Sparse Power Efficient Topology for Wireless Networks

Xiang-Yang Li * Peng-Jun Wan * Yu Wang * Ophir Frieder *

Abstract

Due to the nodes' limited resource in the wireless ad hoc networks, the scalability is crucial for network operations. One effective approach is to maintain only a linear number of links using a localized construction method. In this paper, we consider how to combine several well-known proximity graphs including Gabriel graph and Yao graph to construct power efficient networks. These graphs are sparse and can be constructed locally in an efficient way. We propose two different methods to construct power efficient networks. Firstly, we construct a network topology by using the Gabriel structure and the Yao structure, which has at most $O(n)$ edges and each node has a bounded out-degree. Secondly, we construct a graph by using the Yao structure and then using the reverse of the Yao structure. The constructed topology is guaranteed to be connected if the original unit disk graph is connected. It also has a bounded node degree. The experimental results show that it has a bounded power stretch factor in practice. It has been shown that broadcasting based on the Euclidean minimum spanning tree uses energy no more than a constant factor of the optimum. Our experimental results show that the two graph structures proposed in this paper also have a bounded total broadcasting energy cost in practice.

Keywords: Wireless ad hoc networks, topology control, power consumption, network optimization.

1 Introduction

Due to the nodes' limited resource in wireless ad hoc networks, the scalability is crucial for network operations. One effective approach is to maintain only a linear number of links using a localized construction method. However, this sparseness of the constructed network topology should not compromise too much on the power consumptions on both unicast and broadcast/multicast communications. In this paper, we study how to construct a sparse network topology efficiently for a set of static wireless nodes such that every unicast route in the constructed network topology is power efficient. Here a route is *power efficient* for unicasting if its energy consumption is no more than a constant factor of the least energy needed to connect the source and the destination. A network topology is said to be power efficient if there is a power efficient route to connect any two nodes.

We consider a wireless ad hoc network consisting of a set V of wireless nodes distributed in a two-dimensional plane. Each wireless node has an omnidirectional antenna. This is attractive for a single transmission of a node can be received by many nodes within its vicinity. In the most common power-attenuation model, the power needed to support a link uv is $\|uv\|^{\beta}$, where $\|uv\|$ is the distance between u and v, β is a real constant between 2 and 4 dependent on the wireless transmission environment. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes if and only if their Euclidean distance is at most one. The size of the unit disk graph could be as large as the square order of the number of network nodes. Given a unicasting or multicasting request, the *power efficient routing problem* is to find a route whose energy consumption is within a small constant factor of the optimum route. Notice that the time complexity of computing the shortest path connecting two nodes is proportional to $O(m + n \log n)$, where m is the number of links in the network and n is the number of nodes if a centralized algorithm is used. Consequently the power efficient routing over this unit disk graph is unscalable because here m

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could be as large as $O(n^2)$.

Recently, Rodoplu and Meng [9] described a distributed protocol to construct a topology, which is guaranteed to contain the least energy path connecting any pair of nodes in the unit disk graph. However, their protocol is not time and space efficient. Recently, [7] improved their result by giving an efficient localized algorithm to construct a new network topology that is guaranteed to be a subgraph of the graph constructed by Rodoplu and Meng [9]. They proved that the constructed topology is sparse, i.e., it has a linear number of edges.

A further trade-off can be made between the sparseness of the topology and its power efficiency. Recently, Wattenhofer *et al.* [13] tried to address this trade-off. Unfortunately, their algorithm is problematic and their result is erroneous which was discussed in detail in [8]. In [8], Li *et al.* studied the power efficiency property of several well-known proximity graphs including the relative neighborhood graph, the Gabriel graph and the Yao graph. These graphs are sparse and can be constructed locally in an efficient way. They showed that the power stretch factor of the Gabriel graph is always one, and the power stretch factor of the Yao graph is bounded from above by a real constant while the power stretch factor of the relative neighborhood graph could be as large as the network size minus one. Notice that all of these graphs do not have constant bounded node degrees. They further proposed another sparse topology, namely the sink structure, that has both a constant bounded node degree and a constant bounded power stretch factor. An efficient localized algorithm [8] is presented for constructing this topology.

In this paper, we present some new localized algorithms to construct sparse and power efficient topologies. We show that several combinations of the Yao graph and the Gabriel graph are power-efficient and have at most $O(n)$ edges while each node has a bounded out-degree. In addition, we show that the topology constructed by using the Yao structure and the reverse of the Yao structure is a connected graph if the unit disk graph is connected. We also conduct experiments to show that this topology is power efficient in practice.

The rest of the paper is organized as follows. In Section 2, we first give some definitions and review some results related to the network topology control. In Section 3, we propose two methods to combine some well-known geometry structures to construct network topologies. One method guarantees a bounded power stretch factor in theory, the other guarantees a bounded node degree in theory. We found that both structures have a bounded power stretch factor and a bounded node degree in practice. In addition, the broadcasting schemes based on these two structures consume energy no more than a constant factor of the minimum necessary in practice. We conclude our paper in Section 4 by discussing some possible future works.

