

# Point-of-Interest Recommender Systems: A Separate-Space Perspective

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**Abstract**—With the rapid development of Location-based Social Network (LBSN) services, a large number of Point-Of-Interests (POIs) have been available, which consequently raises a great demand of building personalized POI recommender systems. A personalized POI recommender system can significantly assist users to find their preferred POIs and help POI owners to attract more customers. However, it is very challenging to develop a personalized POI recommender system because a user’s checkin decision making process is very complex and could be influenced by many factors such as social network and geographical distance. In the literature, a variety of methods have been proposed to tackle this problem. Most of these methods model user’s preference for POIs with integrated approaches and consider all candidate POIs as a whole space. However, by carefully examining a longitudinal real-world checkin data, we find that the whole space of users’ checkins actually consists of two parts: social friend space and user interest space. The social friend space denotes the set of POI candidates that users’ friends have checked-in before and the user interest space refers to the set of POI candidates that are similar to users’ historical checkins, but are not visited by their friends yet. Along this line, we develop separate models for the both spaces to recommend POIs. Specifically, in social friend space, we assume users would repeat their friends’ historical POIs due to the preference propagation through social networks, and propose a new Social Friend Probabilistic Matrix Factorization (SFPMF) model. In user interest space, we propose a new User Interest Probabilistic Matrix Factorization (UIPMF) model to capture the correlations between a new POI and one user’s historical POIs. To evaluate the proposed models, we conduct extensive experiments with many state-of-the-art baseline methods and evaluation metrics on the real-world data set. The experimental results firmly demonstrate the effectiveness of our proposed models.

## I. INTRODUCTION

Recent years have witnessed a rapid prevalence of location-based social network (LBSN) services, such as Foursquare, Jiebang, and Facebook Places, that can significantly facilitate users’ outdoor activities by providing a large number of nearby Point-of-Interests (POIs) in a real-time fashion. A variety of user interaction data with these LBSN services, such as searching locations, providing checkin information, building connections among online users, have been accumulated, which provides a good opportunity for developing personalized POI recommender systems. Indeed, the accurate and personalized POI recommendation is a crucial demand in LBSN services. First, given the massive locations, it is very difficult for users to find their preferred ones in an efficient way. A personalized POI recommender

system would help users easily find relevant POIs without spending too much time on searching, particularly when a user is in a new region. Also, it is very challenging for POI owners to deliver right POIs to various users. A personalized POI recommender system is able to not only ease the burden, but also attract more customers with the recommended POIs.

Although developing personalized POI recommender systems can greatly benefit both users and POI owners, it is still a very challenging problem. In fact, a user’s checkin decision making process is very complex and can be influenced by many different kinds of factors. First, a user’s checkins can be greatly influenced by a group of friends. Some of the friends have positive impact on a user’s checkin at a particular location, while others may influence negatively. Even with the same group of friends, two users may be affected by these friends in different ways. Also the distance of a POI might have influence on user’s preference for it. Usually a user would like to prefer a nearby POI rather than another one far away. Thus, modeling the influence of social friends and geographical distance on checkin behaviors is critical for developing personalized POI recommender systems. In addition, whether a user would check in at a POI or not may depend on specific purpose. For instance, when people want to have lunch, they would like to choose those POIs relevant to food rather than sights.

In the literature, some related works have been proposed to incorporate social influence into POI recommendations. For example, [1] assumes that users and their friends share the similar interests and then places a social regularization term on learning user feature vectors. [2] proposes a geo-social correlation model to capture four types of social correlations of users’ checkin behaviours, i.e. local friends, distant friends, local non-friends and distant non-friends. On the other hand, [3] models both local and global social relations by dividing ‘friend’ into two categories: one is the friend who builds connections with each other on LBSN and another refers to the user who has high global reputations. [4] predicts the preference of a user for a POI by collaborating the preferences of his friends on this POI. Most of these methods integrate social influence and personal interest together and model user’s preference for POI with an integrated approach. For instance, one common integration way is using social networks to regularize users’ interest. With such an integrated approach, all candidate POIs are considered as a whole and a user’s preference

for one candidate POI is predicted based on an integrated function. However, by carefully examining the real-world checkin data, we find that users’ checkins mainly consist of two groups of checkins. First, 30% of checkins are those that have been checked-in by direct friends. In other words, many users like to repeat their friends’ checkins. Second, the rest of checkins are very similar as users’ historically checked-in POIs. To this end, we propose to divide the whole recommendation space into two parts: social friend space and user interest space in this paper. The social friend space refers to the set of POI candidates that users’ friends have checked-in before. The user interest space denotes the set of POI candidates that have not been visited by their friends before, but are very similar to users’ historical checkins. Different from previous works that use a unified model to infer users’ preference for a POI, we develop separate models for the two spaces.

Specifically, in social friend space, we assume that users choose a new POI due to the preference propagation with the visitors on this POI as the initial injectors in the whole social network, and propose a new Social Friend Probabilistic Matrix Factorization (SFPMF) to factorize the preference propagation influence into user and factor feature vectors. The checkin probability is then modeled by the overall preference influence propagated from the initial injectors. On the other hand, in user interest space, we suppose that user’s decision on a new POI would be affected by his historical POIs, and propose another User Interest Probabilistic Matrix Factorization (UIPMF) to capture the correlations between the new POI and his historical POIs. Furthermore, we design two strategies for recommendations. One is novelly building two recommender systems for the sake of the different perspectives in two different spaces. Another is regularly utilizing an Integrated Social Friend and User Interest model (namely ISU) to combine them together due to their overlap. To evaluate the proposed models, we conduct extensive experiments with the real-world data set and compare our models with many state-of-the-art models based on different validation metrics. The experimental results demonstrate the superiority of our models.

**Overview.** The rest of this paper is organized as follows: In Section II, we introduce our models in details, including model specifications and parameter estimations. Section III demonstrates the experimental results. In Section IV, we summarize the related works. Finally, we draw conclusion in Section V.

## II. METHODOLOGIES

Our goal is to recommend the new (unvisited) locations for users. Based on the assumption that a new recommended POI for one user is either one of his friends’ historical ones or similar to his own historical ones, we divide the whole recommendation space into social friend space and user interest space. In this section, we first introduce the social friend space and design a new Social Friend Probabilistic

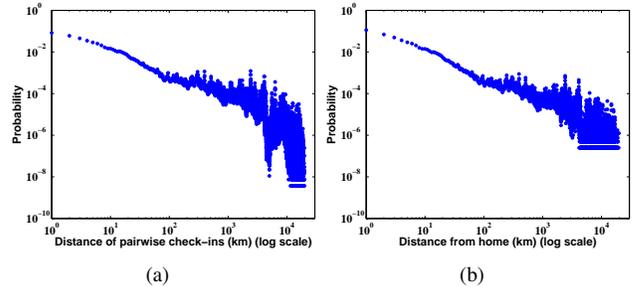


Figure 1: Probability of check-ins as a function of distance of pairwise check-in distance (Left) and from home (Right).

