Optimizing the Critical Path of Distributed Dataflow Graph Algorithms

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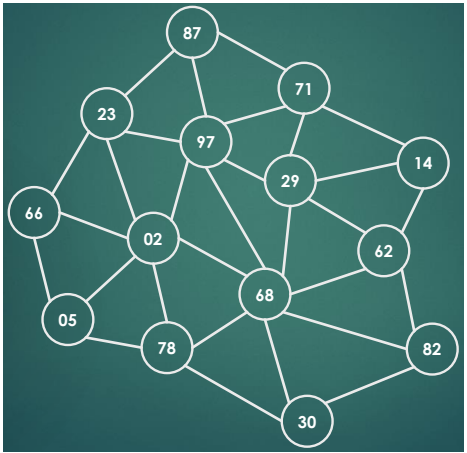
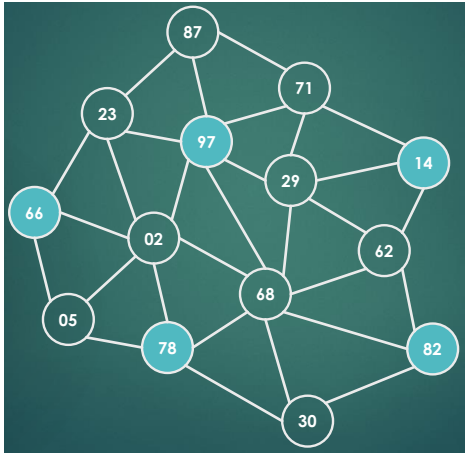
Luby's Algorithm is an Example of a Dataflow Algorithm

Luby's Algorithm for Maximal Independent Set

- Each vertex v picks unique random number $r(v)$ uniformly in $[0; 1)$
- v sends $r(v)$ to each of its neighbors u
- v notes which neighbors u have the property $r(u) > r(v)$
- v marks its own state as `unknown`
- v awaits a message from each of its neighbors u if $r(u) < r(v)$
- If the state of u is `marked`, the state of v is changed to `unmarked`
- After receiving messages from all neighbors u, if the state of v is `unknown`, the state of v is changed to `marked`
- v sends its state to all neighbors u, such that $r(u) > r(v)$
- All vertices in the `marked` state are a maximal independent set

Luby's Algorithm is an Example of a Dataflow Algorithm





Example from Randomized Algorithms (the University of Utah)

Distributed Dataflow Algorithms

- Only use local information
- Processing order of vertices is generated randomly
- Once the order is picked the vertices are processed from low to high in each neighborhood
- Cost to determine other desirable properties is too high
- We are interested in these methods as a model for distributed graph algorithms

Examples

- Luby's Algorithm for Maximal Independent Set
- Jones-Plassmann Algorithm for Graph Coloring

