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The energy loss, ΔE, is given by

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(5.52 - 0.9)^3}{4(5.52)(0.9)} = 4.96 \text{ m}$$

and the initial energy, E_1, is given by

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.9 + \frac{(12.5/0.9)^2}{2(9.81)} = 10.73 \text{ m}$$

Therefore, the fraction of initial energy lost is

$$\frac{\Delta E}{E_1} = \frac{4.96}{10.73} = \boxed{0.462}$$

3.41. The head loss, h_L, is defined by

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

Dividing by y_1 gives

$$1 + \frac{V_1^2}{2gy_1} = \frac{y_2}{y_1} + \frac{V_2^2}{2gy_1} + \frac{h_L}{y_1}$$

which can be put in the form

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{V_1^2}{2gy_1} - \frac{V_2^2}{2gy_1} \tag{1}$$

Define

$$\text{Fr}_1^2 = \frac{V_1^2}{gy_1} \tag{2}$$

Combining Equations 1 and 2 gives

$$\begin{aligned}
\frac{h_L}{y_1} &= 1 - \frac{y_2}{y_1} + \frac{\text{Fr}_1^2}{2}\left[1 - \frac{V_2^2}{gy_1\text{Fr}_1^2}\right] \\
&= 1 - \frac{y_2}{y_1} + \frac{\text{Fr}_1^2}{2}\left[1 - \frac{V_2^2}{gy_1\left(\frac{V_1^2}{gy_1}\right)}\right] \\
&= 1 - \frac{y_2}{y_1} + \frac{\text{Fr}_1^2}{2}\left[1 - \frac{V_2^2}{V_1^2}\right]
\end{aligned}$$

Since $V_1 = q/y_1$ and $V_2 = q/y_2$, then

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\text{Fr}_1^2}{2}\left[1 - \frac{(q/y_2)^2}{(q/y_1)^2}\right]$$

which simplifies to

$$\boxed{\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\text{Fr}_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]}$$

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3.42. The general hydraulic jump equation is given by

$$\frac{Q^2}{gA} + A\bar{y} = \text{constant} \tag{1}$$

For a trapezoidal channel,

$$\bar{y} = \frac{\left(\frac{by^2}{2}\right) + \left(\frac{my^3}{3}\right)}{by + my^2} \tag{2}$$

Combining Equations 1 and 2 yields

$$\frac{Q^2}{g(by+my^2)} + (by+my^2)\frac{\left(\frac{by^2}{2}\right) + \left(\frac{my^3}{3}\right)}{by + my^2} = \text{constant}$$

which simplifies to

$$\frac{Q^2}{gy(b+my)} + \left(\frac{by^2}{2}\right) + \left(\frac{my^3}{3}\right) = \text{constant}$$

which demonstrates that

$$\boxed{\frac{Q^2}{gy_1(b+my_1)} + \left(\frac{by_1^2}{2}\right) + \left(\frac{my_1^3}{3}\right) = \frac{Q^2}{gy_2(b+my_2)} + \left(\frac{by_2^2}{2}\right) + \left(\frac{my_2^3}{3}\right)}$$

3.43. From the given data: $Q = 21$ m^3/s, $b = 2$ m, $m = 1$, and $y_1 = 1$ m. The momentum equations requires that

$$\frac{Q^2}{gA_1} + A_1\bar{y}_1 = \frac{Q^2}{gA_2} + A_2\bar{y}_2 \tag{1}$$

where

$$\begin{aligned}
A_1 &= by_1 + y_1^2 = 2(1) + 1^2 = 3 \text{ m}^2 \\
V_1 &= \frac{Q}{A_1} = \frac{21}{3} = 7 \text{ m/s} \\
A_2 &= by_2 + y_2^2 = 2y_2 + y_2^2 \\
V_2 &= \frac{Q}{A_2} = \frac{21}{2y_2 + y_2^2}
\end{aligned}$$

and

$$\begin{aligned}
\bar{y}_1 &= \frac{\sum A\bar{y}}{\sum A} = \frac{1(2)(0.5) + \frac{(1)1}{2}\frac{1}{3} + \frac{(1)1}{2}\frac{1}{3}}{3} = 0.44 \text{ m} \\
\bar{y}_2 &= \frac{2y_2\left(\frac{y_2}{2}\right) + 2\left[\frac{1}{2}y_2^2\frac{y_2}{3}\right]}{2y_2 + y_2^2} = \frac{y_2 + y_2^2/3}{2 + y_2}
\end{aligned}$$

