J. Hill 2010

For a fluid in rigid-body motion subjected to linear acceleration (from earlier)

$$\frac{dF}{dV} = -\nabla P + \rho \vec{g} = \rho \vec{a}$$ Eqn 2.2 in book

$\rho \vec{g}$ ← body force per unit vol.

$-\nabla P$ ← pressure force per unit vol.

with components

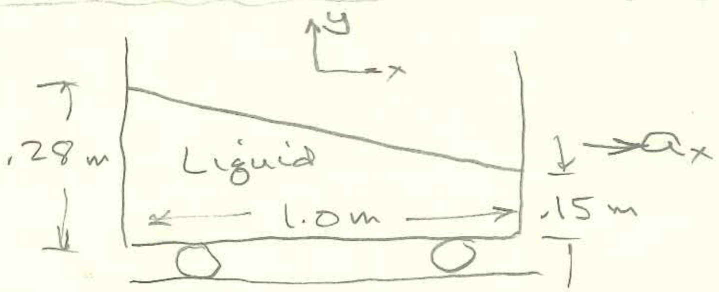
$$-\frac{\partial P}{\partial x} + \rho g_x = \rho a_x$$

$$-\frac{\partial P}{\partial y} + \rho g_y = \rho a_y$$

$$-\frac{\partial P}{\partial z} + \rho g_z = \rho a_z$$

The tank shown is given a constant horiz. accel.: The fluid is observed to assume the shape shown.

Find $a_x$ and the shape of the free surf.




$g_y = -g$

$$-\left(\vec{i}\frac{\partial P}{\partial x} + \vec{j}\frac{\partial P}{\partial y} + \cancel{\vec{k}\frac{\partial P}{\partial z}}^{\,0}\right) + \rho\left(\vec{i} g_x + \vec{j} g_y + \cancel{\vec{k} g_z}^{\,0}\right) = \rho\left(\vec{i} a_x + \cancel{\vec{j} a_y}^{\,0} + \cancel{\vec{k} a_z}^{\,0}\right)$$

$$-\vec{i}\frac{\partial P}{\partial x} - \vec{j}\frac{\partial P}{\partial y} - \vec{j}\rho g = \vec{i}\rho a_x$$

$\vec{i}: \quad \frac{\partial P}{\partial x} = -\rho a_x$

$\vec{j}: \quad \frac{\partial P}{\partial y} = -\rho g$

Find eq'n of free surf.

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy = 0$$

since $P$ = const. on free surf.

substit. $-\rho a_x dx - \rho g_y dy = 0$

$$\left.\frac{dy}{dx}\right)_{\text{free surf}} = -\frac{a_x}{g} = \text{straight line}$$

$$= \frac{.13}{1.0} \Rightarrow a_x = .13g = 1.27\ \frac{m}{s^2}$$