Choice of Random Order Affects Algorithm Runtime

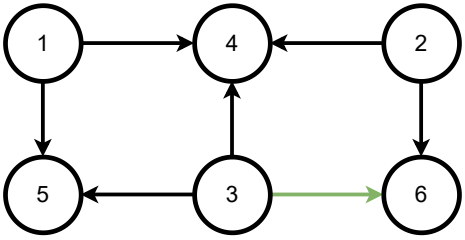


Figure 1: Lucky Draw

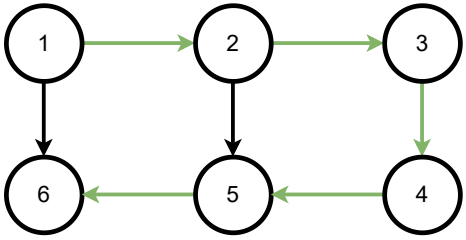


Figure 2: Unlucky Draw

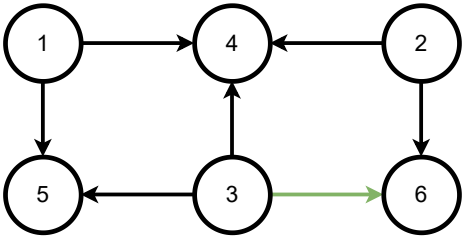


Figure 1: Lucky Draw

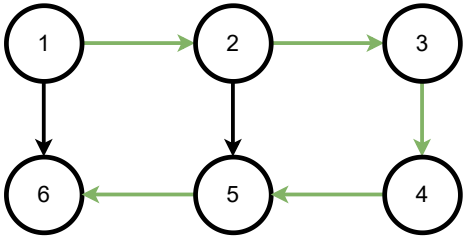


Figure 2: Unlucky Draw

This talk

How can we avoid unlucky draws?

Outline

Developing a Dataflow Model for Distributed Graph Algorithms

Model

- Let $G = \{V, E\}$ be an undirected graph and $w : V \to \mathbb{Z}^+$ be a weight function
- Assign $r(v)$ to each vertex v with your algorithm of choice
- Construct directed graph $\bar{G}$ by orienting the existing edges from low $r(v)$ to high $r(v)$
- Calculate length of critical path of $\bar{G}$

Critical Path

Longest weighted path in $\bar{G}$

Objective

Minimizing the length of the critical path (which minimizes the algorithm execution time)

The Weight Function is Non-trivial

Special Case: $w(v) = 1$

- Execution time is dominated by latency.
- The critical path is the number of phases for the graph.
- Critical path is the same as longest path using euclidean distance

Special Case: $w(v) = \delta(v)$

- Bandwidth or the cost of algorithms on the vertices themselves dominates the total execution time of the algorithm.
- Each vertex sends and receives $\delta(v)$ messages
- Most dataflow algorithms have each vertex do $O(\delta(v))$ computations
- Largest Degree First Order closely resembles that of Largest Processing Time First

The Centralized Problem is Coloring Vertices with Intervals

General Results on Arbitrary Graphs

- NP-Complete
- No Constant Approximation

On Stencil Graphs (IPDPS 2022)

- Studied the Interval Vertex Coloring Problem on 9-pt 2D stencils and 27-pt 3D stencils
- Proved the problem of interval coloring of the 27-pt 3D stencil is NP-Complete
- Developed and evaluated greedy heuristics based on analyses of special cases
- Showed number of colors correlates with real application runtime

Distributed Dataflow Graph Algorithms (PDCO 2023)

- Provide a model for dataflow algorithms as a distributed graph coloring problem
- Understand behavior of algorithms using different methods of generating partial orders

Deriving Better Partial Orders for Distributed Graph Algorithms

`Uniform` (aka draw in $[0; 1)$)

- Existing method of random number generation in dataflow algorithms

`Linear` (aka draw in $[0; \delta(v))$)

- v is guaranteed to be after all vertices u, such that $\delta(u) = \delta(v) - 1$ with probability $\frac{1}{\delta(v)}$
- Good approximation of Largest Degree First with vertices of dramatic difference in degrees
- Poor approximation when $\Delta(G)$ is large and G has many vertices of large degrees

`Exponential` (aka draw in $[0; 2^{\delta(v)})$)

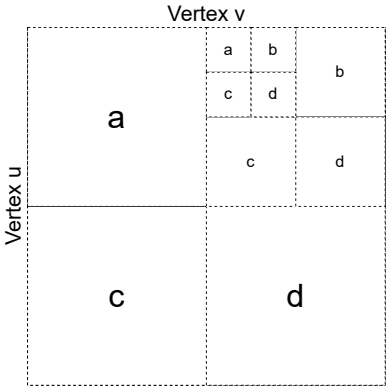
- v is guaranteed to be after all vertices u, such that $\delta(u) = \delta(v) - 1$ with probability $> \frac{1}{2}$
- Better approximation of Largest Degree First

- Communication and Computational cost is the same for each algorithm
- Sampling uniformly in those intervals despite naming conventions