Substituting into Equation 1 gives

$$\frac{21^2}{9.81(3)} + 3(0.44) = \frac{21^2}{9.81(2y_2 + y_2^2)} + y_2^2 + \frac{y_2^3}{3}$$

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which yields

$$\boxed{y_2 = 2.59 \text{ m}}$$

The energy equation gives the energy loss, ΔE, as

$$\Delta E = y_1 + \frac{V_1^2}{2g} - y_2 - \frac{V_2^2}{2g} = 1 + \frac{7^2}{2(9.81)} - 2.59 - \frac{\left(\frac{21}{2\times2.59+2.59^2}\right)^2}{2(9.81)} = \boxed{0.748 \text{ m}}$$

3.44. From the given data: $m = 2$, $Q = 0.30$ m^3/s, $y = 15$ cm, and

$$\begin{aligned}
A &= my^2 = 2(0.15)^2 = 0.045 \text{ m}^2 \\
T &= 2my = 2(2)(0.15) = 0.6 \text{ m} \\
D &= \frac{A}{T} = \frac{0.045}{0.6} = 0.075 \text{ m} \\
V &= \frac{Q}{A} = \frac{0.30}{0.045} = 6.67 \text{ m/s} \\
\text{Fr} &= \frac{V}{\sqrt{gD}} = \frac{6.67}{\sqrt{(9.81)(0.075)}} = 7.78
\end{aligned}$$

Since $\boxed{\text{Fr} = 7.78} > 1$, the flow is supercritical.

The hydraulic jump equation is the same as for a trapezoidal channel with $b = 0$, hence

$$\begin{aligned}
\frac{my_1^3}{3} + \frac{Q^2}{gmy_1^2} &= \frac{my_2^3}{3} + \frac{Q^2}{gmy_2^2} \\
\frac{m(0.15)^3}{3} + \frac{(0.30)^2}{(9.81)(2)(0.15)^2} &= \frac{2y_2^3}{3} + \frac{(0.30)^2}{(9.81)(2)y_2^2} \\
0.219 &= 0.667y_2^3 + \frac{0.00459}{y_2^2}
\end{aligned}$$

which yields

$$y_2 = 0.679 \text{ m} \quad \text{or} \quad 0.145 \text{ m}$$

Since the downstream flow is subcritical, $\boxed{y_2 = 0.679 \text{ m}}$.

3.45. From given data: $Q = 10$ m^3/s, $b = 5.5$ m, $S_o = 0.0015$, $n = 0.038$, $y_2 = 2.2$ m.

(a) Using the direct-integration method,

$$\begin{aligned}
y_1 &= y_2 - \frac{S_o - \left(\frac{nQ\bar{P}^{2/3}}{\bar{A}^{5/3}}\right)^2}{1 - \frac{\bar{V}^2}{g\bar{y}}}(x_2 - x_1) \\
&= y_2 - \frac{S_o - \left(\frac{nQ(b+2\bar{y})^{2/3}}{(b\bar{y})^{5/3}}\right)^2}{1 - \frac{Q^2}{gb^2\bar{y}^3}}(x_2 - x_1) \\
&= 2.2 - \frac{0.0015 - \left(\frac{(0.038)(10)(5.5+2\bar{y})^{2/3}}{(5.5\bar{y})^{5/3}}\right)^2}{1 - \frac{10^2}{(9.81)(5.5)^2\bar{y}^3}}(100 - 0) \qquad (1)
\end{aligned}$$