Matrix Factorization (SFPMF) model. Then we propose another novel User Interest Probabilistic Matrix Factorization (UIPMF) model for user interest space. Finally, we present the recommendation strategies and models’ estimations. For the simplicity of presentation, some main mathematical notations are shown in Table I. The terms **location** and **POI** are used interchangeably in this paper. **Friends** indicate **direct friendship**.

Table I: Mathematical Notations.

| Symbol      | Description   |
|-------------|---|
| $M$         | number of locations   |
| $N$         | number of users   |
| $L$         | the set of all locations  |
| $L_i$       | the set of all locations checked-in by user $i$   |
| $L_i^{c_j}$ | the set of locations that user $i$ checked-in before location $j$ and have the same category as $c_j$ |
| $F_i$       | the set of all friends of user $i$  |
| $\Psi_j$    | the set of all users who have checked-in location $j$   |
| $\Theta_i$  | the set of all locations checked-in by friends of user $i$  |

### A. Social Friend Space

In this section, we introduce the framework of SFPMF model, and present a  $P^3MF$  model to compute the checkin preference propagation probability for users.

1) **The SFPMF Framework:** Many works have demonstrated the importance of social network in recommender system [1][5]. To examine the influence of friends on users’ checkin behaviors, we depict the histogram of the ratio of checkins that repeat friends’ historical ones in Figure 2(d). There are two surprising observations: (1) Over 50% users like to repeat those locations checked-in by their friends prior to their first checkin at them; (2) More than 30% checkins are those that have been visited by friends. The results exhibit an important checkin behavior trend that users are probable to choose a POI from a set of POIs that his friends have visited before. Therefore, we believe the social network is a significant factor that affects user’s decision on POIs. Furthermore, it is also crucial to recommend a new POI for one user from a collection of POIs having been checked-in by his friends, because it is able to encourage users to have more checkins due to the trust to their friends and the similar POI taste as them. The inherent characteristics of checkin data and the evident benefit motivate us to recommend users with their friends’

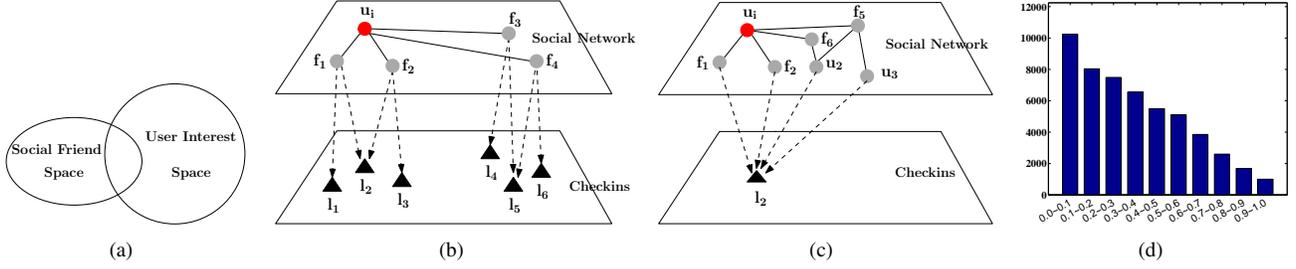


Figure 2: (a) Relationship between social friend space and user interest space. (b) An example of user’s social networks and checkins. The circle is user and the triangle is location. The red circle is the target user. The solid line indicates friendship and the dashed line indicates checkin behavior. (c) An example of users checking-in the same location and their social networks. (d) The x-axis is the ratio of checkins visited by friends; the y-axis is the number of corresponding users.

historically checked-in POIs. The problem in social friend space can be formally defined as:

*Definition 1 (Problem in Social Friend Space):* Given a set of candidate locations  $\Theta_i$ , i.e.  $\{l : l \in (\cup_{k \in F_i} L_k) \setminus L_i\}$ , which have been checked-in by the friends of user  $i$  but are new for him, the prediction is to find the location that user  $i$  would most likely prefer to check-in at the next time.

For example, Figure 2(b) shows that the target user  $u_i$  has friends  $\{f_1, f_2, f_3, f_4\}$  who have checked-in locations  $\{l_1, l_2, l_3, l_4, l_5, l_6\}$ , and these locations are never visited by the target user before. We aim to predict the probabilities of these POIs that he would check-in and recommend the POI with the highest probability for him.

In social friend space, we assume one user would repeat a location that his friends have checked-in before mainly due to the preference propagation in the whole social network. For each location  $j$ , the set of visitors  $\Psi_j$  are regarded as checkin preference injectors. They would likely tell their friends about this location, who might also tell their friends, and finally ”everyone” knows. Thus, a checkin event is propagated in the whole network in such word-of-mouth way with these visitors as the initial preference injectors. For example, in Figure 2(c),  $\{f_1, f_2, u_2, u_3\}$  are the visitors of location  $l_2$  and will propagate the preference for this location in the whole network. The target user  $u_i$  may be influenced by  $f_1, f_2, u_2$  and  $u_3$  with different probabilities. Specifically,  $u_i$  knows the location  $l_2$  from  $u_2$  likely due to (1) He hears from  $u_2$  directly although they are not friends explicitly in the online social network but they are real friends off line; (2)  $u_2$  tells  $f_6$ , and  $f_6$  tells him then. *The weight that the user is influenced by the checkin event about a POI naturally determines the probability that he prefers to check-in this POI.* Based on this analysis, we propose the Social Friend Probabilistic Matrix Factorization (SFPMF) model. Let us define  $P_{iv}^I$  as the probability that the preference is propagated from the user  $v$  to the user  $i$ . Therefore the probability that the user  $i$  chooses to check-in the location  $j$  is obtained by:

$$P_{ij}^S = 1 - \prod_{v \in \Psi_j} (1 - P_{iv}^I). \quad (1)$$

Note that the influence cannot be simply computed as the summation over all the influences propagated by the initial injectors because these injectors may have correlations [6].

Different from traditional online product consuming, one user has a very small chance to go to check-in a far away location due to the limited transportation even though he is interested in it. For instance, a user living at California would not go to check-in a restaurant in New York for the sake of long distance. In the example shown in Figure 2(b), user  $u_i$  has more chance to visit the locations in the left side than those in the right side, because the locations in the left side are much closer to him. Thus, a POI’s distance significantly affects the user’s checkin decision process. Some works [4] propose to leverage a power law distribution to model checkin probability and distance of any pair of visited POIs based on the observation shown in Figure 1(a). However, computing the distance of each POI with user’s all historical POIs is inefficient, especially when the number of POIs is tremendous. To address this issue, we first adopt the method in [7] to locate user’s home location from his all historical checkins. We find that user’s checkin probability and the distance between POI and his home also follow a power law distribution shown in Figure 1(b). Thus let us define the probability of a user to check-in a  $d$ -km far away POI as:

$$Pr(d) = a \cdot d^b, \quad (2)$$

where  $a$  and  $b$  are the parameters of power law distribution, and could be learned by maximum likelihood estimation. Then the probability of user  $i$  to check-in location  $j$  due to the geographical influence is defined as:

$$P_{ij}^G = Pr(d(j, h_i)), \quad (3)$$

where  $h_i$  is the home location of user  $i$ , and  $d(j, h_i)$  indicates the distance between the POI  $j$  and the home of user  $i$ .

