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where

$$\begin{aligned} A_1 &= (b_1 + m_1 y_1)y_1 = (2 + 2 \times 1)(1) = 4 \text{ m}^2 \\ A_2 &= (b_2 + m_2 y_2)y_2 = (2.5 + 2 \times 1)(1) = 4.5 \text{ m}^2 \\ V_1 &= \frac{Q}{A_1} = \frac{8.4}{4} = 2.10 \text{ m/s} \\ V_2 &= \frac{Q}{A_2} = \frac{8.4}{4.5} = 1.87 \text{ m/s} \end{aligned}$$

Substituting into the energy equation gives

$$1 + \frac{2.10^2}{2(9.81)} - 1 - \frac{1.87^2}{2(9.81)} = (100)(S_f - 0.001)$$

which simplifies to

$$S_f = 0.00147$$

and the head loss, h_L, is given by

$$h_L = LS_f = (100)(0.00147) = \boxed{0.147 \text{ m}}$$

The power, P, dissipated is

$$P = \gamma_w Q h_L = (9.79)(8.4)(0.147) = \boxed{12.1 \text{ kW}}$$

where $\gamma_w = 9.79$ kN/m^3 at 20°C.

3.24. The Darcy-Weisbach equation can be written as

$$h_f = \frac{\bar{f}L}{D}\frac{\bar{V}^2}{2g}$$

Defining

$$S = \frac{h_f}{L} \quad \text{and} \quad \bar{R} = \frac{D}{4}$$

and substituting into the Darcy-Weisbach equation gives

$$S = \frac{\bar{f}}{4R}\frac{\bar{V}^2}{2g}$$

3.25. $Q = 30$ m^3/s, $w = 5$ m, and for a rectangular channel

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

where

$$q = \frac{Q}{w} = \frac{30}{5} = 6 \text{ m}^2/\text{s}$$

Hence

$$y_c = \left(\frac{6^2}{9.81}\right)^{1/3} = \boxed{1.54 \text{ m}}$$

Therefore, when the depth of flow is 3 m, $y_c < 3$ m and the flow is $\boxed{\text{subcritical}}$.

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3.26. From the given data: $Q = 50$ m^3/s, $b = 4$ m, and $m = 1.5$. Under critical flow conditions

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

which gives

$$\frac{50^2}{9.81} = \frac{(4y_c + 1.5y_c^2)^3}{4 + 2(1.5)y_c}$$

Solving by trial and error yields

$$\boxed{y_c = 1.96 \text{ m}}$$

When $y = 3$ m, the Froude number, Fr, is given by the relation

$$\begin{aligned} \text{Fr}^2 &= \frac{Q^2 T}{g A^3} \\ &= \frac{(50)^2(4 + 2 \times 1.5 \times 3)}{(9.81)(4 \times 3 + 1.5 \times 3^2)^3} = 0.19 \end{aligned}$$

hence

$$\boxed{\text{Fr} = 0.45}$$

and the flow is $\boxed{\text{subcritical}}$.

3.27. From the given data: $w_1 = 2$ m, $Q = 3$ m^3/s, $y_1 = 1.2$ m, and $w_2 = w_1 - 0.4$ m $= 1.6$ m. Conservation of energy requires that

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

where

$$\begin{aligned} V_1 &= \frac{Q}{A_1} = \frac{Q}{w_1 y_1} = \frac{3}{(2)(1.2)} = 1.25 \text{ m/s} \\ V_2 &= \frac{Q}{A_2} = \frac{Q}{w_2 y_2} = \frac{3}{1.6 y_2} = \frac{1.875}{y_2} \text{ m/s} \end{aligned}$$

Substituting into the energy equation gives

$$\begin{aligned} 1.2 + \frac{1.25^2}{2(9.81)} &= y_2 + \frac{(1.875/y_2)^2}{2(9.81)} \\ 1.28 &= y_2 + \frac{0.179}{y_2^2} \end{aligned}$$

Solving for y_2 gives

$$y_2 = 0.47 \text{ m}, 1.14 \text{ m}$$

These depths correspond to supercritical and subcritical flow conditions respectively. Since the upstream flow is subcritical, the flow in the constriction must also be subcritical, hence

$$\boxed{y_2 = 1.14 \text{ m}}$$

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When choking occurs at the constriction,

$$y = y_c \quad \text{and} \quad \text{Fr} = 1 = \frac{V}{\sqrt{gD}}$$

and the energy equation gives

$$\begin{aligned} y_1 + \frac{V_1^2}{2g} &= y_c + \frac{V_c^2}{2g} \\ 1.28 &= \left(\frac{q^2}{9.81}\right)^{1/3} + \frac{1}{2(9.81)}\left[\frac{q}{(q^2/9.81)^{1/3}}\right]^2 \end{aligned}$$

which yields

$$q = 2.47 \text{ m}^2/\text{s}$$

and

$$w_2 = \frac{Q}{q} = \frac{3}{2.47} = \boxed{1.21 \text{ m}}$$

3.28. From the given data: $Q = 1$ m^3/s, $b = 1$ m, and $y_1 = 1$ m. The flow is choked when there is critical flow in the constriction. The upstream specific energy, E_1, is given by

