On the Network Lifetime of Wireless Sensor Networks Under Optimal Power Control

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Abstract- In this paper we evaluate the effect of power control on the lifetime of wireless sensor networks that are equipped with fixed energy resources. We assume a data collection traffic pattern where all sensor nodes forward data to a centralized base station (sink). Asynchronous low power listening is assumed, which significantly reduces energy wastage by utilizing sleep cycles of the radio. In such networks, overhearing causes significant energy losses, especially if the network density is high. Our calculations demonstrate the benefits of applying power control and the spatial variations of energy consumption in the network under optimal power control.

Keywords: Wireless sensor networks, optimal power-control, network lifetime.

I. INTRODUCTION

Wireless sensor networks (WSNs) comprise of a set of nodes that are equipped with integrated low power processor, memory, and a radio module in addition to one or more sensors. Typically, sensor nodes are powered by fixed energy resources such as a battery, and consequently, their utility is limited by the lifetime of the battery. Hence it is critical that all operations in the nodes, including networking functions and sensing applications, are performed under considerations for conserving energy. This resulted in a number of novel approaches for energy-efficient networking protocols, particularly for medium access control (MAC), since the radio component consumes the maximum amount of power in the sensor nodes. Another popular approach for energy conservation is transmission power control, which reduces the energy consumed for transmissions as well as from overhearing.

In this work we develop a mathematical model to evaluate the network lifetime of wireless sensor networks under optimal power control. We assume a data collection traffic pattern, which is typical for a large number of environmental monitoring applications. Our calculations are based on the fact that nodes apply a link-quality based routing protocol, such as collection tree protocol (CTP) [1], to forward data periodically to the sink. We assume the nodes apply an asynchronous low power listen scheme that applies periodic sleep and wake cycles for conserving energy in the radio. In such networks, power control can significantly reduce the energy consumption in the nodes. We first calculate the optimal transmission range of the nodes so that the overall current consumption is minimized. We then apply this result to calculate the network lifetime assuming that all nodes apply transmit power to achieve this optimal transmission range. Our objective is to determine the effect of transmission power control on the lifetime of the network that is primarily affected by energy consumed in transmissions, receiving, and overhearing.

The rest of the paper is organized as follows. In section II, we describe our system model. Section III describes the calculation of the optimal transmission range of the nodes so that the overall current consumption is minimized. In section IV, we calculate the network lifetime using this optimal transmission range assuming that the transmit power can be controlled based on this transmission range. Results are shown in section V. We conclude our paper section VI.

II. SYSTEM MODEL

We assume a data collection network, in which sensor nodes periodically sense some physical parameters and forward these data to the sink. For exchanging different controlling parameters among themselves, nodes broadcast periodic beacons as well. In large scale WSNs that do not use transmission scheduling, synchronous sleep and wake cycles is difficult to implement. In such a network to conserve energy, nodes use low-power listening (LPL) [2], [3] where a node periodically checks (polls) the wireless channel for an incoming packet. If there is no transmission on the channel, it switches off the radio until the next poll. Otherwise it stays on to receive the incoming packet. In LPL, the sender prepends the message with a preamble that is long enough to span the complete length of the poll interval to ensure that the receiving node observes it regardless of when it wakes up. Because of this long preamble length (for both beacons and data packets), the effect of overhearing becomes costly. In such scenarios, transmission power control can significantly reduce the effect of overhearing.

Under these assumptions, we came up with an experimentally validated model [4] of the estimated current consumption
of a node as

\[ I = \frac{I_B T_B}{T_B} + M I_D T_D + S I_B T_B + O I_D T_D + F I_D T_D + R I_D T_D + \frac{P}{T_D} + \mathcal{P} I_R T_P \]  

(1)

where \( I_s \) and \( T_x \) represent the current drawn and the duration, respectively, of the event \( x \); and \( T_B \) represents the beacon interval. Transmission/reception of beacon packets is denoted by \( B_t/B_r \), data transmit/receive is denoted by \( D_t/D_r \) and processing and sensing are denoted as \( P \) and \( S \), respectively. \( O \) and \( F \) are the overhearing and forwarding rate respectively and \( S \) is the number of neighbors. \( M \) is the rate at which a node transmits its own packets and \( R \) is the reception rate. \( \mathcal{P} \) is the number of times a node wakes up and checks the channel. We assume \( \mathcal{P} = 8 \) in this paper. Under these assumptions, we first calculate the optimal transmission range for multi-hop transmission to minimize the overall power consumption.

