

3.4. The shear stress, τ_o , on the perimeter of the channel is given by

$$\tau_o = \gamma R S_o \quad (1)$$

From the given data $b = 5$ m, $y = 1.8$ m, $m = 1.5$, and the geometric properties of the channel are

$$\begin{aligned} A &= by + my^2 = 5(1.8) + 1.5(1.8)^2 = 13.86 \text{ m}^2 \\ P &= b + 2\sqrt{1 + m^2}y = 5 + 2\sqrt{1 + 1.5^2}(1.8) = 11.49 \text{ m} \\ R &= \frac{A}{P} = \frac{13.86}{11.49} = 1.21 \text{ m} \end{aligned}$$

From the given data, $\tau_o = 3.5$ N/m², and since $\gamma = 9790$ N/m², Equation 1 gives the maximum allowable slope, S_o , as

$$S_o = \frac{\tau_o}{\gamma R} = \frac{3.5}{(9790)(1.21)} = \boxed{0.00030}$$

For the excavated channel, $k_s = 3$ mm = 0.003 m, and at 20°C, the density, ρ , and dynamic viscosity, μ , of water are given by $\rho = 998.2$ kg/m³, and $\mu = 0.00100$ N·s/m². Substituting these data into Equation 3.36 gives the flowrate, Q , as

$$\begin{aligned} Q &= -2A\sqrt{8gRS_o} \log_{10} \left(\frac{k_s}{12R} + \frac{0.625\mu}{\rho R^{3/2}\sqrt{8gS_o}} \right) \\ Q &= -2(13.86)\sqrt{8(9.81)(1.21)(0.00030)} \log_{10} \left(\frac{0.003}{12(1.21)} + \frac{0.625(0.00100)}{(998.2)(1.21)^{3/2}\sqrt{8(9.81)(0.00030)}} \right) \\ &= 17.2 \text{ m}^3/\text{s} \end{aligned}$$

Therefore, for the given flow depth restrictions in the channel, the flow capacity of the channel is $\boxed{17.2 \text{ m}^3/\text{s}}$.

3.5. From the given data: $b = 8$ m, $S_o = 0.0001$, $k_s = 2$ mm = 0.002 m, and $Q = 15$ m³/s. At 20°C, $\rho = 998.2$ kg/m³, $\mu = 0.00100$ N·s/m², and, for a rectangular channel,

$$A = by \quad \text{and} \quad R = \frac{by}{2y + b}$$

Substituting into Equation 3.36 gives

$$\begin{aligned} Q &= -2A\sqrt{8gRS_o} \log_{10} \left(\frac{k_s}{12R} + \frac{0.625\mu}{\rho R^{3/2}\sqrt{8gS_o}} \right) \\ 15 &= -2(8y)\sqrt{8(9.81)\left(\frac{8y}{2y+8}\right)(0.0001)} \log_{10} \left(\frac{0.002}{12\left(\frac{8y}{2y+8}\right)} + \frac{0.625(0.00100)}{(998.2)\left(\frac{8y}{2y+8}\right)^{3/2}\sqrt{8(9.81)(0.0001)}} \right) \end{aligned}$$

which yields

$$y = 2.25 \text{ m}$$

Therefore, the uniform-flow depth in the channel is $\boxed{2.25 \text{ m}}$.

3.6. From Equation 3.38, taking $g = 9.81 \text{ m/s}^2$,

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8(9.81)}{f}} = \frac{8.86}{\sqrt{f}} \quad (1)$$

Equation 3.43 can be written in the form

$$\frac{1}{\sqrt{f}} = 2.97 \frac{R^{1/6}}{d^{1/6}} \quad (2)$$

where k_s has been replaced by the characteristic roughness height d . Combining Equations 1 and 2 yields

$$C = \frac{R^{1/6}}{0.038d^{1/6}} \quad (3)$$

Comparing Equation 3 with Equation 3.39 demonstrates that Manning's n can be expressed in the form

$$\boxed{n = 0.038d^{1/6}}$$

where d is in meters.

