S = Stroke
ℓ = rod length

b = bore

let $r = \frac{V(\pi)}{V(0)}$

A

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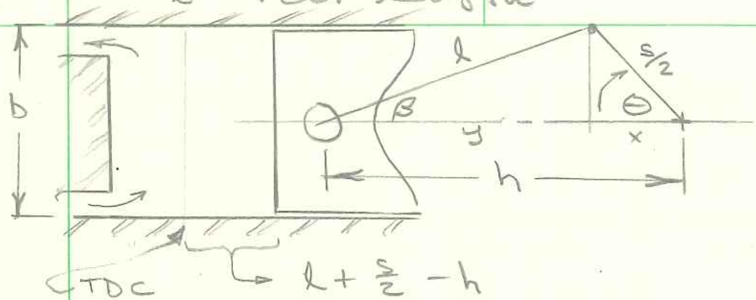


$$\epsilon = \frac{S}{2\ell} = \frac{1}{R}$$

$$h = \ell\cos\beta + \frac{S}{2}\cos\theta$$

$V(0)$ = clearance volume

At any crank angle $V(\theta) = V(0) + \frac{\pi}{4}b^2\left(\ell + \frac{S}{2} - h\right)$

so at $\theta = \pi$, $h = \ell - \frac{S}{2}$

$$V(\pi) = V(0) + \frac{\pi}{4}b^2 S$$

and $$\frac{V(\theta)}{V(0)} = 1 + \frac{\pi}{4}\frac{b^2}{V(0)}\left(\ell + \frac{S}{2} - h\right)$$

$$= 1 + \frac{\pi}{4V(0)}\frac{b^2 S}{2}\left(\frac{2\ell}{S} + 1 - \frac{2h}{S}\right) \quad (1)$$

but $$r_v = \frac{V(\pi)}{V(0)} = 1 + \frac{\pi b^2 S}{4V(0)} \Rightarrow \frac{\pi b^2 S}{4V(0)} = r_v - 1 \quad (2)$$

Substituting (2) into (1) gives

$$\frac{V(\theta)}{V(0)} = 1 + \frac{r_v - 1}{2}\left(\frac{2\ell}{S} + 1 - \frac{2h}{S}\right)$$

$$= 1 + \frac{r_v - 1}{2}\left[\frac{2\ell}{S} + 1 - \frac{2}{S}\left(\ell\cos\beta + \frac{S}{2}\cos\theta\right)\right] \quad (3)$$

From figure, $\ell\sin\beta = \frac{S}{2}\sin\theta$

$$\sin\beta = \epsilon\sin\theta$$

$$\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \epsilon^2\sin^2\theta} \quad (4)$$

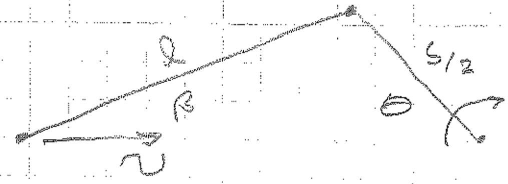
Substituting (4) into (3) gives

$$\frac{V(\theta)}{V(0)} = 1 + \frac{r_v - 1}{2}\left(\frac{1}{\epsilon} + 1 - \frac{1}{\epsilon}\sqrt{1 - \epsilon^2\sin^2\theta} - \cos\theta\right) \quad (5)$$

$\boxed{2.14}$ Pulkrabek

Eq'n (5) gives cylinder volume as a function of crank angle and geometry.

Need piston speed as f'n of θ, geom.



① $h = l\cos\beta + \frac{s}{2}\cos\theta$

② $\frac{s}{2}\sin\theta = l\sin\beta$

(2a) $$v = -\frac{dh}{dt} = l\sin\beta\,\frac{d\beta}{dt} + \frac{s}{2}\sin\theta\,\frac{d\theta}{dt}$$

differentiate ② $$\frac{s}{2}\cos\theta\,\frac{d\theta}{dt} = l\cos\beta\,\frac{d\beta}{dt}$$ $\left\{\text{want to get rid of } \beta\right\}$

Solve for $\frac{d\beta}{dt}$ $$\frac{d\beta}{dt} = \frac{s}{2l}\,\frac{\overset{\omega}{\dot{\theta}}\cos\theta}{\cos\beta}$$

To express β in terms of θ $\sin^2\beta + \cos^2\beta = 1$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$

③ $$\frac{d\beta}{dt} = \frac{s}{2l}\,\frac{\omega\cos\theta}{\sqrt{1-\sin^2\beta}}$$

subs. from ② $\sin^2\beta = \left(\frac{s}{2l}\right)^2\sin^2\theta$

$$= \frac{s}{2l}\,\frac{\omega\cos\theta}{\sqrt{1-\frac{s^2}{4l^2}\sin^2\theta}}$$

$$= \frac{\omega\cos\theta}{\sqrt{\left(\frac{2l}{s}\right)^2-\sin^2\theta}}$$

substitute into ② and ③ into (2a)

$$v = l\left(\frac{s}{2l}\sin\theta\right)\frac{\omega\cos\theta}{\sqrt{\left(\frac{2l}{s}\right)^2-\sin^2\theta}} + \frac{s}{2}\,\omega\sin\theta$$

$$v = \frac{s}{2}\,\omega\sin\theta\left[1 + \frac{\cos\theta}{\sqrt{\left(\frac{2l}{s}\right)^2-\sin^2\theta}}\right]$$

$$v = \frac{s}{2}\,\omega\sin\theta\left[1 + \frac{\cos\theta}{\sqrt{\frac{1}{\varepsilon^2}-\sin^2\theta}}\right]$$ Eq'n 2.5