2 Preliminaries

2.1 Geometry Structures

Let V be a set of n wireless nodes distributed in a two-dimensional plane. These nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes if and only if their Euclidean distance is at most one. We say a node u can see another node v in a graph G if edge $uv \in G$ and the Euclidean distance $||uv||$ between u and v is less than 1. Notice that here G could be a directed graph so edge uv could also be a directed edge. The *constrained relative neighborhood graph* over a (directed) graph G, denoted by $RNG(G)$, is defined as follows. It has an (directed) edge uv iff $uv \in G$ and there is no point $w \in V$ such that u can see w and w can see v. The *constrained Gabriel graph* over a (directed) graph G, denoted by $GG(G)$, has an (directed) edge uv iff $uv \in G$ and the open disk using uv as a diameter does not contain any node w from V such that both (directed) edges uw and wv are in G. The *constrained Yao graph* over a (directed) graph G with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(G)$, is defined as follows. At each node u, any k equal-separated rays originated at u define k equal cones. In each cone, choose the shortest (directed) edge $uv \in G$, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily. If we add the link $\overrightarrow{vu}$ instead of the link $\overrightarrow{uv}$, the graph is denoted by $\overleftarrow{YG}_k(G)$, which is called the *reverse of Yao graph*. Let $YG_k(G)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k(G)$. See the following Figure 1 for an illustration of selecting edges incident on u in the Yao graph.

These graphs extend the conventional definitions of corresponding ones for the completed Euclidean graph; see [5, 6, 14]. Notice that in all of the definitions, when the graph G itself is a directed

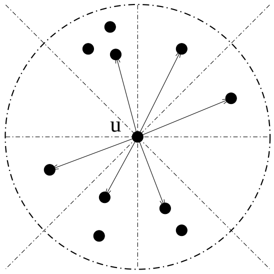

Figure 1: The narrow regions are defined by 8 equal cones. The closest node in each cone is a neighbor of u.

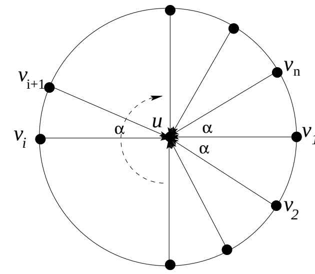

Figure 2: Node u in the relative neighborhood graph has degree n.

graph, all edges in the defined graphs carry their directions also. All these graphs are subgraphs of G. Moreover, the following lemmas had been proved.

Lemma 1 *$RNG(G)$ is a subgraph of $GG(G)$.*

Lemma 2 *If G is $UDG(V)$, $RNG(G) \subseteq YG_k(G)$.*

However, for a general graph G, it is not guaranteed that $RNG(G)$ is a subgraph of $YG_k(G)$.

Lemma 3 *If $UDG(V)$ is connected, it contains the Euclidean minimum spanning tree $EMST(V)$.*

Lemma 4 *If G is $UDG(V)$, $YG(G)$, $GG(G)$ and $RNG(G)$ contain $EMST(V)$ as a subgraph.*

For simplicity, when G is $UDG(V)$, we use $RNG(V)$, $GG(V)$ and $YG(V)$ instead of $RNG(UDG(V))$, $GG(UDG(V))$ and $YG(UDG(V))$ respectively. These graphs are sparse: $|RNG(V)| \leq 3n - 10$, $|GG(V)| \leq 3n - 8$, and $|\overrightarrow{YG}_k(V)| \leq kn$. [1] The sparseness implies that the average node degree is bounded by a constant. However the maximum node degree of the relative neighborhood graph $RNG(V)$ and the Gabriel graph $GG(V)$ and the maximum node in-degree of the Yao graph $\overrightarrow{YG}_k(V)$ could be as large as $n - 1$ as shown in Figure 2. It places n points of V on the unit circle centered at a node $u \in V$. It is not difficult to show that each edge uv_i belongs to $RNG(V)$, $GG(V)$ and $YG_k(V)$.

The configuration given by Figure 2 also shows that there is no geometry structure that has a constant bounded node degree and contains the least energy consumption path for any pair of nodes. Notice that if such structure exists, node u in Figure 2 has to maintain the connection to each node v_i, $1 \leq i \leq n$, because uv_i is the least energy consumption path for nodes u and v_i in $UDG(V)$.

The length stretch factor[2] of a graph G is defined as the maximum ratio of the total edge length of the shortest path connecting any pair of nodes in G to their Euclidean distance. The same analyses by Bose *et al.* [3] implied that the length stretch factor of $RNG(V)$ is at most $n - 1$ and the length stretch factor of $GG(V)$ is at most $\frac{4\pi\sqrt{2n-4}}{3}$. Several papers showed that the Yao graph $YG_k(V)$ has a length stretch factor at most $\frac{1}{1-2\sin\frac{\pi}{k}}$.

2.2 Power Stretch Factor

The following definitions are proposed in [8]. However, for the completeness of the presentation, we still include them here. Consider any unicast $\pi(u, v)$ in G (could be directed) from a node $u \in V$ to another node $v \in V$:

$$\pi(u, v) = v_0 v_1 \cdots v_{h-1} v_h, \ where\ u = v_0,\ v = v_h.$$

Here h is the number of hops of the path π. The total *transmission power* $p(\pi)$ consumed by this path π is defined as

$$p(\pi) = \sum_{i=1}^{h} \|v_{i-1} v_i\|^{\beta}$$

Let $p_G(u, v)$ be the least energy consumed by all paths connecting nodes u and v in G. The path in G connecting u, v and consuming the least energy $p_G(u, v)$ is called the *least-energy path* in G for u and v. When G is the unit disk graph $UDG(V)$, we will omit the subscript G in $p_G(u, v)$.