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2g(by_1)^2} = 1.0 + \frac{1^2}{2(9.81)(1 \times 1)^2} = 1.05 \text{ m}$$

At the constriction, $\text{Fr}_c^2 = 1$ which leads to

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

Substituting given data

$$\frac{1^2}{9.81} = \frac{(by_c)^3}{b}$$

which leads to

$$(by_c)^2 = \frac{0.102}{y_c} \tag{1}$$

The energy equation requires that

$$\begin{aligned} y_c + \frac{Q^2}{2gA_c^2} &= 1.05 \\ y_c + \frac{(1)^2}{2(9.81)(by_c)^2} &= 1.05 \\ y_c + \frac{0.0510}{(by_c)^2} &= 1.05 \qquad (2) \end{aligned}$$

Combining Equations 1 and 2 gives

$$y_c + \frac{0.0510}{0.102/y_c} = 1.05$$

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or

$$y_c = 0.70 \text{ m}$$

which leads to

$$\boxed{b = 0.55 \text{ m}}$$

3.29. From the given data: $b_1 = 10.0$ m, $y_1 = 1.00$ m, $Q = 8$ m^3/s, $b_2 = 6$ m, and $L = 7$ m.

(a) According to the energy equation

$$E_1 = E_2 + \frac{V_1^2}{2g} \tag{1}$$

where

$$\begin{aligned}
V_1 &= \frac{Q}{b_1 y_1} = \frac{8}{(10.0)(1.00)} = 0.800 \text{ m/s} \\
\frac{V_1^2}{2g} &= \frac{(0.800)^2}{2(9.81)} = 0.0326 \text{ m} \\
E_1 &= y_1 + \frac{V_1^2}{2g} = 1.00 + 0.0326 = 1.0326 \text{ m} \\
E_2 &= y_2 + \frac{Q^2}{2g(b_2 y_2)^2} = y_2 + \frac{8^2}{2(9.81)(6y_2)^2} = y_2 + \frac{0.0906}{y_2^2}
\end{aligned}$$

Substituting into the energy equation, Equation 1, gives

$$1.0326 = y_2 + \frac{0.0906}{y_2^2} + 0.0326$$

which simplifies to

$$1.00 = y_2 + \frac{0.0906}{y_2^2}$$

which yields the following positive solutions

$$y_2 = 0.383 \text{ m}, \quad 0.884 \text{ m}$$

Since

$$\text{Fr}_1^2 = \frac{V_1^2}{gy_1} = \frac{0.800^2}{(9.81)(1.00)} = 0.065$$

the upstream flow is subcritical, and therefore the flow in the constriction must also be subcritical, and hence

$$\boxed{y_2 = 0.884 \text{ m}}$$

(b) To assess the effect of the energy loss, the depth of flow in the constriction must be calculated without including the energy loss. According to the energy equation

$$E_1 = E_2 \tag{2}$$

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where

$$\begin{aligned} E_1 &= 1.0326 \text{ m} \\ E_2 &= y_2 + \frac{0.0906}{y_2^2} \end{aligned}$$

Substituting into the energy equation, Equation 2, gives

$$1.0326 = y_2 + \frac{0.0906}{y_2^2}$$

which yields the following positive solutions

$$y_2 = 0.371 \text{ m}, \quad 0.924 \text{ m}$$

Since the upstream flow is subcritical, the flow in the constriction must also be subcritical, and hence

$$\boxed{y_2 = 0.924 \text{ m}}$$

Therefore, if energy losses are neglected the calculated flow depth is in error by $(0.924 - 0.884)/0.884 \times 100 = 4.5\%$. This effect is not very significant.

(c) According to the energy equation

$$E_1 = E_2 + \frac{V_1^2}{2g} \tag{3}$$

where $V_1 = 0.800$ m/s, and

$$\begin{aligned} E_1 &= 1.0326 \text{ m} \\ E_2 &= y_2 + \frac{Q^2}{2g(b_2 y_2)^2} = y_2 + \frac{8^2}{2(9.81)(4.5y_2)^2} = y_2 + \frac{0.1612}{y_2^2} \end{aligned}$$

