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This condition is never satisfied. Taking $n = 0.015$ requires that

$$(0.015)^6\sqrt{\left(\frac{5y}{2y+5}\right)(0.0005)} \geq 9.6 \times 10^{-14}$$

which yields

$$y = 0.15 \text{ m}$$

Therefore, the minimum flow depth for fully turbulent flow $\boxed{\text{varies}}$ depending on the roughness coefficient of the channel. If the channel is smooth ($n = 0.011$), then the flow is never fully turbulent, while if the channel is rough ($n = 0.015$), fully turbulent flow occurs when the flow depth is greater than or equal to 0.15 m.

3.8. The Darcy-Weisbach equation gives the average velocity, V, as

$$V = \sqrt{\frac{8g}{f}}\sqrt{RS_o}$$

From the given data, $y = 2.20$ m, $k_s = 2$ mm $= 0.002$ m, $b = 3.6$ m, $m = 2$, $S_o = 0.0006$, and hence the flow area, A, wetted perimeter, P, and hydraulic radius, R, are given by

$$\begin{aligned} A &= by + my^2 = (3.6)(2.20) + (2)(2.20)^2 = 17.6 \text{ m}^2 \\ P &= b + 2\sqrt{1+m^2}y = 3.6 + 2\sqrt{1+2^2}(2.20) = 13.4 \text{ m} \\ R &= \frac{A}{P} = \frac{17.6}{13.4} = 1.31 \text{ m} \end{aligned}$$

Assuming the flow is fully turbulent the friction factor, f, can be estimated using Equation 3.28 where

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left[\frac{12R}{k_s}\right] = 2\log_{10}\left[\frac{12(1.31)}{0.002}\right] = 7.79$$

which leads to

$$f = 0.016$$

The mean velocity can now be estimated as

$$V = \sqrt{\frac{8g}{f}}\sqrt{RS_o} = \sqrt{\frac{8(9.81)}{0.016}}\sqrt{(1.31)(0.0006)} = 1.96 \text{ m/s}$$

and the corresponding flowrate, Q, is given by

$$Q = AV = (17.6)(1.96) = 34.5 \text{ m}^3/\text{s}$$

This flowrate was obtained by assuming that the flow in the channel is hydraulically rough (fully turbulent), in which case the friction factor does not depend on the Reynolds number of the flow. This assumption can now be checked by re-calculating the friction factor using the calculated flowrate. At 20°C, the kinematic viscosity, ν, of water is 1.00×10^{-6} m^2/s, and the Reynolds number, Re, is therefore given by

$$\text{Re} = \frac{V(4R)}{\nu} = \frac{(1.96)(4 \times 1.31)}{1.00 \times 10^{-6}} = 1.03 \times 10^7$$

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The friction factor can now be estimated by the general expression for the friction factor given by Equation 3.29 where

$$\begin{aligned}\frac{1}{\sqrt{f}} &= -2\log_{10}\left[\frac{k_s}{12R} + \frac{2.5}{\text{Re}\sqrt{f}}\right] \\ &= -2\log_{10}\left[\frac{0.002}{12(1.31)} + \frac{2.5}{1.03\times 10^7\sqrt{f}}\right] \\ &= -2\log_{10}\left[1.27\times 10^{-4} + \frac{2.43\times 10^{-7}}{\sqrt{f}}\right]\end{aligned}$$

which by trial and error yields

$$f = 0.016$$

Since this is the same friction factor as originally estimated, the flow is indeed hydraulically rough and the estimated velocity and flowrate are $\boxed{1.96 \text{ m/s}}$ and $\boxed{34.5 \text{ m}^3\text{/s}}$.

The Manning's equation gives the average velocity, V, as

$$V = \frac{1}{n}R^{2/3}S_o^{1/2}$$

Table 3.1 indicates that a mid-range roughness coefficient for concrete is $n = 0.015$. The average velocity given by the Manning equation is

$$V = \frac{1}{0.015}(1.31)^{2/3}(0.0006)^{1/2} = \boxed{1.96 \text{ m/s}}$$

and the corresponding flowrate, Q, is

$$Q = AV = (17.6)(1.96) = \boxed{34.5 \text{ m}^3\text{/s}}$$

Hence, in this case, the Darcy-Weisbach and Manning equations give the $\boxed{\text{same results}}$.