[1] Here $|G|$ denotes the number of edges of a graph G.

[2] Some researchers call it *dilation ratio*, *spanning ratio*.

Let H be a subgraph of G. The *power stretch factor* of the graph H with respect to G is then defined as

$$\rho_H(G) = \max_{u,v \in V} \frac{p_H(u,v)}{p_G(u,v)}$$

If G is a unit disk graph, we use $\rho_H(V)$ instead of $\rho_H(G)$. For any positive integer n, let

$$\rho_H(n) = \sup_{|V|=n} \rho_H(V).$$

When the graph H is clear from the context, it is dropped from notation. For the remainder of this section, we review some basic properties of the power stretch factor, which are studied in [8].

Lemma 5 *For a constant δ, $\rho_H(G) \leq \delta$ iff for any link v_iv_j in graph G but not in H, $p_H(v_i, v_j) \leq \delta \|v_iv_j\|^\beta$.*

The above lemma implies that it is sufficient to analyze the power stretch factor of H for each link in G but not in H.

Lemma 6 *For any $H \subseteq G$ with a length stretch factor δ, its power stretch factor is at most δ^β for any graph G.*

Therefore a geometry structure H with a constant length stretch factor δ implies that its power stretch factor is no more than δ^β. In particular, a graph with a constant bounded length stretch factor must also have a constant bounded power stretch factor. But the reverse is not necessarily true. Finally, the power stretch factor has the following monotonic property.

Lemma 7 *If $H_1 \subset H_2 \subset G$ then the power stretch factors of H_1 and H_2 satisfy $\rho_{H_1}(G) \geq \rho_{H_2}(G)$.*

2.3 Localized Algorithm

Due to the limited resources of the wireless nodes, it is preferred that the underlying network topology can be constructed in a localized manner. Stojmenovic *et al.* first define what is a localized algorithm in several pioneering papers[4, 11, **?**]. Here a distributed algorithm constructing a graph G is a *localized algorithm* if every node u can exactly decide all edges incident on u based only on the information of all nodes within a constant hops of u (plus a constant number of additional nodes' information if necessary). It is easy to see that the Yao graph $YG(V)$, the relative neighborhood graph $RNG(V)$ and the Gabriel graph $GG(V)$ can be constructed locally. However, the Euclidean minimum spanning tree $EMST(V)$ can not be constructed by any localized algorithm. In this paper, we are interested in designing localized algorithms to construct sparse and power efficient network topologies.

3 Results

In this section, we study the power stretch factor of several sparse geometry structures for unit disk graph although our results usually hold for general graphs. Then we give a new method to construct a sparse network with a bounded node degree and it has a bounded power stretch factor in practice. At the end, we will show our simulation results on these sparse geometry structures.

3.1 Yao and Gabriel Graph

It is easy to show that the Gabriel graph over the unit disk graph $UDG(V)$ has a power stretch factor 1 always. In addition, the number of edges in $GG(V)$ is less than $3n$ given n wireless nodes V because $GG(V)$ is a subgraph of the Delaunay triangulation of V. The Yao graph $YG_k(V)$ has at most kn edges and has a length stretch factor at most $\frac{1}{1-2\sin\frac{\pi}{k}}$. Then from Lemma 6, we know that its power stretch factor is no more than $(\frac{1}{1-2\sin\frac{\pi}{k}})^\beta$. In [8], they proved a stronger result.

Theorem 8 *The power stretch factor of the Yao graph $YG_k(V)$ is at most $\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$.*

We then give two methods to combine the Gabriel and the Yao structures.

First Yao then Gabriel graph. For setting up a power-efficient wireless networking topology, each node u finds all its neighbors in $\overrightarrow{YG}_k(V)$, which can be done in linear time proportional to the number of nodes within its transmission range. To further reduce the number of edges, we can apply the Gabriel graph structure to the constructed Yao graph $\overrightarrow{YG}_k(V)$. A directed edge $\overrightarrow{uv}$ in $\overrightarrow{YG}_k(V)$ survives if and only if, for any node w contained in the open disk using segment uv as diameter, one of the directed edges $\overrightarrow{uw}$ and $\overrightarrow{wv}$ is not in $\overrightarrow{YG}_k(V)$. The power stretch factor of the constructed network topology is also at most $\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$ and the out-degree of each node is at most k. Let $\overrightarrow{GYG}_k(V)$ be the constructed topology. The number of edges of $\overrightarrow{GYG}_k(V)$ is bounded by $O(kn)$.

First Gabriel then Yao graph. On the other hand, we can also first construct the Gabriel graph and then apply the Yao structure over the Gabriel graph. Let $\overrightarrow{YGG}_k(V)$ denote the constructed graph. Because the Gabriel graph $GG(V)$ has a power stretch factor equal to one, the power stretch factor of $\overrightarrow{YGG}_k(V)$ is therefore the same as that of the Yao graph $\overrightarrow{YG}_k(V)$. The node out-degree is also bounded by k. Moreover, the number of edges in $\overrightarrow{YGG}_k(V)$ is bounded by $3n$, which is a bound on the number of edges in $GG(V)$.

The experimental performances of these two graphs $\overrightarrow{GYG}_k(V)$ and $\overrightarrow{YGG}_k(V)$ are presented in Subsection 3.4.