Then Eq. (1) is refined with the geographical influence:

$$P_{ij}^S \propto Pr(d(j, h_i))(1 - \prod_{v \in \Psi_j} (1 - P_{iv}^I)). \quad (4)$$

Specifically, we further introduce a Preference Propagation Probabilistic Matrix Factorization ( $P^3MF$ ) model to compute the probability  $P_{iv}^I$  in the following section.

Table II: The objective functions for the proposed two models.

$$\operatorname{argmin}_{X,Y} \lambda_t \sum_{i=1}^N \sum_{v=1}^N I_{iv} (\hat{T}_{iv} - \beta X_i^T Y_v + (1 - \beta) \sum_{f \in F_{v,-i}} G_{vf} X_i^T Y_f)^2 + \lambda_x \|X\|_F^2 + \lambda_y \|Y\|_F^2 \quad (5)$$

$$\operatorname{argmin}_{U,V} \lambda_r \sum_{i=1}^N \sum_{j=1}^M I_{ij} (\hat{R}_{ij} - \alpha U_i^T V_j + (1 - \alpha) \sum_{l \in L_i^{c_j}} S_{jl} U_i^T V_l)^2 + \lambda_u \|U\|_F^2 + \lambda_v \|V\|_F^2 \quad (6)$$

2) **The  $P^3MF$  Model:** Different from traditional probabilistic matrix factorization [8], we construct a user-user matrix, where each rating  $T_{iv}$  indicates the preference propagated from user  $v$  to user  $i$ . The observed rating  $T_{iv}$  is a function of frequency that user  $i$  repeats the checkins of his friend  $v$ , which will be discussed in Section III-D1. Let  $X \in R^{Z \times N}$  and  $Y \in R^{Z \times N}$  be the latent user and factor feature matrices, with column vectors  $X_i$  and  $Y_v$  representing the  $Z$ -dimensional user-specific and factor-specific feature vectors of user  $i$  and user  $v$ , respectively. Specifically, the factor feature vector captures the properties of user  $v$ , such as age and activity level, and the user feature vector indicates the user's preference for corresponding properties.

The process of the preference propagation from user  $v$  to user  $i$  consists of two parts: (1) User  $v$  influences user  $i$  directly; (2) The friends of user  $v$  (not including user  $i$  if they are also friends) are influenced by him, and then affect user  $i$ . To model this propagation process, we define the predicted rating  $\hat{T}_{iv}$  that user  $i$  is influenced by user  $v$  as:

$$\hat{T}_{iv} = \beta X_i^T Y_v + (1 - \beta) \sum_{f \in F_{v,-i}} G_{vf} X_i^T Y_f, \quad (7)$$

where  $\beta \in [0, 1]$  is the tuning parameter to control the direct influence from friend  $v$ , and  $F_{v,-i}$  indicates the set of friends of user  $v$  excluding user  $i$ .  $G_{vf}$  is the information transmission probability from user  $v$  to his friend  $f$ . As the connection between friends is undirected, we define  $G_{vf}$  as:

$$G_{vf} = \frac{1}{|F_{v,-i}|}$$

It is worth noting that  $G_{vf}$  is different from  $T_{fv}$ , where the latter one is the preference propagated from user  $v$  to user  $f$  which contains both direct and indirect influence ways.

The rating is assumed to be drawn from a Gaussian distribution with the mean as shown in Eq. (7) and the precision as  $\lambda_t$ . We also place zero-mean spherical Gaussian priors on user and factor feature vectors with the precision as  $\lambda_x$  and  $\lambda_y$ , respectively. Therefore, based on a Maximum-a-Posteriori (MAP) estimation, we obtain the objective function about  $X$  and  $Y$  in Eq. (5), where  $\|\cdot\|_F$  denotes the Frobenius norm, and  $I_{iv}$  is the indicator function that is equal to 1 if user  $i$  checks-in a POI that his friend  $v$  checked-in before and equal to 0 otherwise.

In  $P^3MF$  model, we address the preference propagation influence in social network based on user's checkin behavior.

The preference propagated from user  $v$  to user  $i$  is dependent on the influence of himself and his friends. Recursively, the influence of the friend is further affected by his friends. Different from [9] which focuses on modeling the trust propagation for the user feature vector, we directly factorize the preference propagation influence into latent user and factor feature vectors. In addition, the checkin preference propagation is also affected by the geographical distance, i.e. the closer two users live physically, the more possibly they will interplay. Therefore, the checkin preference  $P_{iv}^I$  propagated from user  $v$  to user  $i$  is obtained based on  $\hat{T}_{iv}$  and geographical influence in Eq. (3):

$$P_{iv}^I \propto Pr(d(h_i, h_v))g(\hat{T}_{iv}), \quad (8)$$

where  $g(\cdot)$  is logistic function to bound the value to  $[0, 1]$ , and  $Pr(\cdot)$  is the geographical influence.

### B. User Interest Space

In user interest space, we recommend users with the POIs that have not been visited by their friends before but are similar to their own historical checkins. Therefore, the problem is formally defined as:

*Definition 2 (Problem in Social Friend Space):* Given a set of candidate locations  $\{l : l \in L \setminus (\Theta_i \cup L_i)\}$ , which have not been checked-in by the friends of user  $i$  and are new for him, the prediction is to find the location that user  $i$  would most likely prefer to check-in at the next time.

Probabilistic matrix factorization factorizes the observed rating into user and location latent space, and leverages them for rating prediction [8]. Similarly we have a user-location matrix, where the observed rating  $R_{ij}$  is a function of frequency that user  $i$  checked-in the location  $j$ . Let  $U \in R^{K \times N}$  and  $V \in R^{K \times M}$  be the latent user and location feature matrices, with column vectors  $U_i$  and  $V_j$  representing the  $K$ -dimensional user-specific and location-specific feature vectors of user  $i$  and location  $j$ , respectively. In traditional PMF, it assumes user's preference for a POI is the dot product of this user's and this POI's latent vectors. However, it ignores the strong correlations among user's all POIs. A user may prefer to choose a POI that is very similar to his historical ones. Therefore, we propose a User Interest Probabilistic Matrix Factorization (UIPMF) model to characterize the user's preference for a POI by using his preferences for the historical POIs that have the same category as this POI. In other words, user  $i$  has his special preference for the POI  $j$ , and at the same time, he is also

influenced by his historical POIs that have the same category as this POI. Thus, the predict rating denoted as  $\hat{R}_{ij}$  of user  $i$  for POI  $j$  is defined as:

$$\hat{R}_{ij} = \alpha U_i^T V_j + (1 - \alpha) \sum_{l \in L_i^{c_j}} S_{jl} U_i^T V_l, \quad (9)$$

where  $\alpha \in [0, 1]$  is the tuning parameter. The closer a pair of POIs are, the stronger correlation should be taken into account for a user's POI decision making process. Hence, the similarity between POI  $j$  and POI  $l$  denoted as  $S_{jl}$  is measured by leveraging power law property in Eq. (2) and normalized as:

$$S_{jl} = \frac{Pr(d(j, l))}{\sum_{p \in L_i^{c_j}} Pr(d(j, p))},$$

where  $d(j, l)$  is the distance between POI  $j$  and POI  $l$ .

