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and the average energy slope, $\bar{S}$, between sections 1 and 2 is given by

$$\bar{S} = \frac{1}{2}(S_1 + S_2) = \frac{1}{2}(0.4193 + 2247) = 0.3220$$

Applying the energy equation using the direct step method yields

$$\begin{aligned} L &= \frac{\left[y + \frac{V^2}{2g}\right]_2^1}{\bar{S} - S_o} \\ &= \frac{\left[0.305 + \frac{16.4^2}{2(9.81)}\right] - \left[0.376 + \frac{13.3^2}{2(9.81)}\right]}{0.3220 - 0} \\ &= 14.4 \text{ m} \end{aligned}$$

Therefore, the hydraulic jump occurs $\boxed{14.4 \text{ m}}$ downstream of the gate.

Installation of baffle blocks or other energy-dissipation structures to increase friction losses at the gate outlet would cause a drowned hydraulic jump to occur at the gate.

3.89. From the given data: $y_g = 0.20$ m, $y_1 = 1.7$ m, and $b = 1.5$ m. Assuming $C_c = 0.61$, Equation 3.216 gives

$$\eta = C_c \frac{y_g}{y_1} = 0.61 \frac{0.20}{1.7} = 0.0718$$

and, for free-flow conditions, Equation 3.211 gives the discharge coefficient, C_d, as

$$C_d = \frac{C_c}{\sqrt{1+\eta}} = \frac{0.61}{\sqrt{1 + 0.0718}} = 0.589$$

The free-flow discharge through the gate is given by Equation 3.210 as

$$Q = C_d b y_g \sqrt{2gy_1} = 0.589(1.5)(0.20)\sqrt{2(9.81)(1.7)} = \boxed{1.02 \text{ m}^3/\text{s}}$$

The distinguishing condition for the tailwater depth is given by Equation 3.215 as

$$y_3 = \frac{C_c y_g}{2}\left[\sqrt{1 + \frac{16}{\eta(1+\eta)}} - 1\right] = \frac{0.61(0.20)}{2}\left[\sqrt{1 + \frac{16}{0.0718(1 + 0.0718)}} - 1\right] = \boxed{0.82 \text{ m}}$$

Therefore, the flow depth through the gate will be equal to 1.02 m^3/s as long as the tailwater depth is less than or equal to 0.82 m. If the tailwater depth exceeds 0.0.82 m, then the flow through the gate will be submerged and the flow will be reduced.

3.90. The energy equation (Equation 3.217) can be put in the form

$$y = y_1 + \frac{Q^2}{2yb^2y_1^2} - \frac{Q^2}{2yb^2y_2^2} \tag{1}$$

and the momentum equation (Equation 3.218) can be put in the form

$$y^2 = y_3^2 + \frac{2Q^2}{gb^2y_3} - \frac{2Q^2}{gb^2y_2} \tag{2}$$

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Combining Equations (1) and (2) gives

$$y_3^2 + \frac{2Q^2}{gb^2}\left(\frac{1}{y_3} - \frac{1}{y_2}\right) = \left[y_1 + \frac{Q^2}{2gb^2}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)\right]^2$$

which simplifies to

$$\frac{Q^4}{4g^2b^4}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)^2 + \frac{Q^2}{gb^2}\left(\frac{1}{y_1} - \frac{y_1}{y_2^2} - \frac{2}{y_3} + \frac{2}{y_2}\right) + (y_1^2 - y_3^2) = 0 \tag{3}$$

Define the following relations

$$\xi = \left(\frac{1}{\eta} - 1\right)^2 + 2(\lambda - 1) \tag{4}$$

$$\lambda = \frac{y_1}{y_3} \tag{5}$$

$$y_2 = C_c y_g \tag{6}$$

$$\eta = C_c\frac{y_g}{y_1} = \frac{y_2}{y_1} \tag{7}$$

Combining Equations (3) to (7) yields

$$\left[\frac{\eta^2}{4C_c^2y_g^2}\left(\frac{1}{\eta^2} - 1\right)^2\right]\left(\frac{Q^2}{gb^2y_1}\right)^2 - \xi\left(\frac{Q^2}{gb^2y_1}\right) + \frac{C_c^2y_g^2}{\eta^2}\left(1 - \frac{1}{\lambda^2}\right) = 0 \tag{8}$$

Equation (8) is a quadratic equation in $Q^2/(gb^2y_1)$, and applying the quadratic formula to Equation (8) gives

$$\frac{Q^2}{gb^2y_1} = \frac{\xi \pm \sqrt{\xi^2 - \left(\frac{1}{\eta^2} - 1\right)^2\left(1 - \frac{1}{\lambda^2}\right)}}{\frac{\eta^2}{2C_c^2y_g^2}\left(\frac{1}{\eta^2} - 1\right)^2}$$

which simplifies to

$$\boxed{Q = C_c\frac{\left[\xi - \sqrt{\xi^2 - \left(\frac{1}{\eta^2} - 1\right)^2\left(1 - \frac{1}{\lambda^2}\right)}\right]^{1/2}}{\frac{1}{\eta} - \eta}by_g\sqrt{2gy_1}}$$

3.91. From the given data: $y_1 = 5$ m, $b = 8$ m, $y_g = 1.40$ m, and $y_3 = 4.0$ m. It must first be determined whether free-flow or submerged flow conditions exist. The distinguishing condition is given by Equation 3.215, where the contraction coefficient, C_c, can be assumed equal to 0.61, and η is given by Equation 3.216 as

$$\eta = C_c\frac{y_g}{y_1} = 0.61\frac{1.40}{5} = 0.17$$

Substituting into Equation 3.215 gives the distinguishing condition as

$$y_3 = \frac{C_cy_g}{2}\left[\sqrt{1 + \frac{16}{\eta(1+\eta)}} - 1\right] = \frac{0.61(1.40)}{2}\left[\sqrt{1 + \frac{16}{0.17(1+0.17)}} - 1\right] = 3.43 \text{ m}$$