Outline

Construction of RMAT Graphs

- 2^n nodes
- Recursively split square matrix into 4 quadrants: a, b, c, d
- Each quadrant has an associated probability that a given edge will fall into that quadrant: $a + b + c + d = 1$
- Edges are generated one at a time and placed in a quadrant recursively following those probabilities until the edge is placed in a 1×1 submatrix.
- $ef * 2^n$ edges



RMAT Graphs Study

Methodology

- Sampled RMAT parameter space with constant *ef*
- Computed critical path length for each algorithm
- Calculated 95% confidence intervals
- Computed pairwise ratios of critical paths
- Conducted Z-Test to validate statistical significance

Results

- `Exponential` path ¡ `Linear` path ¡ `Uniform` path
- `Exponential` was never worse than `Uniform`
- At best, `Exponential` was 50% better
- On average, `Exponential` was about 10% better

a	b	c	d	Uniform CI	Exponential CI	Linear CI	U/E	U/L	L/E
0.30	0.28	0.28	0.14	[2944; 2947]	[2623; 2625]	[2850; 2853]	1.123	1.033	1.087
0.40	0.24	0.24	0.12	[4950; 4953]	[4680; 4683]	[4859; 4862]	1.058	1.019	1.038
0.50	0.20	0.20	0.10	[6968; 6971]	[6643; 6647]	[6845; 6848]	1.049	1.018	1.030
0.60	0.16	0.16	0.08	[8353; 8357]	[7949; 7952]	[8175; 8178]	1.051	1.022	1.028
0.70	0.12	0.12	0.06	[9211; 9214]	[8738; 8740]	[8987; 8990]	1.054	1.025	1.029
0.30	0.28	0.17	0.25	[2111; 2113]	[2064; 2066]	[2103; 2105]	1.023	1.004	1.019
0.30	0.28	0.25	0.17	[2633; 2635]	[2437; 2439]	[2583; 2585]	1.080	1.019	1.060
0.30	0.28	0.34	0.08	[3782; 3785]	[3112; 3115]	[3510; 3513]	**1.215**	1.077	1.128
0.30	0.35	0.14	0.21	[2625; 2627]	[2047; 2049]	[2402; 2404]	**1.282**	1.093	**1.173**
0.30	0.35	0.21	0.14	[3064; 3067]	[2464; 2467]	[2825; 2827]	**1.243**	1.085	1.146
0.30	0.35	0.28	0.07	[3959; 3962]	[3221; 3225]	[3644; 3647]	**1.229**	1.086	1.131
0.30	0.42	0.11	0.17	[3632; 3635]	[2181; 2183]	[2880; 2883]	**1.665**	**1.261**	**1.321**
0.30	0.42	0.17	0.11	[3821; 3824]	[2433; 2436]	[3061; 3064]	**1.570**	**1.248**	**1.258**
0.30	0.42	0.22	0.06	[4343; 4346]	[3110; 3113]	[3685; 3688]	**1.396**	**1.178**	**1.185**