3.7. Hydraulically rough flow conditions occur in open channels when

$$\frac{u_* k_s}{\nu} \geq 100 \quad (1)$$

where

$$u_* = \sqrt{gRS_f} \quad (2)$$

Equation 3.45 can be rearranged and put in the form

$$k_s = d = 3.32 \times 10^8 n^6 \quad (3)$$

Substituting Equations (2) and (3) into Equation (1) and noting that $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$ at 20°C and $g = 9.81 \text{ m/s}^2$ yields

$$\frac{\sqrt{9.81} \sqrt{RS_f} \times 3.32 \times 10^8 n^6}{1 \times 10^{-6}} \geq 100$$

which simplifies to

$$\boxed{n^6 \sqrt{RS_f} \geq 9.6 \times 10^{-14}} \quad (4)$$

From the given data: $b = 5 \text{ m}$, and $S_o = 0.05\% = 0.0005$. For a concrete channel, Table 3.1 indicates that $n = 0.011$ is a conservative estimate of n . Since

$$R = \frac{A}{P} = \frac{by}{2y + b} = \frac{5y}{2y + 5} \quad (5)$$

Equation (4), can be combined with Equation (5) to give the following condition for fully turbulent flow,

$$(0.011)^6 \sqrt{\left(\frac{5y}{2y + 5}\right)} (0.0005) \geq 9.6 \times 10^{-14}$$

This condition is never satisfied. Taking $n = 0.015$ requires that

$$(0.015)^6 \sqrt{\left(\frac{5y}{2y+5}\right)} (0.0005) \geq 9.6 \times 10^{-14}$$

which yields

$$y = 0.15 \text{ m}$$

Therefore, the minimum flow depth for fully turbulent flow varies depending on the roughness coefficient of the channel. If the channel is smooth ($n = 0.011$), then the flow is never fully turbulent, while if the channel is rough ($n = 0.015$), fully turbulent flow occurs when the flow depth is greater than or equal to 0.15 m.

3.8. The Darcy-Weisbach equation gives the average velocity, V , as

$$V = \sqrt{\frac{8g}{f}} \sqrt{RS_o}$$

From the given data, $y = 2.20$ m, $k_s = 2$ mm = 0.002 m, $b = 3.6$ m, $m = 2$, $S_o = 0.0006$, and hence the flow area, A , wetted perimeter, P , and hydraulic radius, R , are given by

$$\begin{aligned} A &= by + my^2 = (3.6)(2.20) + (2)(2.20)^2 = 17.6 \text{ m}^2 \\ P &= b + 2\sqrt{1+m^2}y = 3.6 + 2\sqrt{1+2^2}(2.20) = 13.4 \text{ m} \\ R &= \frac{A}{P} = \frac{17.6}{13.4} = 1.31 \text{ m} \end{aligned}$$

Assuming the flow is fully turbulent the friction factor, f , can be estimated using Equation 3.28 where

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left[\frac{12R}{k_s} \right] = 2 \log_{10} \left[\frac{12(1.31)}{0.002} \right] = 7.79$$

which leads to

$$f = 0.016$$

The mean velocity can now be estimated as

$$V = \sqrt{\frac{8g}{f}} \sqrt{RS_o} = \sqrt{\frac{8(9.81)}{0.016}} \sqrt{(1.31)(0.0006)} = 1.96 \text{ m/s}$$

and the corresponding flowrate, Q , is given by

$$Q = AV = (17.6)(1.96) = 34.5 \text{ m}^3/\text{s}$$

This flowrate was obtained by assuming that the flow in the channel is hydraulically rough (fully turbulent), in which case the friction factor does not depend on the Reynolds number of the flow. This assumption can now be checked by re-calculating the friction factor using the calculated flowrate. At 20°C, the kinematic viscosity, ν , of water is 1.00×10^{-6} m²/s, and the Reynolds number, Re , is therefore given by

$$Re = \frac{V(4R)}{\nu} = \frac{(1.96)(4 \times 1.31)}{1.00 \times 10^{-6}} = 1.03 \times 10^7$$

The friction factor can now be estimated by the general expression for the friction factor given by Equation 3.29 where

$$\begin{aligned} \frac{1}{\sqrt{f}} &= -2 \log_{10} \left[\frac{k_s}{12R} + \frac{2.5}{\text{Re}\sqrt{f}} \right] \\ &= -2 \log_{10} \left[\frac{0.002}{12(1.31)} + \frac{2.5}{1.03 \times 10^7 \sqrt{f}} \right] \\ &= -2 \log_{10} \left[1.27 \times 10^{-4} + \frac{2.43 \times 10^{-7}}{\sqrt{f}} \right] \end{aligned}$$

which by trial and error yields

$$f = 0.016$$

Since this is the same friction factor as originally estimated, the flow is indeed hydraulically rough and the estimated velocity and flowrate are 1.96 m/s and $34.5 \text{ m}^3/\text{s}$.

