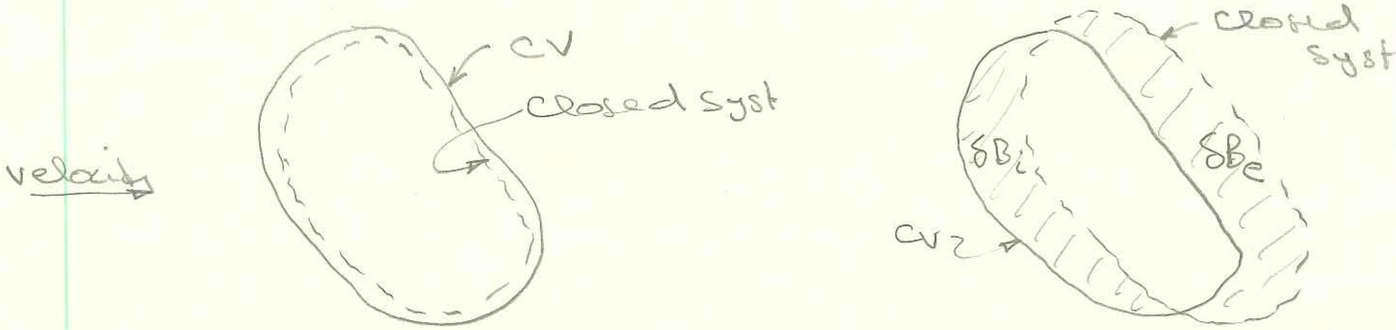
REYNOLDS TRANSPORT THEOREM

J. H. D

Let B be any extensive property and b be B/mass.
Consider a control volume and closed system which are coincident at time $= t$.



time $= t$ time $= t + \delta t$

At $t + \delta t$, $B_{syst} = B_{cv} - \delta B_i + \delta B_e$

so that $$\frac{dB_{SYST}}{dt} = \lim_{\delta t \to 0} \left[\frac{(B_{cv} - \delta B_i + \delta B_e)_{t+\delta t} - B_{cv,t}}{\delta t} \right]$$

or $$\frac{dB_{SYST}}{dt} = \underbrace{\lim_{\delta t \to 0} \left[\frac{B_{cv,t+\delta t} - B_{cv,t}}{\delta t} \right]}_{\frac{dB_{cv}}{dt} = \frac{d}{dt}\int_V \rho b \, dV} - \lim_{\delta t \to 0} \left[\frac{\delta B_e - \delta B_i}{\delta t} \right]$$

and $\lim_{\delta t \to 0} \frac{\delta B_e}{\delta t}$ = rate of flow of property B out

$\lim_{\delta t \to 0} \frac{\delta B_i}{\delta t}$ = rate of flow of property B into CV

So $\dot{B}_i = \rho b \vec{V} \cdot \vec{n} \, dA$ when negative
$\dot{B}_e = \rho b \vec{V} \cdot \vec{n} \, dA$ when positive

$$\boxed{\frac{dB_{syst}}{dt} = \underbrace{\frac{d}{dt} \int_{VOL} \rho b \, dV}_{= 0 \text{ for steady flow}} + \int_{SURF} \rho b \vec{V} \cdot \vec{n} \, dA}$$

J. Hill

If B = mass, $\frac{dm}{dt}_{SYST} = 0$ by definition

and $\frac{d}{dt}\int \rho \frac{m}{m} dV = 0$ (steady flow)

so $\int_{SURF} \rho \frac{m}{m} \vec{V} \cdot \vec{n}\, dA = 0 = \int_S \rho \vec{V} \cdot \vec{n}\, dA$

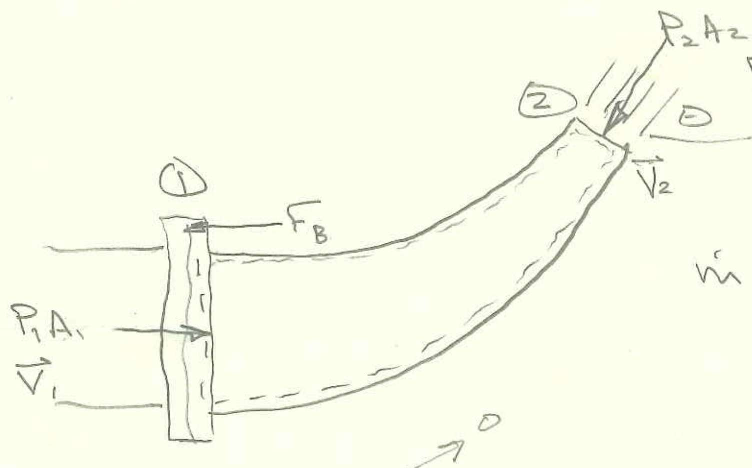
If B = momentum $m\vec{V}$, $b = \vec{V}$

$$\frac{d(m\vec{V})}{dt} = \boxed{F = \int_{SURF} \rho \vec{V}\, \vec{V} \cdot \vec{n}\, dA}$$ steady flow

For x-direction

$$\sum F_x = \int \rho \vec{V}_x\, \vec{V} \cdot \vec{n}\, dA$$

All x-dir. forces incl. body forces



$P_2 = P_{ATM} = 0$ (gage)

TOP VIEW

$$\dot{m} = \int_A \rho \vec{V} \cdot \vec{n}\, dA$$

$$\sum F_x = P_1 A_1 - \cancel{P_2 A_2 \cos\theta} - F_B = \vec{V}_1(-\dot{m}) + \vec{V}_2 \cos\theta\, (\dot{m})$$

$$F_B = P_1 A_1 + \dot{m} \vec{V}_1 - \dot{m} \vec{V}_2 \cos\theta$$