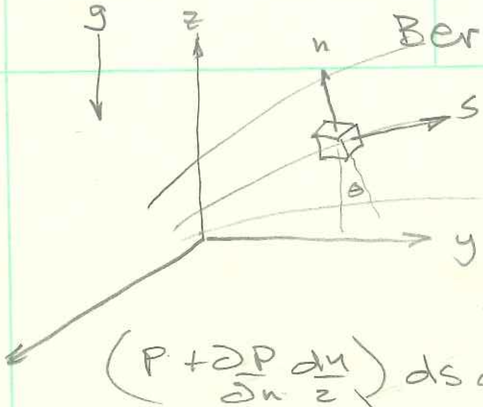
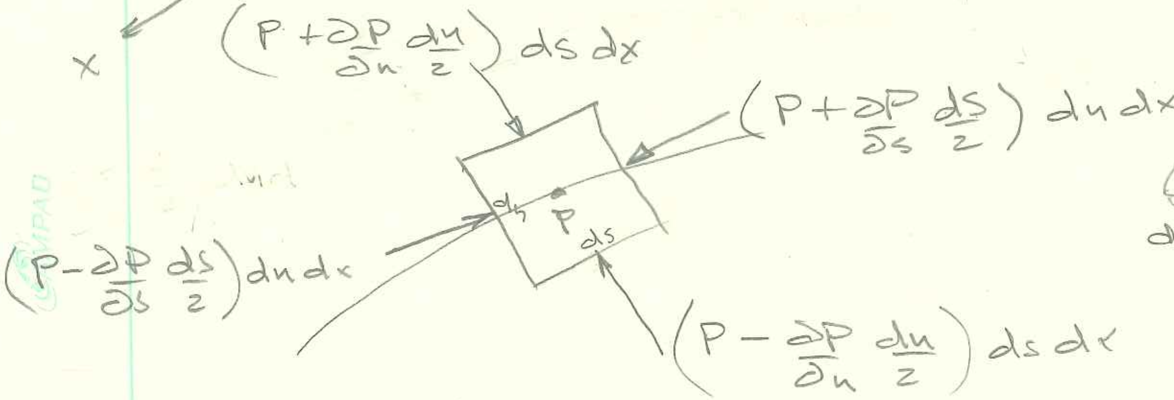
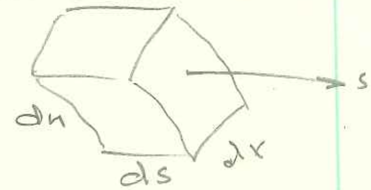
# Bernoulli (yet again)

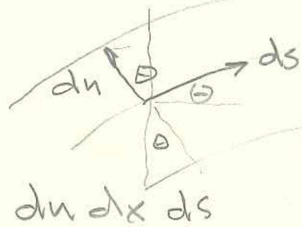
J. Hill 2010




consider a fluid element moving in without friction in the y-z plane with pressure forces shown. Motion is in the s direction.




Along a streamline (s-direction)

$$\left(P - \frac{\partial P}{\partial s}\frac{ds}{2}\right) dn\,dx - \left(P + \frac{\partial P}{\partial s}\frac{ds}{2}\right) dn\,dx - \rho g \sin\theta\, dn\,dx\,ds$$
$$= \rho\, ds\,dn\,dx\, a_s$$

divide by $ds\, dn\, d\theta$

$$-\frac{\partial P}{\partial s} - \rho g \sin\theta = \rho a_s$$

but $dn \cos\theta = dz = ds \sin\theta$
so, $\sin\theta = \frac{dz}{ds}$

Along SL.

$$-\frac{1}{\rho}\frac{dP}{ds} - g\frac{dz}{ds} = a_s$$
$$= \vec{V}_s \frac{d\vec{V}_s}{ds}$$

$\vec{V}_s = \vec{V}(s,t)$

$$dV = \frac{\partial \vec{V}}{\partial s} ds + \frac{\partial \vec{V}}{\partial t} dt$$

$$\frac{dV_s}{dt} = a_s = \vec{V}_s \frac{d\vec{V}_s}{ds} + \cancel{\frac{\partial \vec{V}_s}{\partial t}}$$ steady

Euler's Eq'n

$$\boxed{\frac{1}{\rho}\frac{dP}{ds} + g\frac{dz}{ds} = -\vec{V}_s \frac{d\vec{V}_s}{ds}}$$

$$\frac{dP}{\rho} + g\,dz = -\vec{V}_s\, d\vec{V}_s$$

for $\rho$ = const.

$$\boxed{\frac{P}{\rho} + \frac{\vec{V}^2}{2} + gz = \text{constant}}$$

Bernoulli's Eq'n
- frictionless
- constant density
- no work

J. Hill

now <u>normal</u> to streamline

$$\left(P - \frac{\partial P}{\partial n}\frac{dn}{2}\right) ds\, dx - \left(P + \frac{\partial P}{\partial n}\frac{dn}{2}\right) ds\, dx - \rho g \cos\theta\, dn\, dx\, ds$$

$$= \rho a_n\, dn\, dx\, ds$$

$$-\frac{\partial P}{\partial n} - \rho g \overbrace{\cos\theta}^{\frac{dz}{dn}} = \rho a_n \text{, but } a_n = -\frac{\bar{V}_s^2}{r}$$

$$\boxed{\frac{1}{\rho}\frac{\partial P}{\partial n} + g\frac{\partial z}{\partial n} = \frac{\bar{V}_s^2}{r}}$$

Euler's Eq'n normal to streamline

in a horizontal plane, $\frac{dz}{dn} = \frac{dz}{ds} = 0$

So $\frac{dP}{\rho} = -\bar{V}_s d\bar{V}_s$ and $\frac{1}{\rho}\frac{dP}{dr} = \frac{\bar{V}^2}{r}$

combine $\vec{V} d\vec{V} + \frac{\vec{V}^2}{r} dr = 0$

separate var. $\frac{d\vec{V}}{\vec{V}} + \frac{dr}{r} = 0$

integrate $\ln \vec{V} + \ln \vec{r} = \text{constant}$

$$\ln(r\vec{V}) = \text{constant}$$

$$\boxed{r\vec{V} = \text{constant}}$$