation (8) gives

$$10 = \frac{1}{0.022}\frac{(5y_2+2y_2^2)^{5/3}}{(5+4.47y_2^{2/3}}(0.005)^{1/2}$$

which yields

$$y_2 = 0.713 \text{ m} \tag{9}$$

From the given data, $L = 10$ m, and combining Equations (1), (6), (7), and (9) gives

$$\begin{aligned} &\frac{10}{2}\left[0.00168\frac{(2+2y_1)^{4/3}}{y_1^{10/3}} + 0.00168\frac{(2+2\times 0.713)^{4/3}}{0.713^{10/3}} - 2(0.005)\right] \\ &\qquad = y_1 + \frac{10^2}{2(9.81)(2y_1)^2} - 0.713 - \frac{10^2}{2(9.81)(2\times 0.713)^2} \end{aligned}$$

which yields

$$y_1 = 0.677 \text{ m}$$

Therefore, the depth of flow at the culvert entrance is $\boxed{0.68 \text{ m}}$ and the depth of flow at the culvert exit is $\boxed{0.71 \text{ m}}$. Since the culvert is 2 m × 2 m, then the culvert has $\boxed{\text{adequate capacity}}$ when the flow is 10 m^3/s. It is interesting to note that the critical flow depth in the culvert is 1.36 m, the normal flow depth is 1.33 m, and hence there is a S3 water surface profile in the culvert.

3.101. A flow scenario in which the culvert entrance is not submerged and the exit is submerged is illustrated below:

The flow capacity of the culvert could be calculated as follows: (1) assume a flow, Q; (2) use backwater calculations to find where the water surface intersects the top of the pipe; (3) using this intersection point and the tailwater elevation, calculate the flow through the pressurized portion of the pipe; (4) repeat steps 1 to 3 until the flow in step 1 equals the flow in step 3. This would yield the capacity of the culvert under the given headwater and tailwater conditions.