The Manning's equation gives the average velocity, V , as

$$V = \frac{1}{n} R^{2/3} S_o^{1/2}$$

Table 3.1 indicates that a mid-range roughness coefficient for concrete is $n = 0.015$. The average velocity given by the Manning equation is

$$V = \frac{1}{0.015} (1.31)^{2/3} (0.0006)^{1/2} = 1.96 \text{ m/s}$$

and the corresponding flowrate, Q , is

$$Q = AV = (17.6)(1.96) = 34.5 \text{ m}^3/\text{s}$$

Hence, in this case, the Darcy-Weisbach and Manning equations give the same results.

The Manning equation can be taken as valid when $n^6 \sqrt{RS_o} \geq 9.6 \times 10^{-14}$ and $2.5 < R/d < 250$, where d is the characteristic roughness height corresponding to $n = 0.015$. These conditions are required for fully turbulent flow conditions to exist and for $n/d^{1/6}$ to be approximately constant. In this case,

$$n^6 \sqrt{RS_o} = (0.015)^6 \sqrt{(1.31)(0.0006)} = 3.19 \times 10^{-13}$$

and, taking $d = (n/0.038)^6 = (0.015/0.038)^6 = 0.0038 \text{ m}$ gives

$$\frac{R}{d} = \frac{1.31}{0.0038} = 345$$

These former result indicates that the flow is fully turbulent, and the latter result indicates that $n/d^{1/6}$ may not be constant in this case and therefore the Manning equation may not be strictly applicable.

3.9. $S_o = 0.0001$, $k_s = 1 \text{ mm} = 0.001 \text{ m}$, $Q = 18 \text{ m}^3/\text{s}$.

$$\begin{aligned} A &= 5y + 2y^2 \\ P &= 5 + 2\sqrt{5}y \end{aligned}$$

which gives

$$R = \frac{A}{P} = \frac{5y + 2y^2}{5 + 2\sqrt{5}y} \quad (1)$$

Assume the flow is fully turbulent, then

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{k_s}{12R} \right) = -2 \log_{10} \left(\frac{0.001}{12R} \right) \quad (2)$$

The Darcy-Weisbach uniform-flow equation is

$$Q = A \sqrt{\frac{8g}{f} R S_o}$$

which can be written as

$$15 = (5y + 2y^2) \sqrt{\frac{8(9.81)}{f} R (0.0001)} = 0.0886(5y + 2y^2) \sqrt{\frac{R}{f}} \quad (3)$$

Solving Equations 1 to 3 simultaneously yields $y = 2.18 \text{ m}$, $R = 1.38 \text{ m}$, and $f = 0.014$. Since $V = Q/A = 18/20.4 = 0.88 \text{ m/s}$ (where $A = 20.4 \text{ m}^2$), then the Reynolds number, Re , can be estimated by

$$Re = \frac{V(4R)}{\nu} = \frac{(0.88)(4 \times 1.38)}{1.00 \times 10^{-6}} = 4.85 \times 10^6$$

According to the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{k_s}{12R} + \frac{2.5}{Re\sqrt{f}} \right)$$

which leads to

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{0.001}{12 \times 1.38} + \frac{2.5}{4.85 \times 10^6 \sqrt{f}} \right)$$

and solving for f gives

$$f = 0.014$$

Since this is the same value of f obtained by assuming fully turbulent flow, then fully turbulent flow is verified and the uniform depth of flow is 2.18 m. These results indicate that the flow is hydraulically rough.

Comparing the Manning and Darcy-Weisbach equation gives the following relation between the Manning roughness coefficient, n , and the Darcy friction factor, f ,

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

Taking $g = 9.81 \text{ m/s}^2$, $f = 0.014$, and $R = 1.38 \text{ m}$ yields $n = 0.014$. To be valid, the Manning equation requires fully turbulent flow conditions where $n^6 \sqrt{RS_o} \geq 9.6 \times 10^{-14}$. In this case

$$n^6 \sqrt{RS_o} = (0.014)^6 \sqrt{(1.38)(0.0001)} = 9.2 \times 10^{-14}$$

Since $n^6 \sqrt{RS_o} < 9.6 \times 10^{-14}$, the Manning equation is not valid.