III. OPTIMAL TRANSMISSION RANGE CALCULATION

Let us assume that the current drawn by the receiver electronics in the receiving mode is \( I_{Rr} = \alpha_{12} \). In transmit mode, the amount of current drawn is dependent on the transmit power. Assuming that optimal power control is applied, to transmit a packet over a distance \( d \) with a path loss exponent of \( n \), the current drawn is

\[ I_{Dt} = \alpha_{11} + \alpha_3 d^n \]  

(2)

where \( \alpha_{11} \) is the current consumed by the transmitter electronics, \( \alpha_3 \) accounts for current dissipation in the transmit op-amp. The duration of a packet transmission and reception is proportional to the packet length. We assume that both the data packets and the beacon packets are of same length, thus \( T_{Dt} = T_{Dr} = T_{Br} = T_b \). \(^{1}\) Thus, the amount of current consumed by a relay node that receives a packet and transmit it \( d \) meters onward is,

\[ I_{relay}(d) = (\alpha_{11} + \alpha_3 d^n + \alpha_{12}) T_l \]  

(3)

Also let us assume that \( \rho \) is the node density (i.e. number of nodes in an unit area), then the expected number of nodes that overhear the transmission is given by \( \pi d^2 \rho \). \(^{2}\) When \( \pi d^2 \rho > 2 \), the expected number of overhearing is positive, otherwise it is zero. We deduce 2 because the transmitter and the receiver are not counted as the nodes that overhear. Thus the current consumed for overhearing while transmitting a packet is given by \( I_{ov} = (\pi d^2 \rho - 2) \alpha_{12} T_l \).

Let us first calculate the total current consumed to transmit a packet from \( A \) to \( B \) with \( K - 1 \) relays between them as shown in Figure 1(a). The distance between \( A \) and \( B \) is \( D \). Thus the overall energy consumed is given by

\[ I_T(D) = \sum_{i=1}^{K} (I_{relay}(d_i) + I_{ov}(d_i)) = \sum_{i=1}^{K} I_R(d_i) \]  

(4)

\(^{1}\)The reason behind this assumption is that with low-power operation, nodes send a long preamble before actual transmission.

Theorem 1: Given \( D \) and \( K \), \( I_T(D) \) is minimized when all hop-distances are made equal to \( \frac{D}{K} \).

Proof: The proof is done in the similar line as done in [5]. Note that \( I_R(d) \) is strictly convex as \( \frac{d^2 I_R}{d^2} > 0 \). Thus from Jensen’s inequality, we can write

\[ I_R \left( \sum_{i=1}^{K} \frac{d_i}{K} \right) \leq \sum_{i=1}^{K} I_R(d_i) \]

\[ \Rightarrow K I_R \left( \frac{D}{K} \right) \leq \sum_{i=1}^{K} I_R(d_i) \]

\[ \Rightarrow K I_R \left( \frac{D}{K} \right) \leq I_T(D) \]  

(6)

which completes the proof.

Thus, the minimum energy consumption for sending a packet to a distance \( D \) using \( K \) hops is given by \( I_T(D) = \left( \alpha_1 K + \alpha_2 K \left( \frac{D}{K} \right)^2 + \alpha_3 K \left( \frac{D}{K} \right)^n \right) T_l \). Differentiating \( I_T(D) \) with respect to \( K \) are setting to zero, we get \( \alpha_1 - \alpha_2 \left( \frac{D}{K} \right)^2 + (n-1) \alpha_3 \left( \frac{D}{K} \right)^n = 0 \). If \( K_{\text{opt}} \) is the optimal value of \( K \), then the characteristic distance \( d_m = \frac{D}{K_{\text{opt}}} \).

Replacing \( d_m \) in the previous equation, we get

\[ \alpha_1 - \alpha_2 d_m^2 + (n-1) \alpha_3 d_m^n = 0 \]  

(7)

By solving this equation 7, we get \( d_m \) in terms of \( \alpha_1 \), \( \alpha_2 \), \( \alpha_3 \), \( n \). More importantly \( d_m \) is independent of \( D \). Also to ensure the connectivity, there should be at least 3 nodes in the area of \( \pi r^2 \), where \( r \) is the transmission range, i.e.

\[ \pi r^2 \rho \geq 3 \Rightarrow r \geq \sqrt{\frac{3}{\pi \rho}} \]  

(8)

Thus the optimal transmission range is \( d_o = \max \{d_m, r_{\text{min}}\} \) where \( r_{\text{min}} = \sqrt{\frac{3}{\pi \rho}} \).

Theorem 2: If the maximum current drawn by a radio to transmit at its maximum transmit power is \( I_{t_{\text{max}}} \) and the current drawn in the receive mode is \( I_r \), then \( d_o = r_{\text{min}} \) as long as the \( I_{t_{\text{max}}} < 4 I_r \).