Substituting into the energy equation, Equation 3, gives

$$1.0326 = y_2 + \frac{0.1612}{y_2^2} + \frac{0.800^2}{2(9.81)}$$

which does not have any positive solutions. Therefore, the flow is choked and critical flow exists within the constriction. Under critical flow conditions,

$$\begin{aligned} \frac{Q^2}{g} &= \frac{A^3}{T} \\ \frac{8^2}{9.81} &= \frac{(4.5y_2)^3}{4.5} \end{aligned}$$

which yields

$$\boxed{y_2 = 0.686 \text{ m}}$$

(d) Since the flow is choked, the constriction influences the upstream flow depth. Under critical flow conditions,

$$E_2 = \frac{3}{2}y_2 = \frac{3}{2}(0.686) = 1.028 \text{ m}$$

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According to the energy equation

$$E_1 = E_2 + \frac{V_1^2}{2g} \tag{4}$$

or

$$y_1 + \frac{V_1^2}{2g} = 1.028 + \frac{V_1^2}{2g}$$

which yields

$$\boxed{y_1 = 1.028 \text{ m}}$$

3.30. From given data: $b = 3$ m, $Q = 4$ m^3/s, $y_1 = 1.5$ m, and

$$V_1 = \frac{Q}{by_1} = \frac{4}{3(1.5)} = 0.889 \text{ m/s}$$

Applying the energy equation,

$$\begin{aligned}
y_1 + \frac{V_1^2}{2g} &= y_2 + \frac{V_2^2}{2g} + \Delta z \\
1.5 + \frac{0.889^2}{2(9.81)} &= y_2 + \frac{(4/3y_2)^2}{2(9.81)} + 0.15 \\
1.54 &= y_2 + \frac{0.0906}{y_2^2} + 0.15
\end{aligned}$$

Solving this equation for y_2 gives

$$y_2 = 1.34 \text{ m}, 0.29 \text{ m}$$

Since the upstream flow is subcritical, select the subcritical flow depth, where

$$\boxed{y_2 = 1.34 \text{ m}}$$

When choking just occurs,

$$y_2 = y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

where

$$q = \frac{Q}{b} = \frac{4}{3} = 1.33 \text{ m}^2/\text{s}$$

and therefore

$$y_c = \left(\frac{1.33^2}{9.81}\right)^{1/3} = 0.565 \text{ m}$$

and the energy equation can be written as

$$\begin{aligned}
y_1 + \frac{V_1^2}{2g} &= y_c + \frac{(q/y_c)^2}{2g} + \Delta z_m \\
1.54 &= 0.565 + \frac{(1.33/0.565)^2}{2(9.81)} + \Delta z_m
\end{aligned}$$

which giveS

$$\boxed{\Delta z_m = 0.69 \text{ m}}$$

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3.31. Flow in a rectangular open channel is choked when

$$
\begin{aligned}
E_1 &= E_2 + \Delta z_c \\
y_1 + \frac{V_1^2}{2g} &= \frac{3}{2} y_c + \Delta z_c \\
y_1 + \frac{V_1^2}{2g} &= \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} + \Delta z_c \\
y_1 + \frac{V_1^2}{2g} &= \frac{3}{2}\left(\frac{Q^2}{gb^2}\right)^{1/3} + \Delta z_c \\
y_1 + \frac{V_1^2}{2g} &= \frac{3}{2}\left[\frac{(V_1 b y_1)^2}{gb^2}\right]^{1/3} + \Delta z_c \\
y_1 + \frac{V_1^2}{2g} &= \frac{3}{2}\frac{(V_1^{2/3} y_1^{2/3})}{g^{1/3}} + \Delta z_c
\end{aligned}
$$

Dividing by y_1 yields

$$1 + \frac{V_1^2}{2gy_1} = \frac{3}{2}\left(\frac{V_1^2}{gy_1}\right)^{1/3} + \frac{\Delta z_c}{y_1} \tag{1}$$

and defining

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}}$$

then Equation 1 can be written as

$$\boxed{\frac{\Delta z_c}{y_1} = 1 + \frac{\mathrm{Fr}_1^2}{2} - \frac{3}{2}\mathrm{Fr}_1^{2/3}}$$

From Problem 3.30: $b = 3$ m, $Q = 4$ m^3/s, $y_1 = 1.5$ m, and $\Delta z_c = 0.15$ m. Therefore

$$
\begin{aligned}
V_1 &= \frac{Q}{by_1} = \frac{4}{(3)(1.5)} = 0.889 \text{ m/s} \\
\mathrm{Fr}_1 &= \frac{V_1}{\sqrt{gy_1}} = \frac{0.889}{\sqrt{(9.81)(1.5)}} = 0.232
\end{aligned}
$$

which yields

$$
\begin{aligned}
\frac{\Delta z_c}{y_1} &= 1 + \frac{\mathrm{Fr}_1^2}{2} - \frac{3}{2}\mathrm{Fr}_1^{2/3} \\
\frac{\Delta z_c}{1.5} &= 1 + \frac{(0.232)^2}{2} - \frac{3}{2}(0.232)^{2/3}
\end{aligned}
$$

and solving for Δz_c gives

$$\boxed{\Delta z_c = 0.69 \text{ m}}$$