Similarly, the rating is drawn from a Gaussian distribution with the mean as shown in Eq. (9) and the precision as  $\lambda_r$ . We also place zero-mean spherical Gaussian priors on user and location feature vectors with the precision as  $\lambda_u$  and  $\lambda_v$ , respectively. Therefore, the object function is obtained through Maximum-a-Posteriori estimation in Eq. (6).

As discussed earlier, the check-in decision making process of user  $i$  on POI  $j$  is significantly affected by the POI's geographical distance. Thus, similar to Eq. (4), the probability that user  $i$  prefers to check-in POI  $j$  is given as:

$$P_{ij}^U \propto P_{ij}^G g(\hat{R}_{ij}), \quad (10)$$

### C. Strategies for POI Recommendations

In this paper, we propose that the recommendation space could be divided into two parts: social friend space and user interest space, and their relationship is shown in Figure 2(a). Evidently, these two spaces might have overlap because the POIs that friends have checked-in are also possibly similar to users' historical checkins. Thus, we adopt the following two strategies for POI recommendations.

#### • Separated Recommendation.

We build two different recommender systems. One is to adopt SFPMF model to recommend POIs that users' friends have checked-in before. Another one is to adopt UIPMF model to recommend POIs that have not been visited by friends but are very similar to their historical checkins.

#### • Integrated Recommendation.

We propose an Integrated Social Friend and User Interest model (ISU) to integrate SFPMF model and UIPMF model for recommendation in the whole space. The integrated probability  $P_{ij}^R$  that user  $i$  will check-in POI  $j$  is defined as:

$$P_{ij}^R = \gamma P_{ij}^S + (1 - \gamma) P_{ij}^U, \quad (11)$$

where  $\gamma \in [0, 1]$  is the tuning parameter to align two recommendation spaces. Specifically,  $P_{ij}^S$  is computed only on the candidate POIs that user's friends have checked-in, while  $P_{ij}^U$  is calculated for all POIs.

### D. Parameter Estimation

Both  $P^3MF$  model and UIPMF model are matrix factorization based models, and their objective functions are given in Eq. (5) and Eq. (6). Alternating Least Squares (ALS) is a popular optimization method with accurate parameter estimation and fast convergence rate. Thus, we utilize ALS method to compute each latent variable by fixing the other variables when minimizing the object function. As two models have similar formulations, their optimizations are also similar. The optimization process is executed as:

- Randomly initialize each variable of interest.
- Update each of them with the updating equation iteratively until the object function converges.

**The Optimization for the  $P^3MF$  Model:** For the simplicity of inference, we first define the following variables:

$$\tilde{Y}_v^i = \beta Y_v + (1 - \beta) \sum_{f \in F_{vi}} G_{vf} Y_f,$$

$$w_\beta = \beta^2 + (1 - \beta)^2 \sum_{f \in F_{vi}} G_{vf}^2,$$

$$\bar{T}_{iv} = T_{iv} - \beta X_i^T Y_v - (1 - \beta) \sum_{f \in F_{vi}} G_{vf} X_i^T Y_f,$$

$$f(\bar{T}_{iv}) = \beta \bar{T}_{iv} + (1 - \beta) \sum_{f \in F_{vi}} G_{vf} \bar{T}_{if},$$

where  $F_{vi}$  is the set of common friends of user  $v$  and  $i$ . Then the updating equations for  $X$  and  $Y$  are obtained as:

$$X_i = [\lambda_x I_Z + \lambda_t \sum_{v=1}^N I_{iv} \tilde{Y}_v^i \tilde{Y}_v^{iT}]^{-1} \lambda_t \sum_{v=1}^N I_{iv} T_{iv} \tilde{Y}_v^i,$$

$$Y_v = [\lambda_y I_Z + \lambda_t \sum_{i=1}^N I_{iv} w_\beta X_i X_i^T]^{-1} \lambda_t \sum_{i=1}^N I_{iv} [f(\bar{T}_{iv}) + w_\beta X_i^T Y_v] X_i.$$

**The Optimization for the UIPMF Model:** We can obtain the similar definitions about  $\tilde{V}_j^i$ ,  $w_\alpha$ ,  $\bar{R}_{ij}$  and  $f(\bar{R}_{ij})$ :

$$\tilde{V}_j^i = \alpha V_j + (1 - \alpha) \sum_{l \in L_i^{c_j}} S_{jl} V_l,$$

$$w_\alpha = \alpha^2 + (1 - \alpha)^2 \sum_{l \in L_i^{c_j}} S_{jl}^2,$$

$$\bar{R}_{ij} = R_{ij} - \alpha U_i^T V_j - (1 - \alpha) \sum_{l \in L_i^{c_j}} S_{jl} U_i^T V_l,$$

$$f(\bar{R}_{ij}) = \alpha \bar{R}_{ij} + (1 - \alpha) \sum_{l \in L_i^{c_j}} S_{jl} \bar{R}_{il}.$$

Then the updating equations for  $U$  and  $V$  are shown as:

$$U_i = [\lambda_u I_K + \lambda_r \sum_{j=1}^M I_{ij} \tilde{V}_j^i \tilde{V}_j^{iT}]^{-1} \lambda_r \sum_{j=1}^M I_{ij} R_{ij} \tilde{V}_j^i,$$

$$V_j = [\lambda_v I_K + \lambda_r \sum_{i=1}^N I_{ij} w_\alpha U_i U_i^T]^{-1} \lambda_r \sum_{i=1}^N I_{ij} [f(\bar{R}_{ij}) + w_\alpha U_i^T V_j] U_i.$$

### III. EXPERIMENTAL RESULTS

In this section, we will evaluate our proposed models with the real-world data set.

#### A. The Experimental Setup

**Dataset.** In this paper, we use Gowalla data set [10] to evaluate the performance of our models, which contains checkin data ranging from January 2009 to August 2010. Each checkin record in the data set includes user ID, location ID and timestamp, where each location has latitude, longitude and category information. Meanwhile the data set has undirected friendship information. Specifically, we have the creation timestamp for each friendship, which is different from most of data sets used in recent research works. In addition, we remove users who have visited less than 5 locations and more than 1000 locations, and locations which are visited by less than 5 users. The data statistics are shown in Table III.

Table III: Statistics of Data Set.

| #User     | #Location | #Checkin                 | Sparsity                 |
|-----------|-----------|--------------------------|--------------------------|
| 61,578    | 178,062   | 3,257,029                | 0.0297%                  |
| #Train    | #Test     | #Test (SFS) <sup>1</sup> | #Test (UIS) <sup>2</sup> |
| 2,581,882 | 675,147   | 167,546                  | 507,601                  |

In recommendation system, we aim to recommend those unvisited locations for users. Therefore, we split the training and testing data as follows: for each individual user, (1) aggregating the checkins for each individual location; (2) sorting the location according to the first time that user checks in; (3) selecting the earliest 80% to train the model, and using the next 20% as testing. With the dynamic information, we use the social network at the end date of training data for both training and testing. Specifically, there are on average 8.29 friends for each user, and for those users who have friends, there are on average 556.32 locations that their friends have visited before. The observed rating for SFPMF model is a function of frequency that user repeats the checkins of his friends, and we then obtain 372,502 ratings in the training.