The Manning equation can be taken as valid when $n^6\sqrt{RS_o} \geq 9.6\times 10^{-14}$ and $2.5 < R/d < 250$, where d is the characteristic roughness height corresponding to $n = 0.015$. These conditions are required for fully turbulent flow conditions to exist and for $n/d^{1/6}$ to be approximately constant. In this case,

$$n^6\sqrt{RS_o} = (0.015)^6\sqrt{(1.31)(0.0006)} = 3.19\times 10^{-13}$$

and, taking $d = (n/0.038)^6 = (0.015/0.038)^6 = 0.0038$ m gives

$$\frac{R}{d} = \frac{1.31}{0.0038} = 345$$

These former result indicates that the flow is fully turbulent, and the latter result indicates that $n/d^{1/6}$ may not be constant in this case and therefore the Manning equation may not be strictly applicable.

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3.9. $S_o = 0.0001$, $k_s = 1$ mm $= 0.001$ m, $Q = 18$ m^3/s.

$$\begin{aligned} A &= 5y + 2y^2 \\ P &= 5 + 2\sqrt{5}y \end{aligned}$$

which gives

$$R = \frac{A}{P} = \frac{5y + 2y^2}{5 + 2\sqrt{5}y} \qquad (1)$$

Assume the flow is fully turbulent, then

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k_s}{12R}\right) = -2\log_{10}\left(\frac{0.001}{12R}\right) \qquad (2)$$

The Darcy-Weisbach uniform-flow equation is

$$Q = A\sqrt{\frac{8g}{f}RS_o}$$

which can be written as

$$15 = (5y + 2y^2)\sqrt{\frac{8(9.81)}{f}R(0.0001)} = 0.0886(5y + 2y^2)\sqrt{\frac{R}{f}} \qquad (3)$$

Solving Equations 1 to 3 simultaneously yields $y = 2.18$ m, $R = 1.38$ m, and $f = 0.014$. Since $V = Q/A = 18/20.4 = 0.88$ m/s (where $A = 20.4$ m^2), then the Reynolds number, Re, can be estimated by

$$\text{Re} = \frac{V(4R)}{\nu} = \frac{(0.88)(4 \times 1.38)}{1.00 \times 10^{-6}} = 4.85 \times 10^6$$

According to the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k_s}{12R} + \frac{2.5}{\text{Re}\sqrt{f}}\right)$$

which leads to

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{0.001}{12 \times 1.38} + \frac{2.5}{4.85 \times 10^6\sqrt{f}}\right)$$

and solving for f gives

$$f = 0.014$$

Since this is the same value of f obtained by assuming fully turbulent flow, then fully turbulent flow is verified and the uniform depth of flow is $\boxed{2.18 \text{ m}}$. These results indicate that the flow is $\boxed{\text{hydraulically rough}}$.

Comparing the Manning and Darcy-Weisbach equation gives the following relation between the Manning roughness coefficient, n, and the Darcy friction factor, f,

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

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Taking $g = 9.81$ m/s^2, $f = 0.014$, and $R = 1.38$ m yields $n = 0.014$. To be valid, the Manning equation requires fully turbulent flow conditions where $n^6\sqrt{RS_o} \geq 9.6 \times 10^{-14}$. In this case

$$n^6\sqrt{RS_o} = (0.014)^6\sqrt{(1.38)(0.0001)} = 9.2 \times 10^{-14}$$

Since $n^6\sqrt{RS_o} < 9.6 \times 10^{-14}$, $\boxed{\text{the Manning equation is not valid}}$.