3.2 Yao and Sink

Notice that although the directed graphs $\overrightarrow{YG}_k(V)$, $\overrightarrow{GYG}_k(V)$ and $\overrightarrow{YGG}_k(V)$ have a bounded stretch ratio and a bounded out-degree k for each node, some nodes may have a very large in-degree. The nodes configuration given in Figure 2 will result a very large in-degree for node u. Bounded out-degree gives us advantages when apply several routing algorithms. However, unbounded in-degree at node u will often cause large overhead at u. Therefore it is often imperative to construct a sparse network topology such that both the in-degree and the out-degree are bounded by a constant while it is still power-efficient.

Arya *et al.* [1] had given an ingenious technique to generate a bounded degree graph with a constant length stretch factor. In [8], the authors apply the same technique to construct a sparse network topology with a bounded degree and a bounded power stretch factor. The technique is to replace the directed star consisting of all links towards a node u by a directed tree $T(u)$ with u as the sink. Tree $T(u)$ is constructed recursively. See [8] for more detail. Figure 3 illustrates a directed star centered at u and Figure 4 shows the directed tree $T(u)$ constructed to replace the star.

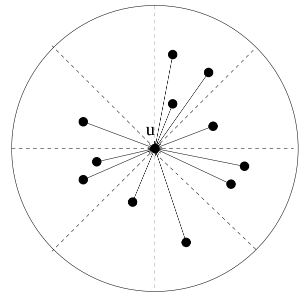

Figure 3: The directed star formed by all links towards to u.

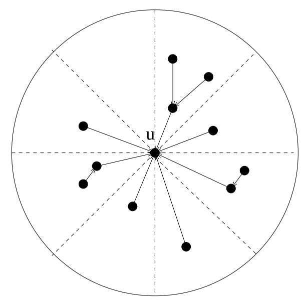

Figure 4: The directed tree $T(u)$ sinked at u.

The union of all trees $T(u)$ is called the *sink structure* $\overrightarrow{YG}^*_k(V)$. They [8] proved that its power stretch factor is at most $(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta})^2$ and its in-degree is bounded by $(k+1)^2-1$. However, the construction of this sink structure $\overrightarrow{YG}^*_k(V)$ is actually more complicated than the previous two methods and the performance gain is not so obvious in practice as shown by our experimental results.

Theorem 9 *The power stretch factor of the graph* $\overrightarrow{YG}^*_k(V)$ *is at most* $(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta})^2$. *The maximum in-degree of the graph* $\overrightarrow{YG}^*_k(V)$ *is at most* $(k+1)^2-1$. *The maximum out-degree is* k.

Notice that the sink structure and the Yao graph structure do not have to have the same number of cones.

3.3 Yao plus Reverse Yao Graph

In this section, we present a new algorithm that constructs a sparse and power efficient topology. Assume that each node v_i of V has a unique identification number $ID(v_i) = i$. The identity of a directed link $\overrightarrow{uv}$ is defined as $ID(\overrightarrow{uv}) = (||uv||, ID(u), ID(v))$. Then we can order all directed links (at most $n(n-1)$ such links) in an increasing order of their identities. Here the identities of two links are ordered based on the following rule: $ID(\overrightarrow{uv}) > ID(\overrightarrow{pq})$ if

1. $||uv|| > ||pq||$ or
2. $||uv|| = ||pq||$ and $ID(u) > ID(p)$ or
3. $||uv|| = ||pq||$, $u = p$ and $ID(v) > ID(q)$.

Correspondingly, the rank $rank(\overrightarrow{uv})$ of each directed link $\overrightarrow{uv}$ is its order in the sorted directed links. Notice that, we actually only have to consider the links with length no more than one. For the remainder of the subsection, we present our new network topology construction algorithm and then show that the constructed network topology is connected.

Algorithm 10 ***Yao+ReverseYao Topology Construction***

1. Each node u divides the space by k equal-sized cones centered at u. We generally assume that the cone is half open and half-close. Node u chooses a node v from each cone so the directed link $\overrightarrow{uv}$ has the smallest $ID(\overrightarrow{uv})$ among all directed links $\overrightarrow{uv}_i$ with v_i in that cone, if there is any. Let $\overrightarrow{YG}_k(V)$ be the union of all chosen directed links.
2. Node u chooses a node v from each cone, if there is any, so the directed link $\overrightarrow{vu}$ has the smallest $ID(\overrightarrow{vu})$ among all directed links computed in the first step in that cone.
3. The union of all chosen directed links in the second step is the final network topology, denoted by $\overrightarrow{YY}_k(V)$.

If the directions of all links are ignored, the graph is denoted as $YY_k(V)$. To prove the correctness of the algorithm, we first show that the resulted network topology is connected if $UDG(V)$ is connected.

Theorem 11 *The directed graph $\overrightarrow{YY}_k(V)$ is strongly connected if $UDG(V)$ is connected and $k > 6$.*

PROOF. Notice that it is sufficient to show that there is a directed path from u to v for any two nodes u and v with $||uv|| \leq 1$. Notice that the Yao graph $\overrightarrow{YG}_k(V)$ is strongly connected. Therefore, we only have to show that for any directed link $\overrightarrow{uv}$ in $\overrightarrow{YG}_k(V)$, there is a directed path from u to v in $\overrightarrow{YY}_k(V)$.