**Experimental Settings.** In the experiments, the parameter  $\beta$ ,  $\alpha$  and  $\gamma$  are set as 0.1, 0.8, and 0.01, respectively. The parameters  $\lambda_t$  and  $\lambda_r$  are set as 0.0001 and the other regularization parameters are set as 0.01. The dimensions of latent factors (i.e.  $Z$  and  $K$ ) in SFPMF and UIPMF models are set as the same. We discuss the rating conversion methods for MF based models in Section III-D1.

#### B. Evaluation Metrics

As POI recommender system only recommends the limited POIs for users, we quantitatively evaluate recommendation models in terms of top-K recommendation performance i.e. Precision@K and Recall@K metrics. We also adopt

<sup>1</sup>We select those checkins in the test which friends have checked-in before to evaluate models in social friend space.

<sup>2</sup>We select those checkins in the test which friends have not checked-in before to evaluate models in user interest space.

MAP metric, the mean of the average precision (AP) over all locations in the testing, to evaluate models' performance. Formally, they are defined as:

$$Precision@K = \frac{1}{N} \sum_{i=1}^N \frac{|S_i(K) \cap T_i|}{K},$$

$$Recall@K = \frac{1}{N} \sum_{i=1}^N \frac{|S_i(K) \cap T_i|}{|T_i|},$$

$$MAP = \frac{1}{N} \sum_{i=1}^N \frac{\sum_{j=1}^{\hat{M}_i} p(j) \times rel(j)}{|T_i|},$$

where  $S_i(K)$  is a set of top-K POIs recommended to user  $i$  excluding those POIs in the training,  $T_i$  is a set of locations that are checked-in by user  $i$  in the testing.  $\hat{M}_i$  is the number of the returned locations in the list for user  $i$ ,  $p(j)$  is the precision of a cut-off rank list from 1 to  $j$ , and  $rel(j)$  is an indicator function that equals to 1 if the location is appearing in the testing, otherwise equals to 0.

#### C. Baseline Methods

To comparatively demonstrate the effectiveness of our proposed models, we compare them with five recommendation models: (1) **USG** [4], taking geographical influence, social network and user interest into account for POI recommendation; (2) **LOCABAL** [3], capturing two types of social relations, i.e. the local friends and the users with high global reputations, for recommendation based on matrix factorization; (3) **RegPMF** [1], assuming that users and their friends share similar interests in the preference and placing a social regularization term on learning latent user feature vectors; (4) **PMF** [8] that assumes the user and location latent vectors to be drawn from Gaussian distribution and estimates a user's preference for a location as the dot product of user-specific and location-specific latent vector; (5) **UC**, user-based collaborative filtering that adopts cosine similarity as the similarity measurement between users.

#### D. Performance Comparisons

First, we evaluate different rating conversion methods for MF based models, and then compare the performance of our proposed models with baseline methods in social friend space, user interest space and the whole recommendation space. Finally, we elaborate the region effect in checkin data to improve the efficiency of POI recommendation.

1) **Performance Comparisons of Rating Conversion Methods:** In the literature, various rating conversion methods are proposed to fit Matrix Factorization (MF) based models for POI recommendation due to the bias of checkin data (i.e. majority ratings are very small and small percentages of ratings are extremely high). We formally compare the following six methods with MF based models:

- **Logistic:** Logistic function  $\frac{1}{1+(e^x)^{-1}}$  is commonly used in recommendation to map each matrix entry into  $[0, 1]$ .
- **Exponential [11]:** A mapping function  $\frac{1}{1+x^{-1}}$  is used to bound each matrix entry into  $[0, 1]$ .

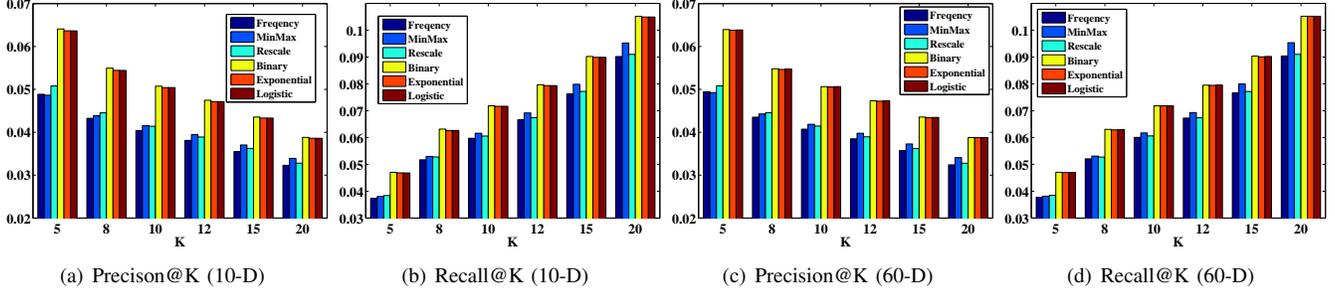


Figure 3: Performance comparisons for different rating conversions on UIPMF with different dimensions.

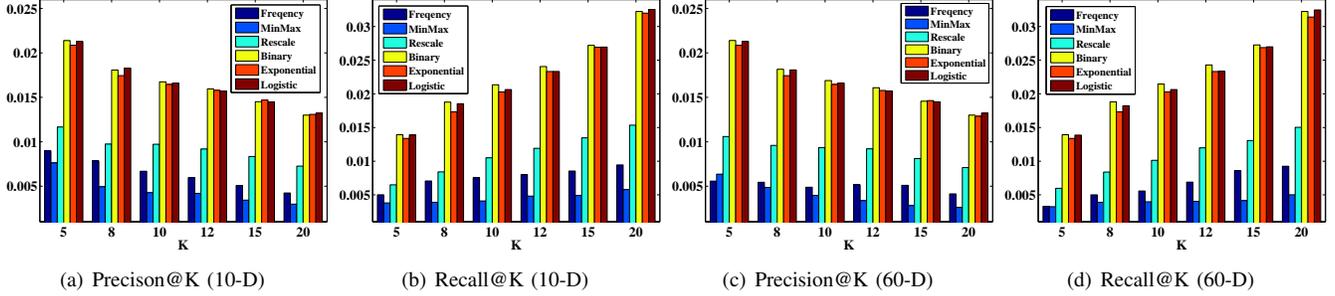


Figure 4: Performance comparisons for different rating conversions on PMF with different dimensions.

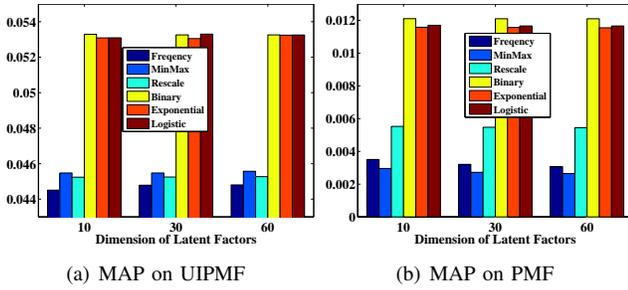


Figure 5: Performance comparisons for different rating conversions in terms of MAP with different dimensions.