3.10. Comparing the Manning and Darcy-Weisbach equations

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

which gives

$$n = \frac{\sqrt{f} R^{1/6}}{\sqrt{8g}} = \frac{f^{1/2} R^{1/6}}{\sqrt{8(9.81)}} = \frac{f^{1/2} R^{1/6}}{8.86}$$

If the friction factor, f , is taken as a constant, the above relation indicates that n must also be a function of the depth (since R is a function of the depth). However, in fully-turbulent flow conditions f is certainly not a constant, and Williamson (1951) has shown that f is proportional to $R^{-1/3}$ (see Equation 3.43). Taking $f \sim R^{-1/3}$, n would be a constant in the above equation. So the answer to the question is no.

3.11. Given: $Q = 20 \text{ m}^3/\text{s}$, $n = 0.015$, $S_o = 0.01$

(a) Manning equation is given by

$$Q = \frac{1}{n} A_n R_n^{2/3} S_o^{1/2} = \frac{1}{n} \frac{A_n^{5/3}}{P_n^{2/3}} S_o^{1/2}$$

where

$$\begin{aligned} A_n &= [b + my_n]y_n = [2.8 + 2y_n]y_n \\ P_n &= b + 2\sqrt{1 + m^2}y_n = 2.8 + 2\sqrt{5}y_n = 2.8 + 4.472y_n \end{aligned}$$

Substituting into the Manning equation yields

$$20 = \frac{1}{0.015} \frac{[(2.8 + 2y_n)y_n]^{5/3}}{(2.8 + 4.472y_n)^{2/3}} (0.01)^{1/2}$$

or

$$\frac{[(2.8 + 2y_n)y_n]^{5/3}}{(2.8 + 4.472y_n)^{2/3}} = 3.0$$

Solving by trial and error yields

$$\boxed{y_n = 0.91 \text{ m}}$$

(b) Comparing the Manning and Darcy-Weisbach equations gives

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n}$$

which leads to

$$f = \frac{8gn^2}{R^{1/3}}$$

In this case

$$A = (2.8 + 2y)y = (2.8 + 2 \times 0.91)(0.91) = 4.2 \text{ m}^2$$

$$P = 2.8 + 4.472(0.91) = 6.87 \text{ m}$$

$$R = \frac{A}{P} = \frac{4.20}{6.87} = 0.611 \text{ m}$$

therefore

$$f = \frac{8(9.81)(0.015)^2}{(0.611)^{1/3}} = 0.0208$$

For fully turbulent, where the Manning equation applies,

$$\begin{aligned} \frac{1}{\sqrt{f}} &= -2 \log \left[\frac{k_s}{12R} \right] \\ \frac{1}{\sqrt{0.0208}} &= -2 \log \left[\frac{k_s}{12(0.611)} \right] \\ 6.93 &= -2 \log [0.136k_s] \end{aligned}$$

which leads to

$$k_s = 0.00249 \text{ m} = \boxed{2.5 \text{ mm}}$$

3.12. From the given information,

$$n = 0.039d^{1/6}$$

where d is in ft. In this case, $d = 30 \text{ mm} = 0.09843 \text{ ft}$, and a 70% error in d is $0.7(0.09843) = 0.06890 \text{ ft}$. Hence, $d = 0.09843 \text{ ft} \pm 0.06890 \text{ ft}$. Hence, the estimated value of n , \bar{n} , is given by

$$\bar{n} = 0.039(0.09843)^{1/6} = 0.027$$

The lower estimate of n , n_L , is given by

$$n_L = 0.039(0.09843 - 0.06890)^{1/6} = 0.022$$

and the upper estimate of n , n_U , is given by

$$n_U = 0.039(0.09843 + 0.06890)^{1/6} = 0.029$$

The maximum percentage error in estimating n is therefore given by

$$\text{error} = \frac{0.027 - 0.022}{0.027} \times 100 = \boxed{19\%}$$

3.13. According to Equation 3.50,

$$\frac{n}{k_s^{1/6}} = \frac{\frac{1}{\sqrt{8g}} \left(\frac{R}{k_s} \right)^{1/6}}{2.0 \log \left(12 \frac{R}{k_s} \right)} \quad (1)$$

Let

$$y = \frac{n}{k_s^{1/6}}$$

$$x = \frac{R}{k_s}$$

and taking $g = 9.81 \text{ m/s}^2$, Equation 1 can be written

$$y = \frac{\frac{1}{\sqrt{8(9.81)}}x^{1/6}}{2.0 \log(12x)} = \frac{0.1129x^{1/6}}{2.0(\log 12 + \log x)} \quad (2)$$

$$= \frac{0.1129x^{1/6}}{2.0(\log 12 + 0.4343 \ln x)} = \frac{0.1129x^{1/6}}{2.158 + 0.8686 \ln x} \quad (3)$$