Proof: Let us assume that \( h^h \) and \( h^{h+1} \) are the overall current consumption when there are \( h \) and \( h + 1 \) hops present
in between A and B. Also \( r_i = \frac{D}{i} \) is the transmission range when there are \( i \) hops in between A and B. To preserve the connectivity, \( \pi r_i^2 \rho \geq 3 \forall i \). Then,

\[
I^h = (I_t^h + (\pi r_i^2 \rho - 2) I_r + I_r) . h . T_i
\]

\[
I^{h+1} = (I_t^{h+1} + (\pi r_{i+1}^2 \rho - 2) I_r + I_r) . (h + 1) . T_i
\]

\[
\Delta^T = I^h - I^{h+1} = \Delta^T + \Delta^R
\]

where

\[
\Delta^T = I^h_h . h . T_i - I^{h+1}_i . (h + 1) . T_i
\]

\[
( (I^h_h - I^{h+1}_h) . h . T_i - I^{h+1}_i . T_i)
\]

\[
\Delta^R = ( (\pi r_{k+1}^2 \rho - 1) . h - (\pi r_{h+1}^2 \rho - 1) . (h + 1) ) . I_r . T_i
\]

\[
(\pi r_i^2 \rho + 1) . I_r . T_i
\]

\[
(3 + 1) . I_r . T_i = 4 . I_r . T_i
\]

When \( \Delta^T \geq 0 \) then \( \Delta^T > 0 \). When \( \Delta^T < 0 \) then \( \Delta^T = (I^h_h - I^{h+1}_h) . h . T_i - I^{h+1}_i . T_i > -I^{h+1}_i . T_i \) as \( I^h_h > I^{h+1}_i \). Thus \( \Delta^R = \Delta^T + \Delta^R > (I^{h+1}_i + 4 . I_r) . T_i \) which is positive if \( I^{h+1}_i < 4 . I_r \). This concludes that if \( I^{h+1}_i < 4 . I_r \), \( I^h_h > I^{h+1}_i \) i.e. increasing the number of hops results in reduced current consumption as long as \( r \geq \sqrt{\frac{3}{\pi \rho}} \). At \( r = \sqrt{\frac{3}{\pi \rho}} \), the current consumption is minimized, i.e. \( d_o = r_{min} \).

Generally for all radios that are used in sensor networks such as CC1000 (used by MICA2 motes) and CC2420 (used by MICAz motes), \( I_{t max}^{cc1000} < 4 . I_r \). For CC1000 radios \( I_{t max}^{cc2420} = 27 \) mA and \( I_r = 10 \) mA whereas for CC2420 radios \( I_{t max}^{cc2420} = 17.4 \) mA and \( I_r = 19.7 \) mA. Thus for these kinds of low power devices, it is always good to use the minimum power that is sufficient to preserve the network connectivity and required quality. Figure 2 shows the variation of \( I_T \) with the increase in number of hops. The maximum number of hops occurs when the distance between each node is \( r_{min} \). From this figure we can observe that when the number of hops is less, overhearing dominates due to high transmission range. With the increase in number of hops, overhearing starts reducing whereas consumptions due to reception and transmission increase as the number of relays increases. This is to be noted that when \( T_i^{max} < 4 . I_r \), \( d_0 = r_{min} \). For transceivers with \( T_i^{max} > 4 . I_r \), \( d_0 \) has to be calculated as \( \max(d_m, r_{min}) \).

IV. NETWORK LIFETIME CALCULATION

In the previous section, we conclude that when the distance between intermediate nodes is \( d_o \), we get the minimum current consumption. With this we calculate the upper limit of the lifetime of a network, where \( N \) sensor nodes are uniformly distributed in an area of \( A \times B \). Now divide this region by rectangular areas of width \( d_o \) as shown in Figure 1(a). Let us define each rectangular region as a cut. Nodes in any cut forward their packets to the nodes that are in their immediately right cut. When there are not enough number of nodes in a cut, the packets sent by the preceding cuts cannot be able to cross that cut, thus the network is partitioned. Now we calculate the energy consumption in each cut. Let us assume that each node generates \( b \) packets/seconds and the beacon rate is \( B \) beacons/seconds. Define the left-most cut as the first cut, the next one as the second and so on. Also let us assume that the right-most one is the \( m \)-th cut, where \( m = \frac{A}{d_o} \). We assume that the nodes in any cut convey the traffic of the nodes in their left cuts. Thus, nodes in the first cut transmits \( b \) packets/seconds, the nodes in the second cut on average transmit \( 2b \) packets/seconds (their own \( b \) packets/seconds + packets generated by the first cut). So, the nodes in the \( i \)-th cut on average transmit \( ib \) packets/seconds. Thus the expected energy consumed for different actions under our assumptions in the \( i \)-th cut can be written as:

\[
T_{bb} = ib(\alpha_{11} + \alpha_3 d_{o}^3) . T_i
\]

\[
T_{bb} = (i - 1) bo_{12} . T_i
\]

\[
T_{bb} = B (\alpha_{11} + \alpha_3 d_{o}^3) . T_i
\]

\[
T_{bb} = (\pi d_{o}^2 \rho - 1) bo_{12} . T_i
\]

Now let us calculate the expected overhearing in the \( i \)-th cut with the help of Figure 1(b). Let us consider a point \( a \) and draw a circle with radius \( d_o \). Thus if we place a node at \( a \), that node overhears all traffic that are forwarded by the nodes that are inside this circle. Nodes that are in \( A_1 \) and \( A_2 \) transmit at \((i - 1)b \), \( ib \) and \((i + 1)b \) packets/seconds. The areas of \( A_1 \), \( A_3 \) and \( A_2 \) can be written as \( d_{o}^2 (\theta_2 - \sin \theta_2 \cos \theta_2) \), \( d_{o}^2 (\theta_1 - \sin \theta_1 \cos \theta_1) \) and \( \pi d_{o}^2 - A_1 - A_2 \) respectively. Thus the expected number of packets that a node at \( a \) overhears in a second is given by:

\[
\nu_{ov} = E[A_1](i - 1)b + E[A_2]b - 1)ib + E[A_3](i + 1)b \text{ for } i < m
\]

\[
E[A_1] = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} d_{o}^2 (\theta_2 - \sin \theta_2 \cos \theta_2) . d\theta_2
\]

\[
E[A_3] = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} d_{o}^2 (\theta_1 - \sin \theta_2 \cos \theta_1) . d\theta_1
\]

\[
E[A_2] = \pi d_{o}^2 - E[A_1] - E[A_2]
\]
Then
\[
T_{ov}^i = \frac{2\sin(\pi T_i(\pi, \sigma_0^2, \sigma_1^2))}{\pi} \quad \text{for } i < m
\]
\[
= \frac{2\sin(\pi T_i(\pi, \sigma_1^2, \sigma_2^2))}{\pi} \quad \text{for } i = m
\]
(11)

Thus the total current consumption for the nodes in the \(i\)-th cut is
\[
T_i = T_{D_t} + T_{D_r} + T_{b_t} + T_{b_r} + T_{o_v} + T_{b} + T_p
\]
(12)

**Expected Lifetime for Same Battery Capacities:** We define the initial battery capacity of each node by \(e_0\) and \(\tau\) is the cut-off capacity, beyond that the sensor mote does not work. Then the expected lifetime of any node in the \(i\)-th cut \(L_i\) can be written as \(L_i = \frac{e_0 - \tau}{\Delta t}\). For any \(i < m\), it can be shown that \(L_i < L_{i-1}\). Now let us compare \(L_i\) for \(i = m - 1\) and \(i = m\). Clearly, \(T_{m-1}^i > T_{m}^i\) and \(T_{D_r} > T_{D_t}^{m-1}\). But \(T_{ov}\) can be greater or less than \(T_{m-1}^i\) based on the values of different parameters. This is because, nodes in the \((m - 1)\)-th cut overhear from transmissions from both \((m - 2)\)-th cut and \(m\)-th cut, whereas nodes in the \(m\)-th cut overhear only from transmissions from \((m - 1)\)-th cut. Thus \(L_i\) is minimum when \(i = m\) or \(i = m - 1\). Thus in case of rechargeable WSNs, we need ensure that a node is recharged at a rate at least as high as \(T_i\), \((i\) is \(m\) or \((m - 1)\)) to avoid network partitioning.

We note that \(T_i\) is the rate of minimum current consumption in the \(i\)-th cut. Recently there is a growing interest of using energy harvesting, such as using solar power to recharge the sensor nodes to prolong the lifetime of the network. Thus to make the network survivable, we need ensure that a node is recharged more than the rate at which its charge is dissipated. That gives an important finding, i.e. we need to ensure that nodes are recharged at a rate at least as high as \(T_i\), \((i\) can be \(m\) or \((m - 1)\)) to avoid network partitioning.