We prove the claim by induction on the ranks of all directed links in $\overrightarrow{YG}_k(V)$.

If the directed link $\overrightarrow{uv}$ has the smallest rank among all links of $\overrightarrow{YG}_k(V)$, then $\overrightarrow{uv}$ will obviously survive after the second step. So the claim is true for the smallest rank.

Assume that the claim is true for all links in $\overrightarrow{YG}_k(V)$ with rank at most r. Then consider a directed link $\overrightarrow{vu}$ in $\overrightarrow{YG}_k(V)$ with $rank(\overrightarrow{vu}) = r + 1$ in $\overrightarrow{YG}_k(V)$.

If $\overrightarrow{vu}$ survives in the second phase, then the claim holds. Otherwise, $\overrightarrow{vu}$ can only be removed by the node u in the second phase. Then there must exist a directed link $\overrightarrow{wu}$ survived with a smaller identity in the same cone as $\overrightarrow{vu}$. In addition, the angle $\angle wuv$ is less than $\theta = \frac{2\pi}{k}$. Here

$$(||wu||, ID(w), ID(u)) < (||vu||, ID(v), ID(u)).$$

Therefore $||wu|| \leq ||vu||$. Because $\angle wuv < \frac{2\pi}{k}$, we have $||wv|| < ||uv||$. Consequently, the identity $(||vw||, ID(v), ID(w))$ of the directed link $\overrightarrow{vw}$ is less than that of the directed link $\overrightarrow{vu}$, which is $(||vu||, ID(v), ID(u))$.

Notice that here the directed link $\overrightarrow{vw}$ is not guaranteed to be in $\overrightarrow{YG}_k(V)$ and our induction is for all directed links in $\overrightarrow{YG}_k(V)$. So we can not directly use the induction. There are two cases here

Case 1: the link $\overrightarrow{vw}$ is in $\overrightarrow{YG}_k(V)$. Then by induction, there is a directed path $v \rightsquigarrow w$ from v to w after the second phase. Consequently, there is a directed path (concatenation of the path $v \rightsquigarrow w$ and the link $\overrightarrow{wu}$) from v to u after the second phase.

Case 2: the link $\overrightarrow{vw}$ is not in $\overrightarrow{YG}_k(V)$. Then we know that there is a directed path $\pi_{\overrightarrow{YG}_k}(v, w) = q_1 q_2 \cdots q_h$ from v to w in $\overrightarrow{YG}_k(V)$, where $q_1 = v$ and $q_h = w$. Using the same proof technique, we can prove that each directed link $q_i q_{i+1}$, $1 \leq i < h$, in $\pi_{\overrightarrow{YG}_k}(v, w)$ has a smaller rank than $\overrightarrow{vw}$, which is r. Here $rank(q_1 q_2 = v q_2) < rank(v, w)$ because the selection method in the first step. And $rank(q_i q_{i+1}) < rank(v, w)$, $1 < i < h$, because

$$||q_i q_{i+1}|| \leq ||q_i w|| < ||q_{i-1} w|| < \cdots < ||q_1 w|| = ||vw||.$$

Then for each link in $q_i q_{i+1}$ in $\pi_{\overrightarrow{YG}_k}(v, w)$, there is a directed path $q_i \rightsquigarrow q_{i+1}$ survived in $\overrightarrow{YY}_k(V)$ after the second phase (this is proved by induction on the rank $rank(q_i q_{i+1})$). The the concatenation of all such paths $q_i \rightsquigarrow q_{i+1}$, $1 \le i < h$, and the directed link $\overrightarrow{wu}$ forms a directed path from v to u in $\overrightarrow{YY}_k(V)$.

This finishes the proof of the strong connectivity theorem. $\square$

It is obvious that both the out-degree and in-degree of a node in $\overrightarrow{YY}_k(V)$ are bounded by k. And our experimental results show that this sparse topology has a small power stretch factor in practice (see the next subsection 3.4). We conjecture that it also has a constant bounded power stretch factor theoretically. The proof of this conjecture or the construction of a counter-example remains a future work.

3.4 Experimental Results

In this section we measure the performances of the new sparse and power efficient topologies by conducting some experiments. In a wireless network, each node is expected to potentially send and receive messages from many nodes. Therefore an important requirement of such network is a strong connectivity. In Section 2 and Section 3, we have shown all these sparse topologies are strongly connected if the unit disk graph $UDG(V)$ is connected. So in our experiments, we randomly generate a set V of n wireless nodes and its $UDG(V)$, and test the connectivity of $UDG(V)$. If it is strongly connected, we construct different topologies from V by various algorithms (some are already studied before, some are newly presented in the previous sections). Then we measure the sparseness and the power efficiency of these topologies by the following criteria: the average and the maximum node degree, and the average and the maximum power stretch factor. Notice that, for a directed topology, we also measure its average and the maximum in-degree. In the experimental results presented here, we choose total $n = 100$ wireless nodes; the number of cones is set to 8 when we construct any graph using the Yao structure (for example, $YG(V)$, $YGG(V)$, $GYG(V)$, $YG^*(V)$ and $YY(V)$); the power attenuation constant $\beta = 2$. We generate 1000 vertex sets V (each with 100 vertices) and then generate the graphs for each of these 1000 vertex sets. The average and the maximum are computed over all these 1000 vertex sets. Figure 5 gives all eight different topologies defined in this paper for the unit disk graph illustrated by the first figure of Figure 5.

