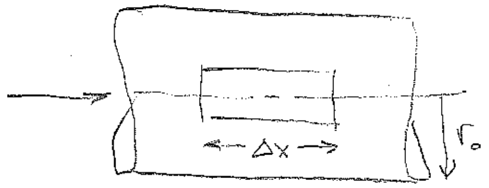
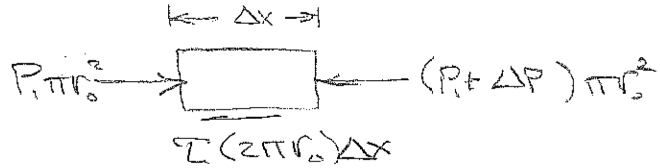
J. Hill

For steady, fully-developed laminar flow





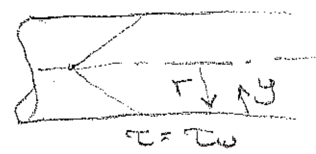
$$\Sigma F_x = P_1 \pi r_o^2 - \tau\, 2\pi r_o \Delta x - (P_1 + \Delta P)\pi r_o^2 = 0$$

$$-\tau\, 2\pi r_o \Delta x - \Delta P \pi r_o^2 = 0$$

$$\frac{\Delta P}{\Delta x} = -\frac{2\tau}{r_o} \Rightarrow \tau = -\frac{r_o}{2}\frac{dP}{dx} = \mu \frac{du}{dy} = -\mu \frac{du}{dr}$$

$$-\mu \frac{du}{dr} = -\frac{r}{2}\frac{dP}{dx}$$

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{dP}{dx}$$



$$\int du = \frac{1}{2\mu}\frac{dP}{dx}\int r\,dr \Rightarrow u = \frac{1}{4\mu}\frac{dP}{dx} r^2 + C_1$$

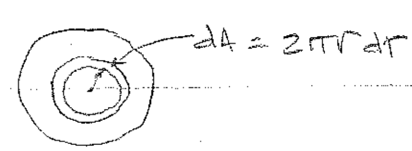
B.C. @ $r = r_o$, $u = 0$ so $C_1 = -\frac{dP}{dx}\frac{r_o^2}{4\mu}$

$$\boxed{u = \frac{dP}{dx}\frac{1}{4\mu}\left(r^2 - r_o^2\right)}$$

$u = u_{MAX}$ @ $r = 0$ $\Rightarrow u_{MAX} = -\frac{dP}{dx}\frac{1}{4\mu} r_o^2$

$$\boxed{\frac{u}{u_{MAX}} = 1 - \left(\frac{r}{r_o}\right)^2} \qquad \frac{dP}{dx} = -\frac{4\mu u_{MAX}}{r_o^2}$$

OR $u = -\frac{dP}{dx}\frac{r_o}{4\mu}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$



can get flow rate Q

$$Q = \int u\,dA = \int_{r=0}^{r_o} u\, 2\pi r\,dr = 2\pi u_{MAX}\int_0^{r_o}\left[1 - \left(\frac{r}{r_o}\right)^2\right] r\,dr$$

$$= 2\pi u_{max}\left[\frac{r^2}{2} - \frac{r^4}{4r_o^2}\right]_0^{r_o}$$

$$= 2\pi u_{MAX}\left[\frac{r_o^2}{2} - \frac{r_o^2}{4}\right] = \boxed{\frac{\pi u_{MAX} r_o^2}{2} = Q}$$

but $u_{AVG} = \frac{Q}{A} = \frac{\pi u_{MAX} r_o^2}{2\pi r_o^2} = \frac{u_{MAX}}{2}$

so $\boxed{u_{MAX} = 2 u_{AVG}}$ for laminar flow