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Since the tailwater elevation (= 4 m) exceeds 3.43 m, then the flow is submerged and Equation 3.219 must be used to calculate the flow through the gate. From the given data, Equations 3.220 and 3.221 give

$$\begin{aligned}\lambda &= \frac{y_1}{y_3} = \frac{5}{4} = 1.25 \\ \xi &= \left(\frac{1}{\eta} - 1\right)^2 + 2(\lambda - 1) = \left(\frac{1}{0.17} - 1\right)^2 + 2(1.25 - 1) = 24.34\end{aligned}$$

Substituting into Equation 3.219 gives

$$\begin{aligned}Q &= C_c \frac{\left[\xi - \sqrt{\xi^2 - \left(\frac{1}{\eta^2} - 1\right)^2\left(1 - \frac{1}{\lambda^2}\right)}\right]^{1/2}}{\frac{1}{\eta} - \eta} b y_g \sqrt{2gy_1} \\ &= 0.61 \frac{\left[24.34 - \sqrt{24.34^2 - \left(\frac{1}{0.17^2} - 1\right)^2\left(1 - \frac{1}{1.25^2}\right)}\right]^{1/2}}{\frac{1}{0.17} - 0.17}(8)(1.40)\sqrt{2(9.81)(5)} \\ &= 38.8 \text{ m}^3/\text{s}\end{aligned}$$

Therefore, under the given headwater, tailwater, and gate-opening conditions, the (submerged) discharge through the gate is $\boxed{38.8 \text{ m}^3/\text{s}}$.

3.92. From the given data: $y_1 = 5$ m, $b = 8$ m, and $Q = 50$ m^3/s under free-flow conditions. Taking $C_c = 0.61$, Equation 3.216 gives

$$\eta = C_c \frac{y_g}{y_1} = 0.61 \frac{y_g}{5} = 0.122 y_g$$

and Equation 3.211 gives

$$C_d = \frac{C_c}{\sqrt{1+\eta}} = \frac{0.61}{\sqrt{1 + 1.22 y_g}}$$

and Equation 3.210 gives the (free) flow through the gate as

$$\begin{aligned}Q &= C_d b y_g \sqrt{2gy_1} \\ 50 &= \frac{0.61}{\sqrt{1 + 1.22 y_g}}(8)(y_g)\sqrt{2(9.81)(5)}\end{aligned}$$

which yields

$$\boxed{y_g = 1.10 \text{ m}}$$

For $y_g = 1.10$ m, and $\eta = 1.22(1.10) = 0.134$, the distinguishing condition is given by Equation 3.215 as

$$\begin{aligned}y_3 &= \frac{C_c y_g}{2}\left[\sqrt{1 + \frac{16}{\eta(1+\eta)}} - 1\right] \\ &= \frac{0.61(1.10)}{2}\left[\sqrt{1 + \frac{16}{0.134(1+0.134)}} - 1\right] \\ &= 3.12 \text{ m}\end{aligned}$$

Since the tailwater depth of 4 m exceeds the distinguishing condition ($y_3 = 3.12$ m), then submerged flow conditions exist and the flow through the gate is given by Equation 3.219. In this case, Equations 3.220 and 3.221 give

$$\lambda = \frac{y_1}{y_3} = \frac{5}{4} = 1.25$$

$$\xi = \left(\frac{1}{\eta} - 1\right)^2 + 2(\lambda - 1) = \left(\frac{1}{0.134} - 1\right)^2 + 2(1.25 - 1) = 42.3$$

and Equation 3.219 gives

$$Q = C_c \frac{\left[\xi - \sqrt{\xi^2 - \left(\frac{1}{\eta^2} - 1\right)^2 \left(1 - \frac{1}{\lambda^2}\right)}\right]^{1/2}}{\frac{1}{\eta} - \eta} b y_g \sqrt{2gy_1}$$

$$= 0.61 \frac{\left[42.3 - \sqrt{42.3^2 - \left(\frac{1}{0.134^2} - 1\right)^2 \left(1 - \frac{1}{1.25^2}\right)}\right]^{1/2}}{\frac{1}{0.134} - 0.134} (8)(1.10)\sqrt{2(9.81)(5)}$$

$$= 28.7 \text{ m}^3/\text{s}$$

Therefore, when the tailwater depth is 4 m, the flow through the gate is submerged and equal to $\boxed{28.7 \text{ m}^3/\text{s}}$.

3.93. Since

$$C_c = 1 - 0.75\left(\frac{\theta}{90}\right) + 0.36\left(\frac{\theta}{90}\right)^2$$

To find θ that minimizes C_c,

$$\frac{dC_c}{d\theta} = -\frac{0.75}{90} + 0.72\left(\frac{\theta}{90}\right)\frac{1}{90} = 0$$

which simplifies to

$$-0.00833 + 0.0000889\theta = 0$$

or

$$\theta = 93.7°$$

Since

$$\frac{d^2C_c}{d\theta^2} > 0$$

then $\theta = 93.7°$ is a minimum. Since θ must be between 0° and 90°, then, within this range, the minimum value of C_c occurs at $\boxed{\theta = 90°}$.

3.94. From the given data: $D = 0.5$ m, $n = 0.013$, $k_e = 0.05$, $C_d = 0.95$, $S_o = 0.02$, and $L = 20$ m. The elevation difference, Δh, between the headwater and the top of the culvert at the exit is given by

$$\Delta h = 0.2 + S_oL = 0.2 + (0.02)(20) = 0.6 \text{ m}$$