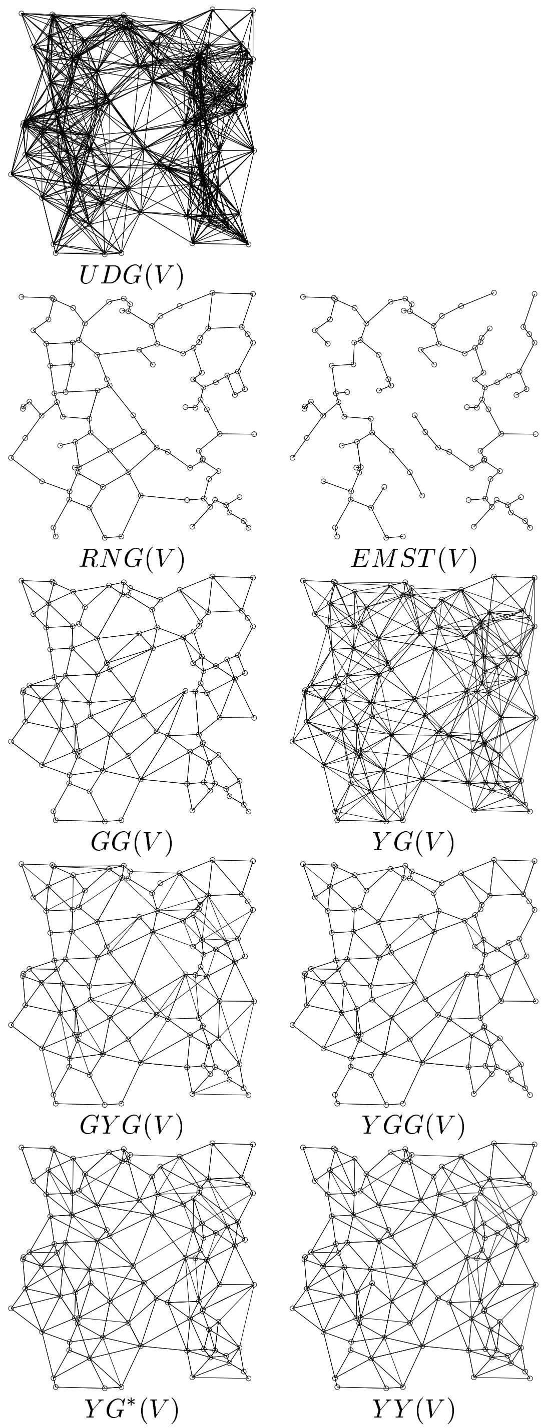

Figure 5: Different topologies generated from the same *unit disk graph* $UDG(V)$.

3.4.1 Node Degree

Before we show the power efficiency of different topologies, we would like to understand the characteristics of the resulting topologies. Figure 5 shows an example of all the topologies generated by different topology control algorithms. The average node degree of each topology is shown in Table 1. The average node degree of the wireless networks should not be too large. Otherwise a node with a large degree has to communicate with many nodes directly. This increases the interference and collision, and increases the overhead at this node. The average node degree should also not be too small either: a low node degree usually implies that the network has a lower fault tolerance and it also tends to increase the overall network energy consumption as longer paths may have to be taken. Thus the average node degree is an important performance metric for the wireless network topology. Table 1 shows that *first Yao then Gabriel graph* $GYG(V)$, *first Gabriel then Yao graph* $YGG(V)$, and the *Yao plus reverse Yao graph* $YY(V)$ have a much less number of edges than the *Yao graph* $YG(V)$. In other words, these graphs are sparser than the Yao graph $YG(V)$, which is also verified by Figure 5. Notice that theoretically, the *sink structure* $YG^*(V)$ has the same number of edges as the Yao graph $YG(V)$. However, the in-degree of each node of the *sink structure* $YG^*(V)$ is bounded from above by a constant. Let d_{avg} and d_{max} be the average and the maximum node degree over all nodes and all undirected graphs respectively. For directed graphs, we ignore the direction of each link. Let O_{avg} and O_{max} be the average and the maximum node out-degree over all nodes and all directed graphs respectively; I_{avg} and I_{max} be the average and the maximum node in-degree over all nodes and all directed graphs respectively. Notice that for a directed graph, its I_{avg} equals to its O_{avg}.

	d_{avg}	d_{max}	I_{avg}	I_{max}	O_{max}
UDG	23.46	50	-	-	-
GG	3.56	9	-	-	-
RNG	2.37	5	-	-	-
$EMST$	1.98	4	-	-	-
YG	9.05	22	6.66	21	8
GYG	4.47	12	3.88	10	8
YGG	3.56	9	3.46	9	8
YG^*	5.94	12	5.53	13	8
YY	5.00	11	4.64	8	8

Table 1: The node degrees of different topologies.