3.10. Comparing the Manning and Darcy-Weisbach equations

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

which gives

$$n = \frac{\sqrt{f}R^{1/6}}{\sqrt{8g}} = \frac{f^{1/2}R^{1/6}}{\sqrt{8(9.81)}} = \frac{f^{1/2}R^{1/6}}{8.86}$$

If the friction factor, f, is taken as a constant, the above relation indicates that n must also be a function of the depth (since R is a function of the depth). However, in fully-turbulent flow conditions f is certainly not a constant, and Williamson (1951) has shown that f is proportional to $R^{-1/3}$ (see Equation 3.43). Taking $f \sim R^{-1/3}$, n would be a constant in the above equation. So the answer to the question is $\boxed{\text{no}}$.

3.11. Given: $Q = 20$ m^3/s, $n = 0.015$, $S_o = 0.01$

(a) Manning equation is given by

$$Q = \frac{1}{n}A_nR_n^{2/3}S_o^{1/2} = \frac{1}{n}\frac{A_n^{5/3}}{P_n^{2/3}}S_o^{1/2}$$

where

$$\begin{aligned} A_n &= [b + my_n]y_n = [2.8 + 2y_n]y_n \\ P_n &= b + 2\sqrt{1+m^2}y_n = 2.8 + 2\sqrt{5}y_n = 2.8 + 4.472y_n \end{aligned}$$

Substituting into the Manning equation yields

$$20 = \frac{1}{0.015}\frac{[(2.8 + 2y_n)y_n]^{5/3}}{(2.8 + 4.472y_n)^{2/3}}(0.01)^{1/2}$$

or

$$\frac{[(2.8 + 2y_n)y_n]^{5/3}}{(2.8 + 4.472y_n)^{2/3}} = 3.0$$

Solving by trial and error yields

$$\boxed{y_n = 0.91 \text{ m}}$$

(b) Comparing the Manning and Darcy-Weisbach equations gives

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

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which leads to

$$f = \frac{8gn^2}{R^{1/3}}$$

In this case

$$\begin{aligned} A &= (2.8 + 2y)y = (2.8 + 2 \times 0.91)(0.91) = 4.2 \text{ m}^2 \\ P &= 2.8 + 4.472(0.91) = 6.87 \text{ m} \\ R &= \frac{A}{P} = \frac{4.20}{6.87} = 0.611 \text{ m} \end{aligned}$$

therefore

$$f = \frac{8(9.81)(0.015)^2}{(0.611)^{1/3}} = 0.0208$$

For fully turbulent, where the Manning equation applies,

$$\begin{aligned} \frac{1}{\sqrt{f}} &= -2\log\left[\frac{k_s}{12R}\right] \\ \frac{1}{\sqrt{0.0208}} &= -2\log\left[\frac{k_s}{12(0.611)}\right] \\ 6.93 &= -2\log[0.136k_s] \end{aligned}$$

which leads to

$$k_s = 0.00249 \text{ m} = \boxed{2.5 \text{ mm}}$$

3.12. From the given information,

$$n = 0.039d^{1/6}$$

where d is in ft. In this case, $d = 30$ mm $= 0.09843$ ft, and a 70% error in d is $0.7(0.09843) = 0.06890$ ft. Hence, $d = 0.09843$ ft $\pm$ 0.06890 ft. Hence, the estimated value of n, $\bar{n}$, is given by

$$\bar{n} = 0.039(0.09843)^{1/6} = 0.027$$

The lower estimate of n, n_L, is given by

$$n_L = 0.039(0.09843 - 0.06890)^{1/6} = 0.022$$

and the upper estimate of n, n_U, is given by

$$n_U = 0.039(0.09843 + 0.06890)^{1/6} = 0.029$$

The maximum percentage error in estimating n is therefore given by

$$\text{error} = \frac{0.027 - 0.022}{0.027} \times 100 = \boxed{19\%}$$

3.13. According to Equation 3.50,

$$\frac{n}{k_s^{1/6}} = \frac{\frac{1}{\sqrt{8g}}\left(\frac{R}{k_s}\right)^{\frac{1}{6}}}{2.0\log\left(12\frac{R}{k_s}\right)} \qquad (1)$$