

3.31. Flow in a rectangular open channel is choked when

$$\begin{aligned} E_1 &= E_2 + \Delta z_c \\ y_1 + \frac{V_1^2}{2g} &= \frac{3}{2}y_c + \Delta z_c \\ y_1 + \frac{V_1^2}{2g} &= \frac{3}{2} \left(\frac{q^2}{g} \right)^{1/3} + \Delta z_c \\ y_1 + \frac{V_1^2}{2g} &= \frac{3}{2} \left(\frac{Q^2}{gb^2} \right)^{1/3} + \Delta z_c \\ y_1 + \frac{V_1^2}{2g} &= \frac{3}{2} \left[\frac{(V_1 b y_1)^2}{gb^2} \right]^{1/3} + \Delta z_c \\ y_1 + \frac{V_1^2}{2g} &= \frac{3}{2} \frac{(V_1^{2/3} y_1^{2/3})}{g^{1/3}} + \Delta z_c \end{aligned}$$

Dividing by y_1 yields

$$1 + \frac{V_1^2}{2gy_1} = \frac{3}{2} \left(\frac{V_1^2}{gy_1} \right)^{1/3} + \frac{\Delta z_c}{y_1} \quad (1)$$

and defining

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$

then Equation 1 can be written as

$$\boxed{\frac{\Delta z_c}{y_1} = 1 + \frac{Fr_1^2}{2} - \frac{3}{2}Fr_1^{2/3}}$$

From Problem 3.30: $b = 3$ m, $Q = 4$ m³/s, $y_1 = 1.5$ m, and $\Delta z_c = 0.15$ m. Therefore

$$\begin{aligned} V_1 &= \frac{Q}{by_1} = \frac{4}{(3)(1.5)} = 0.889 \text{ m/s} \\ Fr_1 &= \frac{V_1}{\sqrt{gy_1}} = \frac{0.889}{\sqrt{(9.81)(1.5)}} = 0.232 \end{aligned}$$

which yields

$$\begin{aligned} \frac{\Delta z_c}{y_1} &= 1 + \frac{Fr_1^2}{2} - \frac{3}{2}Fr_1^{2/3} \\ \frac{\Delta z_c}{1.5} &= 1 + \frac{(0.232)^2}{2} - \frac{3}{2}(0.232)^{2/3} \end{aligned}$$

and solving for Δz_c gives

$$\boxed{\Delta z_c = 0.69 \text{ m}}$$