3.4.2 Power Stretch Factor

Besides strong connectivity, the most important design metric of wireless networks is perhaps the energy efficiency, as it directly affects both the node and the network lifetime. So while our new topologies increase the sparseness, how do they affect the power efficiency of the constructed network? Table 2 summarizes our experimental results of the power stretch factors of these topologies. It shows that the new proposed network topologies still have small power stretch factors. Notice that even the average and the maximum node degree of the new topologies $GYG(V)$, $YGG(V)$, and $YY(V)$ is much smaller than that of $YG(V)$, the average and the maximum power stretch factors of these graphs are at the same level of that of the Yao graph $YG(V)$. Especially, the power stretch factor of the *Yao plus reverse Yao graph* $YY(V)$ is just little bit higher than those of $GYG(V)$ and $YGG(V)$. Remember that $YY(V)$ has a bounded node degree while no other topologies (except $YG^*(V)$) have such a property.

	ρ_{avg}	ρ_{max}	σ_{avg}	σ_{max}
UDG	1.000	1.000	96.756	110.434
GG	1.000	1.000	3.819	4.770
RNG	1.059	3.131	1.694	2.083
$EMST$	1.487	20.788	1.000	1.000
YG	1.002	1.555	12.967	15.615
GYG	1.002	1.555	5.327	7.118
YGG	1.002	1.555	3.628	4.292
YG^*	1.003	1.833	7.302	8.937
YY	1.004	1.833	6.899	8.492

Table 2: The quality measurements of different topologies.

In the Table 2, ρ_{avg} and ρ_{max} are the average and the maximum unicasting power stretch factor over all nodes and all graphs respectively; σ_{avg} and σ_{max} are the average and the maximum multicasting/broadcasting power stretch factor over all graphs respectively, which will be defined later.

3.4.3 Broadcasting (or Multicasting) Power Stretch Factor

The power stretch factor (see Subsection 2.2) discussed previously is defined for the unicasting communications. However, in practice, we also have to consider the broadcast or multicast communications. Wan *et al.*[12] showed that the minimum energy cost of broadcasting or multicasting is related to the total energy cost of all links in the Euclidean minimum spanning tree $EMST$ of the same point set. They proved that a broadcasting method based on the Euclidean minimum spanning tree rooted at the sender uses energy no more than 12 times the minimum energy cost of any broadcasting scheme. Then we formally define the broadcasting (or multicasting) power stretch factor of any topology G as follows.

Definition 12 *The broadcasting (or multicasting) power stretch factor, denoted by σ_G, of a topology $G(V)$ over a point set V is defined as the ratio of the total energy cost of all links in G to that in $EMST$. In other words,*

$$\sigma_G = \frac{\sum_{e \in G} ||e||^\beta}{\sum_{e \in EMST} ||e||^\beta}.$$

Unfortunately, the broadcasting (or multicasting) power stretch factor of any graph structures mentioned above (except $EMST$) could be an arbitrarily large real number theoretically. Figure 6 gives such an example of wireless nodes. Here $||u_i v_i|| = 1$ and $||u_i u_{i+1}|| = ||v_i v_{i+1}|| = \varepsilon$ for a very small positive real number ε. The graph shown in the example is the relative neighborhood graph $RNG(V)$. It is easy to show that

$$\sigma_{RNG(V)} = \frac{\sum_{e \in RNG(V)} ||e||^\beta}{\sum_{e \in EMST(V)} ||e||^\beta} = \frac{n + 2(n-1)\varepsilon^2}{1 + 2(n-1)\varepsilon^2} \to n,$$

when $\varepsilon \to 0$. Notice that all other graph structures (except $EMST(V)$) contain $RNG(V)$ as a subgraph for this node configuration. It then implies our previous claim.

On the other hand, our experiments (see Table 2) show that the broadcasting (or multicasting) power stretch factors of $GYG(V)$, $YGG(V)$ and $YY(V)$ are actually small enough for practical usage. Notice that here the $YGG(V)$ graph has the smallest broadcasting (or multicasting) power stretch factor among the new topologies we presented. It is reasonable because the number of its edges is bounded by $3n$, while the number of edges of the other two graphs $GYG(V)$ and $YY(V)$ is bounded by kn, and $k \geq 6$.

Notice that Arya *et al.* [2] gave a centralized algorithm to construct a graph with bounded node degree and the total edge length is no more than a constant factor of that of $EMST(V)$. Then Arya *et al.* [1] gave another centralized algorithm to construct a graph that satisfies these two conditions in addition that the graph has a bounded length stretch factor. However, it is very complicated to transform their algorithms to a distributed algorithm.

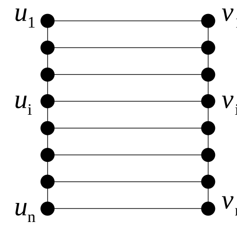

Figure 6: An instance of wireless nodes that every network structure (except $EMST$) has an arbitrarily large broadcasting (or multicasting) power stretch factor.

3.4.4 Special Case Study

Then we study the performances of various structures for the following special node configuration. There are total 100 points: one point u is at the center of the domain; 50 points are distributed on the circle centered at u with radius one; all other 49 points are randomly distributed outside of the circle. Figure 7 illustrates various topology structures generated for this point set. As we expected, all graphs except the *sink structure* $YG^*(V)$ and the *Yao plus the reverse of Yao* $YY(V)$ have a very large node degree at u. Both the *sink structure* $YG^*(V)$ and the *Yao plus the reverse of Yao* $YY(V)$ have a constant bounded node degree. In addition, these two graphs have similar unicasting power stretch factors and broadcasting power stretch factors in our experiments. Notice that, unlike $YG^*(V)$, it is an open problem whether $YY(V)$ has a constant bound on the unicasting power stretch factor theoretically. However, the *Yao plus the reverse of Yao* $YY(V)$ has two advantages over the *sink structure* $YG^*(V)$: (1) it is easier to construct $YY(V)$ than $YG^*(V)$, (2) the node degree bound of $YY(V)$ is not larger than that of $YG^*(V)$.