0.30	0.49	0.08	0.13	[4852; 4855]	[2245; 2246]	[3437; 3441]	**2.160**	**1.411**	**1.530**
0.30	0.49	0.13	0.08	[4775; 4778]	[2505; 2508]	[3325; 3330]	**1.906**	**1.435**	**1.328**
0.30	0.49	0.17	0.04	[5052; 5056]	[3151; 3155]	[3883; 3887]	**1.603**	**1.301**	**1.232**
0.40	0.24	0.14	0.22	[3246; 3249]	[3185; 3188]	[3242; 3245]	1.019	1.001	1.018
0.40	0.24	0.22	0.14	[4571; 4574]	[4377; 4380]	[4509; 4512]	1.044	1.014	1.030
0.40	0.24	0.29	0.07	[5946; 5950]	[5500; 5504]	[5759; 5763]	1.081	1.032	1.047
0.40	0.30	0.12	0.18	[4026; 4029]	[3457; 3460]	[3811; 3814]	**1.165**	1.056	1.102
0.40	0.30	0.18	0.12	[5003; 5006]	[4551; 4555]	[4821; 4825]	1.099	1.038	1.059
0.40	0.30	0.24	0.06	[6141; 6144]	[5652; 5656]	[5932; 5935]	1.086	1.035	1.049
0.40	0.36	0.10	0.14	[4903; 4906]	[3767; 3771]	[4333; 4336]	**1.301**	1.132	1.150
0.40	0.36	0.14	0.10	[5506; 5509]	[4637; 4641]	[5071; 5075]	**1.187**	1.086	1.094
0.40	0.36	0.19	0.05	[6383; 6387]	[5684; 5688]	[6045; 6049]	1.123	1.056	1.063
0.40	0.42	0.07	0.11	[5667; 5670]	[3901; 3906]	[4639; 4643]	**1.452**	**1.221**	**1.189**
0.40	0.42	0.11	0.07	[6196; 6199]	[4927; 4932]	[5478; 5483]	**1.257**	1.131	1.112
0.40	0.42	0.14	0.04	[6670; 6674]	[5681; 5685]	[6132; 6136]	**1.174**	1.088	1.079
0.50	0.20	0.12	0.18	[5009; 5012]	[4881; 4884]	[4981; 4984]	1.026	1.006	1.020
0.50	0.20	0.18	0.12	[6489; 6492]	[6213; 6216]	[6399; 6402]	1.044	1.014	1.030
0.50	0.20	0.24	0.06	[7813; 7816]	[7345; 7368]	[7632; 7635]	1.064	1.024	1.034
0.50	0.25	0.10	0.15	[5687; 5690]	[5230; 5234]	[5515; 5519]	1.087	1.031	1.054
0.50	0.25	0.15	0.10	[6920; 6924]	[6500; 6504]	[6748; 6751]	1.065	1.026	1.038
0.50	0.25	0.20	0.05	[7984; 7987]	[7503; 7507]	[7766; 7770]	1.064	1.028	1.035
0.50	0.30	0.08	0.12	[6294; 6297]	[5524; 5528]	[5943; 5946]	1.139	1.059	1.076
0.50	0.30	0.12	0.08	[7263; 7267]	[6644; 6648]	[6969; 6972]	1.093	1.042	1.049
0.50	0.30	0.16	0.04	[8073; 8076]	[7495; 7499]	[7786; 7789]	1.077	1.037	1.039
0.50	0.35	0.06	0.09	[6812; 6815]	[5778; 5781]	[6278; 6281]	**1.179**	1.085	1.087