- **Binary:** It has two values: 0 and 1. The rating is assigned to 1 if user has checkin at this POI, and assigned to 0 otherwise.
- **Rescale [12]:** Due to the power law distribution of user-location checkin numbers, we could obtain a five-point scale rating with check-in frequency: converting one check-in to 2, two check-ins to 3, three check-ins to 4, and four or more checkins to 5.
- **MinMax [13]:** It is defined as  $\frac{x-1}{max-1}$ , where  $max$  is the maximum frequency value.
- **Frequency:** Rating is the number of user-location checkins.

where  $x$  is the number of user-location checkins. It is worth to noting that for Rescale and Frequency, we use the ratings after minus mean value to fit models; other kinds of ratings are used to fit the models directly. Due to the limited space, we only report the performance of different rating conversion methods on both UIPMF and PMF in terms of precision@K, recall@K and MAP in Figure 3, Figure 4 and Figure 5.

Based on the results, we summarize as following: (1) Two models perform almost consistent with different rat-

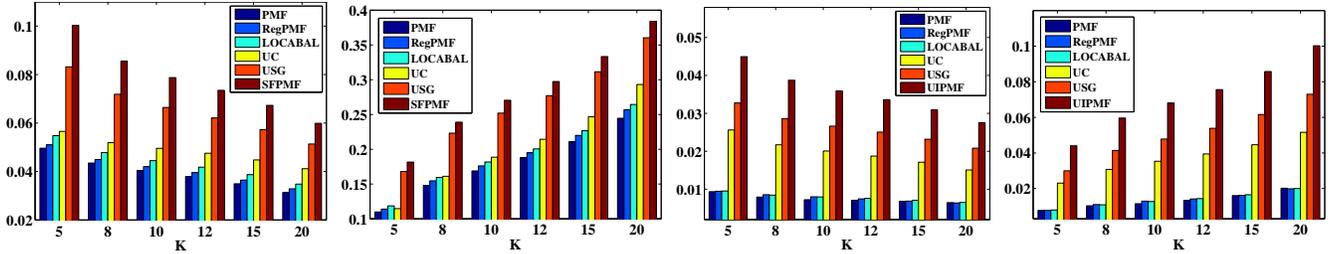
Table IV: Performance comparisons in terms of MAP in social friend space and user interest space.

| Social Friend Space |         |         |         |         |         |
|---------------------|---------|---------|---------|---------|---------|
| SFPMF               | USG     | UC      | LOCABAL | RegPMF  | PMF     |
| <b>0.16825</b>      | 0.14947 | 0.11111 | 0.10406 | 0.10017 | 0.09689 |
| User Interest Space |         |         |         |         |         |
| UIPMF               | USG     | UC      | LOCABAL | RegPMF  | PMF     |
| <b>0.04538</b>      | 0.02996 | 0.02303 | 0.00684 | 0.00666 | 0.00647 |

ing conversion methods, indicating that matrix factorization based models will have consistent performance with different rating conversion methods. (2) Frequency, MinMax and Rescale perform much worse than others, suggesting the bias in checkin data would affect the model's performance. Even though MinMax bounds the rating to  $[0, 1]$ , many zero ratings are possible to explain its bad performance. (3) Logistic, Exponential and Binary methods have very similar performance and are much superior than others. It happens possibly because they constrain the ratings in  $[0, 1]$  to avoid the large fluctuation of ratings. Surprisingly Binary performs very well under this group of regularization parameters. But we observe that this method pronely leads to over-fitting in high dimension under other parameter settings. In the following experiments, we will adopt logistic function as rating conversion method to fit matrix factorization based methods because it is widely used in recommendation and obtains good performance. We only report the performance with the latent factor dimension as 10 due to the similar performance in different dimensions.

#### 2) Performance Comparisons in Social Friend Space:

We compare our proposed SFPMF model with the baseline methods in social friend space, where the testing checkins are only those that friends have visited before (see Test(SFS) in Section III-A). We first compute the probabilities for



(a) Precision@K in social friend space (b) Recall@K in social friend space (c) Precision@K in user interest space (d) Recall@K in user interest space  
Figure 6: Performance comparisons in terms of precision@K and recall@K in social friend space and user interest space.

each user and his candidate locations with different models, and then recommend the top-K locations with the highest probabilities. The candidate locations are those that users' friends have checked-in before. The performances in terms of Precision@K, Recall@K and MAP are shown in Figure 6(a), Figure 6(b) and at the top of Table IV.

It can be observed that all models perform consistently with different metrics. We find that as K increases, the precision decreases while the recall increases. Totally PMF has the worst performance among all methods, indicating that traditional matrix factorization is difficult to work well on checkin data due to the difference between user's behaviors on products consuming and checkins. The POI decision making process is more affected by social network and geographical influence. Both RegPMF and LOCALBAL perform better than PMF. Their improvements indicate that social network is a factor affecting the performance in POI recommender system. Leveraging the assumption that users and their friends share similar interests does improve the recommendation accuracy. It is surprising that UC also works well. It is possible that users' similar interests can help to estimate more accurate ratings in our checkin data. Although USG takes the geographical influence, social network and user interest into account for recommendation, it is not as good as our proposed SFPMF model. It demonstrates the framework of SFPMF and the modeling approach of user's preference propagation assist to improve the prediction accuracy. Furthermore, it illustrates that utilizing the characteristics of social network, i.e. preference propagation from one person to another, can appropriately model user's checkin decision making process in social friend space.

### 3) Performance Comparisons in User Interest Space:

We evaluate the performance of UIPMF model versus various baseline methods in user interest space, where the testings are excluding those that are checked-in by friends in the testing (see Test(UIS) in Section III-A). The performances in terms of Precision@K, Recall@K and MAP are shown in Figure 6(c), Figure 6(d) and at the bottom of Table IV.

From the results, we can see RegPMF and LOCALBAL are only a little better than PMF, but they are much worse than others. It shows the traditional modeling methods with social network fail to achieve accurate recommendations in user interest space. In this space, since users and friends

have no common checkins in the testing, social network does not work in USG model. Hence in fact USG only captures user's interests and geographical influence. Its superior performance exhibits that geographical influence play an important role in location recommendation. The significant improvements of UIPMF compared to USG demonstrate that utilizing user's historical interests to seek a new POI similar to previous ones, as well as exploiting the spatial clustering phenomenon of checkin data are helpful for recommendation.