**Expected Lifetime for Gaussian Distributed Battery Capacities:** In real deployment scenarios, sensor nodes have different battery capacities due to the spatio-temporal variations of network traffic and the amount of remaining energy of the individual nodes, which can be modeled as a Gaussian distribution. Thus we assume that the initial battery capacity of any node is a Gaussian random variable with mean \(\mu\) and standard deviation \(\sigma\). We define the remaining capacity of any node \(k\) in the \(i\)-th cut at time \(t_j\) is \(e_{ki}(t_j)\). The network starts at time \(t_0\) and so \(e_{ki}(t_0) \sim N(\mu, \sigma^2)\). Then the probability that the current consumption of a node in the \(i\)-th cut is greater than \(\tau\) at time \(t_j\) (assume \(t_j - t_0 = \Delta t\) is
\[
p_i = P[e_{ki}(t_j) > \tau] = P[e_{ki}(t_0) - T_i \cdot \Delta t > \tau] = \frac{\pi (\mu - T_i \cdot \Delta t - \mu)}{2\sigma^2}\]
(13)

Thus expected number of nodes at time \(t_j\) in the \(i\)-th cut whose capacity is greater than \(\tau\) is given by \(\sum_{x=1}^{N} x \cdot \frac{\mu x}{\sigma^2} (1 - p_i)^{N-x}\), where \(N = \frac{\mu}{\sigma}\) is the number of nodes in each cut. If we assume that the lifetime of the cut is the time till \(f\) fraction of the nodes stay alive then
\[
\sum_{x=1}^{N} x \cdot \frac{\mu x}{\sigma^2} (1 - p_i)^{N-x} = f \cdot N
\]
(14)

By solving equation (14) we find the expected lifetime of any cut \(i\). Clearly the lifetime is minimum for the \(m\)-th or the \((m - 1)\)-th cut. This is depicted in Figure 3 and the parameters used for the results are listed in Table I. We place 100 nodes uniformly in an area of 200\(\times\)200 meter\(^2\). We consider two cases: First, the initial battery capacities of all nodes are same and equal to 5000 mAHr. Second, the battery capacities are normally distributed with mean of 5000 mAHr and standard deviation of 1000 mAHr. In case of normally distributed battery capacities, the expected lifetime is calculated as the time till the 75% of nodes in a cut survive. Each node transmits a packet and a beacon in every minute. \(\tau\) is assumed to be 0. As shown in Figure 3, with these set of parameters the expected network lifetime is lowest for the nodes in \((m - 1)\)-th cut. Also expected lifetime of nodes in \(m\)-th cut is slightly higher than that of the \((m - 1)\)-th cut, but lower than others.

**V. RESULTS**

In this section we compare the expected network lifetime under two battery capacity distributions (same battery capacities and Gaussian distributed battery capacities) with the same setup as done for Figure 3. Figure 4 shows the variation of lifetime for the \((m - 1)\)-th cut with different number of nodes, placed uniformly in an area of 200\(\times\)200 meters\(^2\). From this figure we can observe that when all nodes have same battery capacity, the lifetime is higher compared to the case when the battery capacities are Gaussian distributed. This is obvious because of the fact that when all nodes in a cut have the same capacity, they all die at the same time. On the other hand in case of Gaussian distributed battery capacities, a fraction of nodes in a cut die faster than others, which reduces the network lifetime. Also the expected network lifetime decreases with the increasing number of nodes because of increased overhearing.

<table>
<thead>
<tr>
<th>(I_{Br})</th>
<th>Values</th>
<th>(T_{Br})</th>
<th>Values</th>
<th>(I_{Dr})</th>
<th>Values</th>
<th>(T_{Dr})</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.7 mA</td>
<td>8 mA</td>
<td>17.4 mA (-0 dBm)</td>
<td>140 ms</td>
<td>10.7 mA (-1 dBm)</td>
<td>140 ms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.5 mA (-7 dBm), 11.2 mA (-10 dBm)</td>
<td>3 ms</td>
<td>15.2 mA (-3 dBm), 13.9 mA (-5 dBm)</td>
<td>7.5 mA</td>
<td>9.9 mA (-15 dBm)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**VI. CONCLUSION**

The fundamental challenge of designing WSN protocols is to maximize the network lifetime. This paper addresses a theoretical calculation of network lifetime for WSNs for different capacity distributions and compare the effects of different parameters on network lifetime under optimal power control scheme. Other than power control, another important factor for controlling overhearing as well as network lifetime is to consider the effects of multiple channels [6], [7]. In future we like to incorporate the effects of these distributed routing

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Table I: Different Parameters for MICA2

- **Var**: Different Parameters
- **Values**: Values for each parameter

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and channel selection schemes on network lifetime on top of optimal transmission power control.

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