The minimum value of $n/k_s^{1/2}(=y)$ occurs when $dy/dx = 0$, where

$$\frac{dy}{dx} = \frac{(2.158 + 0.8686 \ln x)(\frac{1}{6} \times 0.1129x^{-5/6}) - (0.1129x^{1/6})(0.8686x^{-1})}{(2.158 + 0.8686 \ln x)^2} = 0$$

which yields

$$x = 33.63$$

and substituting into Equation 3 yields

$$y = 0.0389$$

Therefore, under fully-rough flow conditions, the minimum value of $n/k_s^{1/6}(=y)$ is 0.0389, or approximately 0.039.

When $n/k_s^{1/6}$ differs by 5% from 0.039,

$$\frac{n}{k_s^{1/6}} = 1.05(0.039) = \frac{\frac{1}{\sqrt{8g}} \left(\frac{R}{k_s}\right)^{\frac{1}{6}}}{2.0 \log \left(12 \frac{R}{k_s}\right)}$$

or

$$0.04095 = \frac{0.1129x^{1/6}}{2.158 + 0.8686 \ln x}$$

which yields

$$x = 6 \quad \text{or} \quad 281$$

Therefore, $n/k_s^{1/6}$ is within 5% of 0.039 when

$$\boxed{6 \leq \frac{R}{k_s} \leq 281}$$

It is noteworthy that this range is narrower than suggested by Yen (1991) and Hager (1999). The reason for this is that the constant value they assumed is a bit higher than 0.039.

3.14. For fully-turbulent flow conditions,

$$\frac{u_* k_s}{\nu} > 70 \quad (1)$$

where u_* is given by Equation 3.31 as

$$u_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{gRS_0} \quad (2)$$

Combining Equations 1 and 2 gives

$$\frac{\sqrt{gRS_0} k_s}{\nu} > 70$$

or

$$k_s \sqrt{RS_0} > \frac{70\nu}{\sqrt{g}}$$

Taking $\nu = 1.00 \times 10^{-6}$ m²/s (at 20°C), and $g = 9.81$ m/s² yields the turbulence condition

$$k_s \sqrt{RS_0} > \frac{70(1.00 \times 10^{-6})}{\sqrt{9.81}}$$

which simplifies to

$$\boxed{k_s \sqrt{RS_0} > 2.2 \times 10^{-5}}$$

For the given trapezoidal channel, $k_s = 3$ mm = 0.003 m, $S_0 = 0.1\% = 0.001$, $b = 3$ m, $m = 2$, and for a flow depth y ,

$$R = \frac{A}{P} = \frac{by + my^2}{b + 2y\sqrt{1 + m^2}} = \frac{3y + 2y^2}{3 + 2y\sqrt{1 + 2^2}} = \frac{3y + 2y^2}{3 + 4.472y}$$

For turbulent flow,

$$\begin{aligned} k_s \sqrt{RS_0} &> 2.2 \times 10^{-5} \\ 0.003 \sqrt{\left(\frac{3y + 2y^2}{3 + 4.472y}\right) (0.001)} &> 2.2 \times 10^{-5} \end{aligned}$$

which requires that

$$y > 0.056 \text{ m}$$

Therefore, flow conditions are fully turbulent when the depth of flow exceeds 0.056 m = $\boxed{5.6 \text{ cm}}$.

At this minimum flow depth,

$$\begin{aligned} R &= \frac{3(0.056) + 2(0.056)^2}{3 + 4.472(0.056)} = 0.0536 \text{ m} \\ \frac{R}{k_s} &= \frac{0.0536 \text{ m}}{0.003 \text{ m}} = 17.9 \end{aligned}$$

Since R/k_s is within the range for $n/k_s^{1/6}$ to be assumed constant, using the Manning equation is $\boxed{\text{appropriate}}$.