4) *Performance Comparisons in the Whole Recommendation Space:* As social friend space and user interest space might have overlap, we can integrate them (namely ISU) to make recommendations in the whole space. We evaluate the performance of ISU model with baseline models in terms of precision@K, recall@K and MAP. We do the test in the whole testing data. The performances are reported in Table V. We summarize the main results as the followings:

- PMF performs the worst among all the models. The sparseness of data may be one reason why it has such bad performance. UC performs better than PMF. Although the data is very sparse, it might have the tendency in our checkin data that similar users have similar interests in the preference of POIs. Thus, UC achieves good performance in our checkin data.
- Both RegPMF and LOCALBAL are better than PMF. It shows social network is helpful for location prediction. Although both of them assume users and their friends have similar interests, they have different modeling approaches. The better performance of LOCALBAL than that of RegPMF reflects that considering the local and global effect of friends is a superior approach to utilize social network information for recommendation.
- USG obtains a better performance than LOCALBAL, RegPMF, UC and PMF, which indicates that both social network and geographical influence can benefit POI recommender system. However, it is worse than our proposed model possibly due to the weak connection of a new POI and one user's historical POIs. Thus it clearly shows our models' effectiveness.
- ISU performs much better than other baseline models, illustrating the effectiveness of (1) modeling user's repeating behaviour for his friends' historical POIs;

Table V: Performance comparisons in terms of Precision, Recall and MAP in the whole recommendation space.

|         | Precision@5    | Recall@5       | Precision@8    | Recall@8       | Precision@10   | Recall@10      | Precision@15   | Recall@15      | MAP            |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| ISU     | <b>0.06464</b> | <b>0.04784</b> | <b>0.05548</b> | <b>0.06400</b> | <b>0.05139</b> | <b>0.07312</b> | <b>0.04414</b> | <b>0.09174</b> | <b>0.05379</b> |
| USG     | 0.03651        | 0.02916        | 0.03488        | 0.04387        | 0.03377        | 0.05255        | 0.03137        | 0.07106        | 0.02847        |
| UC      | 0.02895        | 0.02330        | 0.02744        | 0.03469        | 0.02668        | 0.04159        | 0.02524        | 0.05743        | 0.02427        |
| LOCABAL | 0.02193        | 0.01426        | 0.01866        | 0.01906        | 0.01735        | 0.02198        | 0.01488        | 0.02773        | 0.01231        |
| RegPMF  | 0.02142        | 0.01393        | 0.01846        | 0.01891        | 0.01715        | 0.02182        | 0.01448        | 0.02707        | 0.01190        |
| PMF     | 0.02129        | 0.01389        | 0.01828        | 0.01854        | 0.01660        | 0.02063        | 0.01448        | 0.02695        | 0.01170        |

(2) capturing the influence of user’s own historical checkins on new POIs. It also indicates that both social network and geographical influence contribute to POI recommendation together.

*E. Additional Experiment: Efficiency with Region Effect*

POI recommender system runs with the following procedures: (1) computing the probabilities of all the POIs for each individual user; (2) then recommending top-K POIs with the highest probabilities for each one. However, in reality the number of POIs is more than millions, as a result it becomes very inefficient to take all the POIs as candidates into account for each user in online system. As user checkin activities of LBSNs exhibit a spatial clustering phenomenon, the geographical influence plays an important role in POI recommendation. In other words, one user has a larger chance to check-in a nearby POI than a far away one, which reflects a region effect. To improve the efficiency, we only consider those POIs in a region as the candidates when recommending top-K POIs for each user. The region is simply defined as a circle area with a predefined radius and user’s home location as center. As in user friend space there are only 556.32 locations per user on average which is very small, ISU and UIPMF have on average 177,721.07 and 177,265.40 candidate locations per user, respectively. Hence we only evaluate the performance of with different region radius shown in Table VI. The ratios of candidate POIs under different region radius to the all POIs are reported at the top of Table VI. Note that the number of candidates directly reflects the efficiency of recommendation.

Based on the results, we can conclude that as the radius increases, the performance also increases. It is possible that users sometimes like to travel to long-distance POIs which are out range of the candidates. Thus with the increasing of radius, candidates will contain more true POIs and as a result, lead to the better performance. However, even though only the candidates in a small region are taken into account, both ISU and UIPMF are still much better than the baselines (see Section III-D4 and III-D3 respectively). This motives us to improve the efficiency of recommendation by taking the advantage of region effect. For example, in a real system, we could define the radius as 50km for ISU model, as a result the number of candidates is only 0.022 times as the number of the whole POIs, which not only keeps the good performance, but also significantly improves the recommendation’s efficiency.

IV. RELATED WORK

Related works can be grouped into two categories. The first category throws light on elaborating social network information for recommendation [1][2][3][4][5][14][15][16]. For example, based on the intuition that users and their friends will share the similar interests, [1] places a social regularization term to constrain matrix factorization object functions for learning more accurate user feature vectors. [2] proposes a geo-social correlation model to capture four types of social correlations of users’ checkin behaviours, i.e. local friends, distant friends, local non-friends and distant non-friends. The checkin probability is measured as a combination of these four geo-social correlations, where the corresponding coefficients are learned by a group of features in a logistic regression similar fashion. On the other hand, [3] leverages local and global social relations to assist recommendations. Specifically, in local context, it models the correlations between users and their friends by fitting the similarities between them; while in global context, it utilizes the reputation of a user in the whole social network as the weight to fit the observed ratings. [15][4] predicts the preference of a user for a POI by collaborating the preferences of his friends on this POI.

The second category focuses on modeling geographical influence for recommendation [4][17][18][19][20][21][22][23][24][25]. Specifically, there are several approaches to incorporate geographical distance for location prediction. For example, [17] discovers user’s checkin behaviour follows a two-state (home and work) mixture of Gaussian in geographical distance, assuming a POI that user would choose to check in is next to either his home or work place. Instead of leveraging the fixed two-center Gaussian mixture models, [18] further adopts the multi-center Gaussian model to form the distribution of the distance between a visited location and its center for each individual user. On the other hand, [19] first utilizes the kernel density estimation (KDE) to learn personalized checkin behaviour with POI locations, which is much more flexible. [20] further studies a mixture of adaptive Kernel density estimates to characterize a distribution between checkin probability and distances at different spatial level in order to avoid the data sparsity in individual level for KDE. Meanwhile, [23] considers two types of geographical characteristics: geographical neighborhoods’ and regions’ effect. It assumes locations which are nearest neighborhoods and in the same regions would share similar user preferences. In addition, [4] proposes to use a

Table VI: Performance comparisons with different region radius. Ratio represents the ratio of candidate POIs to all POIs. P@K and R@K represent Precision@K and Recall@K, respectively. All for ISU indicates all POIs excluding users’ training POIs, while All for UIPMF denotes all POIs excluding users’ training POIs and those that their friends have visited before.