0.50	0.35	0.09	0.06	[7537; 7540]	[6729; 6732]	[7108; 7111]	1.120	1.060	1.056
0.50	0.35	0.12	0.03	[8135; 8138]	[7420; 7423]	[7754; 7757]	1.096	1.049	1.045
0.60	0.16	0.10	0.14	[6491; 6495]	[6204; 6208]	[6406; 6409]	1.046	1.013	1.032
0.60	0.16	0.14	0.10	[7774; 7777]	[7418; 7422]	[7631; 7635]	1.048	1.019	1.029
0.60	0.16	0.19	0.05	[9077; 9080]	[8613; 8616]	[8843; 8846]	1.054	1.026	1.027
0.60	0.20	0.08	0.12	[6942; 6945]	[6427; 6431]	[6741; 6744]	1.079	1.030	1.049
0.60	0.20	0.12	0.08	[8257; 8260]	[7803; 7806]	[8044; 8048]	1.058	1.026	1.031
0.60	0.20	0.16	0.04	[9253; 9256]	[8754; 8757]	[8997; 9000]	1.057	1.028	1.028
0.60	0.24	0.06	0.10	[7261; 7264]	[6516; 6519]	[6906; 6909]	1.114	1.052	1.063
0.60	0.24	0.10	0.06	[8597; 8600]	[8041; 8044]	[8308; 8311]	1.069	1.035	1.033
0.60	0.24	0.13	0.03	[9283; 9286]	[8731; 8734]	[8987; 8990]	1.063	1.033	1.029
0.60	0.28	0.05	0.07	[7829; 7832]	[6958; 6962]	[7380; 7383]	1.125	1.061	1.061
0.60	0.28	0.07	0.05	[8537; 8540]	[7838; 7841]	[8157; 8160]	1.089	1.047	1.041
0.60	0.28	0.10	0.02	[9249; 9252]	[8631; 8634]	[8900; 8903]	1.072	1.039	1.031
0.70	0.12	0.07	0.11	[7229; 7232]	[6852; 6856]	[7101; 7105]	1.055	1.018	1.036
0.70	0.12	0.11	0.07	[8854; 8857]	[8424; 8427]	[8659; 8662]	1.051	1.023	1.028
0.70	0.12	0.14	0.04	[9813; 9816]	[9197; 9200]	[9514; 9517]	1.067	1.031	1.034
0.70	0.15	0.06	0.09	[7797; 7800]	[7252; 7255]	[7573; 7576]	1.075	1.030	1.044
0.70	0.15	0.09	0.06	[9093; 9096]	[8589; 8592]	[8850; 8853]	1.059	1.027	1.030
0.70	0.15	0.12	0.03	[10032; 10035]	[9347; 9350]	[9694; 9696]	1.073	1.035	1.037
0.70	0.18	0.05	0.07	[8267; 8270]	[7587; 7591]	[7941; 7945]	1.090	1.041	1.047
0.70	0.18	0.07	0.05	[9183; 9186]	[8599; 8602]	[8878; 8879]	1.068	1.034	1.032
0.70	0.18	0.10	0.02	[10074; 10076]	[9370; 9372]	[9700; 9702]	1.075	1.039	1.035
0.70	0.21	0.04	0.05	[8667; 8669]	[7899; 7902]	[8263; 8266]	1.097	1.049	1.046
0.70	0.21	0.05	0.04	[9168; 9171]	[8503; 8506]	[8798; 8800]	1.078	1.042	1.035
0.70	0.21	0.07	0.02	[9844; 9846]	[9198; 9200]	[9470; 9472]	1.070	1.039	1.030