- 3.15.** From the given data: $y = 4.00$ m, $b = 4$ m, $m = 3$, and $S_o = 0.0001$. The Manning equation is valid under the following conditions,

$$3.6 < \frac{R}{k_s} < 360 \quad (1)$$

and

$$k_s \sqrt{RS_o} > 2.2 \times 10^{-5} \quad (2)$$

Assuming $n = 0.013$ and $n/k_s^{1/6} = 0.040$,

$$k_s = \left(\frac{n}{0.040} \right)^6 = \left(\frac{0.013}{0.040} \right)^6 = 0.00118 \text{ m}$$

and since

$$R = \frac{by + my^2}{b + 2\sqrt{1 + m^2}y} = \frac{4(4) + 3(4)^2}{4 + 2\sqrt{1 + 3^2}(4)} = 2.18 \text{ m}$$

then

$$\frac{R}{k_s} = \frac{2.18 \text{ m}}{0.00118 \text{ m}} = 1847$$

$$k_s \sqrt{RS_o} = (0.00118) \sqrt{(2.18)(0.0001)} = 1.74 \times 10^{-5}$$

Since $R/k_s > 360$ and $k_s \sqrt{RS_o} < 2.2 \times 10^{-5}$, the flow is not fully turbulent and Manning's equation is not applicable.

- 3.16.** From the given data: $Q = 1.8 \text{ m}^3/\text{s}$, $m = 2$, $n = 0.025$, and $S_o = 0.1\% = 0.001$.

(a) Size the channel to accommodate the design flow under normal conditions. Assuming that the flow in the channel can be described by the Manning equation (i.e. fully turbulent)

$$Q = \frac{1}{n} AR^{2/3} S_o^{1/2} \quad (1)$$

Since the lengths of the channel sides are equal to the bottom width, b , then the flow depth, y , is related to the bottom width by the relation

$$y = \frac{b}{\sqrt{1 + m^2}} = \frac{b}{\sqrt{1 + 2^2}} = 0.447b \quad (2)$$

The geometric properties of the channel are

$$A = by + my^2 = b(0.447b) + (2)(0.447b)^2 = 0.847b^2$$

$$P = 3b$$

$$R = \frac{A}{P} = \frac{0.847b^2}{3b} = 0.282b$$

Substituting into the Manning equation, Equation 1, gives

$$1.8 = \frac{1}{0.025} (0.847b^2)(0.282b)^{2/3} (0.001)^{1/2}$$

which yields

$$b = 1.67 \text{ m}$$

In this case,

$$n^6 \sqrt{RS_o} = (0.025)^6 \sqrt{(0.282 \times 1.67)(0.001)} = 5.30 \times 10^{-12} \geq 9.6 \times 10^{-14}$$

and therefore use of the Manning equation is justified, and according to Equation 2 the depth of flow is given by

$$y = 0.447(1.67) = 0.746 \text{ m}$$

The required channel is to have a bottom width of 1.67 m, side slopes of 2:1 (H:V), and a depth of at least 0.746 m.

(b) Let y be the depth of flow when the average shear stress, τ , on the channel lining is equal to the critical shear stress, $\tau_c = 4.0$ Pa. The channel lining then becomes unstable and the geometric properties of the channel are

$$\begin{aligned} A &= by + my^2 = 1.67y + 2y^2 \\ P &= b + 2\sqrt{1 + m^2}y = 1.67 + 2\sqrt{1 + 2^2}y = 1.67 + 4.47y \\ R &= \frac{A}{P} = \frac{1.67y + 2y^2}{1.67 + 4.47y} \end{aligned}$$

The average shear stress, τ , on the perimeter of the channel is given by

$$\tau = \gamma RS_o \tag{3}$$

where $\gamma = 9790 \text{ N/m}^3$. The channel lining is unstable when $\tau = \tau_c = 4.0$ Pa, and Equation 3 gives

$$4.0 = (9790) \frac{1.67y + 2y^2}{1.67 + 4.47y} (0.001)$$

which yields

$$y = 0.625 \text{ m}$$

Therefore, whenever the flow depth exceeds 0.625 m, the channel lining becomes unstable. In terms of flow, the Manning equation gives

$$\begin{aligned} Q &= \frac{1}{n} AR^{2/3} S_o^{1/2} \\ &= \frac{1}{0.025} [1.67(0.625) + 2(0.625)^2] \left[\frac{1.67(0.625) + 2(0.625)^2}{1.67 + 4.47(0.625)} \right]^{2/3} (0.001)^{1/2} = 1.27 \text{ m}^3/\text{s} \end{aligned}$$

Therefore, whenever the flowrate exceeds $1.27 \text{ m}^3/\text{s}$, the channel lining becomes unstable. An alternative lining should be used if the channel is to accommodate the design flow of $1.8 \text{ m}^3/\text{s}$.