|          | Region Radius (km) |       |       |       |       |       |       |       |       |       |        |       |       |       |       |       |
|----------|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|-------|
|          | 5                  | 10    | 25    | 50    | 80    | 100   | 200   | All   | 5     | 10    | 25     | 50    | 80    | 100   | 200   | All   |
|          | ISU                |       |       |       |       |       |       |       | UIPMF |       |        |       |       |       |       |       |
| Ratio    | 0.007              | 0.010 | 0.016 | 0.022 | 0.026 | 0.028 | 0.037 | 1.000 | 0.007 | 0.010 | 0.0158 | 0.022 | 0.026 | 0.027 | 0.036 | 1.000 |
| P@5 (%)  | 5.401              | 5.447 | 5.465 | 5.473 | 5.474 | 5.474 | 5.474 | 6.464 | 4.258 | 4.317 | 4.345  | 4.359 | 4.362 | 4.363 | 4.364 | 6.365 |
| P@8 (%)  | 4.727              | 4.791 | 4.813 | 4.822 | 4.823 | 4.823 | 4.824 | 5.548 | 3.661 | 3.733 | 3.764  | 3.778 | 3.783 | 3.783 | 3.785 | 5.443 |
| P@10 (%) | 4.418              | 4.486 | 4.510 | 4.520 | 4.521 | 4.521 | 4.522 | 5.139 | 3.391 | 3.463 | 3.498  | 3.513 | 3.518 | 3.519 | 3.521 | 5.042 |
| P@15 (%) | 3.844              | 3.923 | 3.954 | 3.965 | 3.968 | 3.968 | 3.969 | 4.414 | 2.910 | 2.987 | 3.027  | 3.041 | 3.047 | 3.048 | 3.050 | 4.331 |
| R@5 (%)  | 4.324              | 4.370 | 4.387 | 4.396 | 4.397 | 4.397 | 4.398 | 4.784 | 4.214 | 4.215 | 4.331  | 4.350 | 4.353 | 4.354 | 4.357 | 4.682 |
| R@8 (%)  | 5.820              | 5.919 | 5.954 | 5.970 | 5.973 | 5.973 | 5.974 | 6.400 | 5.646 | 5.793 | 5.862  | 5.892 | 5.900 | 5.900 | 5.907 | 6.262 |
| R@10 (%) | 6.667              | 6.795 | 6.845 | 6.866 | 6.868 | 6.868 | 6.870 | 7.312 | 6.420 | 6.598 | 6.694  | 6.731 | 6.743 | 6.745 | 6.752 | 7.164 |
| R@15 (%) | 8.363              | 8.569 | 8.658 | 8.695 | 8.703 | 8.703 | 8.705 | 9.174 | 8.016 | 8.293 | 8.445  | 8.500 | 8.517 | 8.520 | 8.529 | 8.991 |
| MAP      | 4.511              | 4.740 | 4.902 | 4.960 | 4.982 | 4.991 | 5.011 | 5.379 | 4.013 | 4.215 | 4.361  | 4.414 | 4.437 | 4.445 | 4.464 | 5.309 |

power law distribution to estimate the check-in probability with the distance of any pair of visited POIs, due to the spatial clustering phenomenon exhibited in LBSNs.

However, our work is different from these existing works. We divide the whole recommendation space into social friend space and user interest space, and then develop separate models in two spaces.

## V. CONCLUSION

We investigate a novel Point-of-Interest recommender system in this paper. Specifically, we divide the recommendation space into social friend space and user interest space. In social friend space, the problem is formulated as recommending one user with new POIs that his friends have checked-in before. A novel SFPMF model is proposed to factorize the preference propagation influence into user and factor feature vectors. In user interest space, the problem is defined as recommending one user with new POIs that have not been visited by his friends but are very similar to his historical ones. Then UIPMF model is developed to capture the connection between one user’s preference for a new POI and his preference for historically visited POIs. Finally, experimental results on a real-world data set effectively demonstrate the improvement of our proposed models over several baseline methods based on many validation metrics.

## VI. ACKNOWLEDGEMENTS

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## REFERENCES

- [1] H. Ma, D. Zhou, C. Liu, M. R. Lyu, and I. King, “Recommender systems with social regularization,” in *WSDM*, 2011.
- [2] H. Gao, J. Tang, and H. Liu, “gscorr: Modeling geo-social correlations for new check-ins on location-based social networks,” in *CIKM*, 2012.
- [3] J. Tang, X. Hu, H. Gao, and H. Liu, “Exploiting local and global social context for recommendation,” in *IJCAI*, 2013.
- [4] M. Ye, P. Yin, W.-C. Lee, and D.-L. Lee, “Exploiting geographical influence for collaborative point-of-interest recommendation,” in *SIGIR*, 2011, pp. 325–334.
- [5] H. Wang, M. Terrovitis, and N. Mamoulis, “Location recommendation in location-based social networks using user check-in data,” in *SIGSPATIAL*, 2013, pp. 374–383.
- [6] G. Li, S. Chen, J. Feng, K. Lee Tan, and W. syan Li, “Efficient location-aware influence maximization,” in *SIGMOD*, 2014.
- [7] Z. Cheng, J. Caverlee, K. Lee, and D. Z. Sui, “Exploring millions of footprints in location sharing services,” in *ICWSM*, 2011.
- [8] R. Salakhutdinov and A. Mnih, “Probabilistic matrix factorization,” in *NIPS*, 2007, pp. 1257–1264.
- [9] M. Jamali and M. Ester, “A matrix factorization technique with trust propagation for recommendation in social networks,” in *RecSys*, 2010.
- [10] S. Scellato, A. Noulas, and C. Mascolo, “Exploiting place features in link prediction on location-based social networks,” in *KDD*, 2011.
- [11] H. Gao, J. Tang, X. Hu, and H. Liu, “Exploring temporal effects for location recommendation on location-based social networks,” in *RecSys*, 2013.
- [12] D. Yang, D. Zhang, Z. Yu, and Z. Wang, “A sentiment-enhanced personalized location recommendation system,” in *HT*, 2013.
- [13] B. Liu and H. Xiong, “Point-of-interest recommendation in location based social networks with topic and location awareness,” in *SDM*, 2013.
- [14] H. Gao, J. Tang, and H. Liu, “Exploring social-historical ties on location-based social networks,” in *ICWSM*, 2012.
- [15] M. Ye, P. Yin, and W.-C. Lee, “Location recommendation for location-based social networks,” in *GIS*, 2010, pp. 458–461.
- [16] I. Konstas, V. Stathopoulos, and J. M. Jose, “On social networks and collaborative recommendation,” in *SIGIR*, 2009.
- [17] E. Cho, S. A. Myers, and J. Leskovec, “Friendship and mobility: User movement in location-based social networks,” in *SIGKDD*, 2011.
- [18] C. Cheng, H. Yang, I. King, and M. R. Lyu, “Fused matrix factorization with geographical and social influence in location-based social networks,” in *AAAI*, 2012.
- [19] J. Zhang and C. Chow, “igsrl: Personalized geo-social location recommendation - a kernel density estimation approach,” in *SIGSPATIAL*, 2013, pp. 334–343.
- [20] M. Lichman and P. Smyth, “Modeling human location data with mixture of kernel densities,” in *KDD*, 2014.
- [21] B. Liu, Y. Fu, Z. Yao, and H. Xiong, “Learning geographical preferences for point-of-interest recommendation,” in *KDD*, 2013.
- [22] D. Lian, C. Zhao, X. Xie, G. Sun, E. Chen, and Y. Rui, “Geomf: Joint geographical modeling and matrix factorization for point-of-interest recommendation,” in *KDD*, 2014.
- [23] Y. Liu, W. Wei, A. Sun, and C. Miao, “Exploiting geographical neighborhood characteristics for location recommendation,” in *CIKM*, 2014, pp. 739–748.
- [24] Q. Yuan, G. Cong, and A. Sun, “Graph-based point-of-interest recommendation with geographical and temporal influences,” in *CIKM*, 2014.
- [25] A. Noulas, S. Scellato, N. Lathia, and C. Mascolo, “Mining user mobility features for next place prediction in location-based services,” in *ICDM*, 2012.