Understanding the Results of the RMAT Graphs Study

Why is `Exponential` better on RMAT Graphs

- RMAT Graphs have the same properties of a social network
- Social networks are "Onion-like" - dense core, but outer layers become less dense
- `Exponential` is similar to Largest First

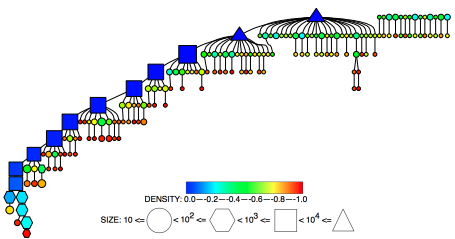


Figure 3: Hierarchy of Dense Subgraphs by Sariyuce et al. (2015)

Building a Real World Application from the RMAT Graphs Study

Real World Application

- Conducted similar experiment on real world graphs from SNAP
- All graphs have small world properties, except roads of Pennsylvania
- `Exponential` was better except on ca-HepPh and roadNet-PA

Name	Vertices	Edges	Max Degree	Clustering Coefficient	Diameter	Uniform CI	Exponential CI	Linear CI	U/E	U/L	L/E
CA-HepPh	89,209	118,521	491	0.6115	13	[1030; 1036]	[1040; 1045]	[1032; 1037]	0.991	0.999	0.992
Email-Enron	36,692	183,831	1,383	0.4970	11	[43437; 43720]	[38836; 38982]	[40688; 41002]	1.120	1.067	1.050
p2p-Gnutella04	10,879	39,994	103	0.0062	9	[911; 925]	[568; 575]	[728; 740]	**1.606**	**1.251**	**1.284**
roadNet-PA	1,090,920	1,541,898	9	0.0465	786	[49; 49]	[49; 50]	[48; 49]	0.990	1.010	0.980
soc-Epinions1	75,888	405,740	3,044	0.1378	14	[94793; 95270]	[88297; 88593]	[89488; 90034]	1.074	1.059	1.015
soc-pokec-relationships	1,632,804	22,301,964	14,854	0.1094	11	[118924; 119528]	[96958; 97239]	[100836; 101775]	**1.228**	**1.177**	1.043
web-Google	916,428	4,322,051	6,332	0.5143	21	[80466; 81618]	[18166; 18192]	[20577; 21084]	**4.458**	**3.891**	1.146
WikiTalk	2,394,385	4,659,565	100,029	0.0526	9	[1352414; 1357165]	[1101248; 1103043]	[1145942; 1151894]	**1.229**	**1.179**	1.042

Results of the Real World Application

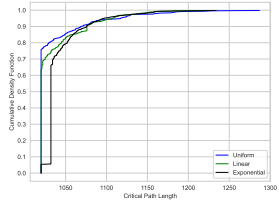


Figure 4: ca-HepPh

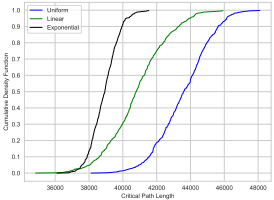


Figure 5: email-Enron

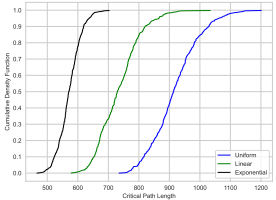


Figure 6: p2p-Gnutella04

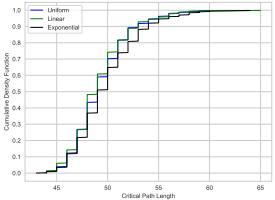


Figure 7: roadNet-PA

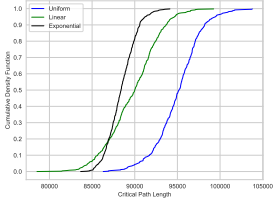


Figure 8: soc-Epinions1

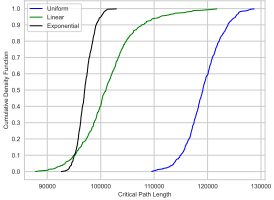


Figure 9: soc-pokec

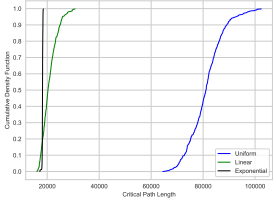


Figure 10: web-Google

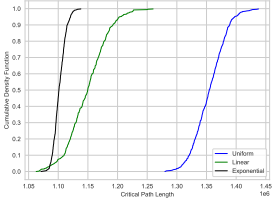


Figure 11: wiki-Talk

Outline

Summary

Conclusion

- We provided a model for dataflow algorithms as a distributed graph coloring problem.
- We presented new ways to generate partial orderings and provided a theoretical argument for why they are sound.
- We studied the behavior of algorithms using different partial orders on both randomly generated RMAT graphs and graphs from real world applications.
- We showed that `Exponential` algorithm yields a shorter critical path than other partial orderings in the study of RMAT graphs and real world applications.

Future Works

- Develop a stronger theoretical argument for why `Exponential` performs better than `Uniform`
- Consider other graph models

Thank you!

Papers

Dante Durrman and Erik Saule. Optimizing the critical path of distributed dataflow graph algorithms. In Proceedings of IPDPS Workshops (IPDPSW); PDCO, 2023.
Dante Durrman and Erik Saule. Coloring the vertices of 9-pt and 27-pt stencils with intervals. In Proc. of IPDPS, May 2022.

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