4 Summary and Future Work

In this paper, we first combine some well-known geometry structures such as the Gabriel graph $GG(V)$ and the Yao graph $YG(V)$ to get the new sparse topologies $GYG(V)$ and $YGG(V)$. These two new topologies are power-efficient and have constant

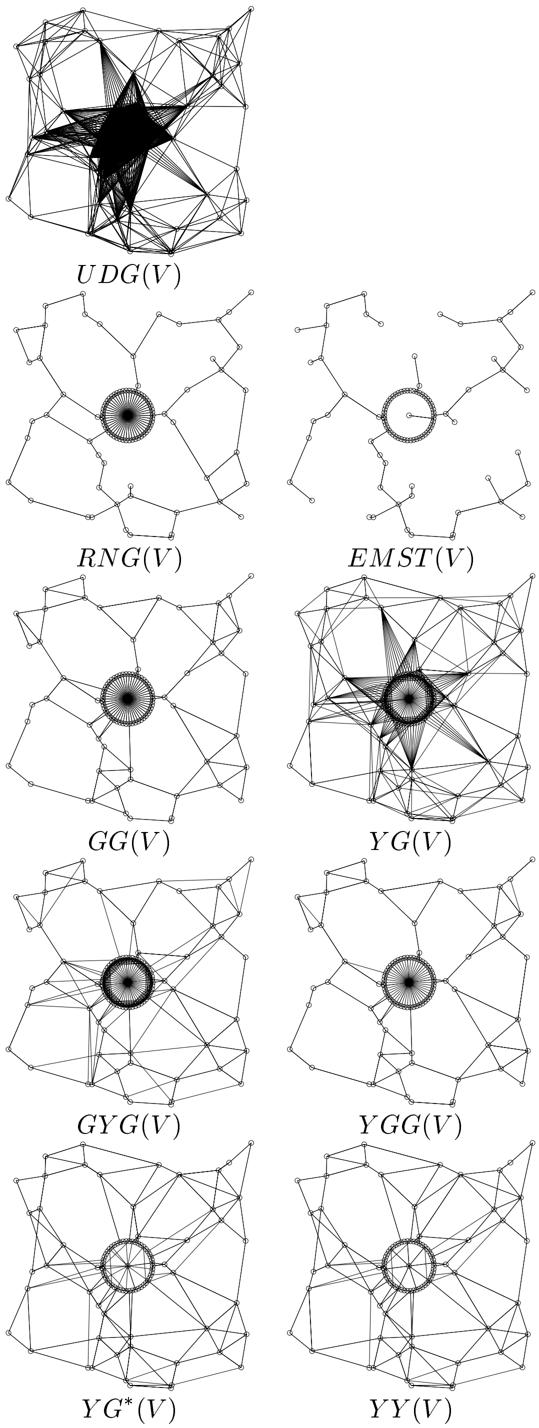

Figure 7: Different topologies generated from the same *unit disk graph* $UDG(V)$.

	d_{avg}	d_{max}	I_{avg}	I_{max}	O_{max}
UDG	45.36	74	-	-	-
GG	3.75	50	-	-	-
RNG	3.16	50	-	-	-
$EMST$	1.98	4	-	-	-
YG	10.01	52	6.63	52	8
GYG	6.02	50	4.38	50	8
YGG	3.75	50	3.26	50	8
YG^*	4.59	14	4.23	11	8
YY	3.69	10	3.47	8	8

Table 3: The node degrees of different topologies.

	ρ_{avg}	ρ_{max}	σ_{avg}	σ_{max}
UDG	1.000	1.000	138.761	118.018
GG	1.000	1.000	4.098	5.082
RNG	1.032	2.471	2.440	2.829
$EMST$	1.327	49.718	1.000	1.000
YG	1.001	1.472	12.988	16.624
GYG	1.001	1.472	5.478	7.460
YGG	1.001	1.472	3.550	4.408
YG^*	1.002	1.472	5.609	6.878
YY	1.002	1.472	5.364	6.622

Table 4: The quality measurements of different topologies.

bounded node out-degrees. However, the node in-degree could be very large theoretically. Then we present an algorithm to construct a new topology called the *Yao plus reverse Yao graph* $YY(V)$, which has a bounded node degree. Our experimental results show that its power stretch factor is very small in practice. In addition, the experiments also show that these three topologies have small broadcasting (or multicasting) power stretch factors. We also found that even the sink structure $YG^*(V)$ has both bounded node degree and unicast power stretch factor theoretically, it is not better than the $YY(V)$ structure in practice. Notice that it is easy to show that $YY(GG(V))$ is always not worse than $YY(V)$ and $YG^*(GG(V))$ is always not worse than $YG^*(V)$. We did not conduct the experiments on them because we are more interested in the structures of YY and YG^*.

Even the graph $YY(V)$ has a bounded degree and a good unicasting and broadcasting power stretch factor in practice, it is still an open problem whether it has a bounded unicasting power stretch factor theoretically. We also leave it as a future work to design an efficient localized algorithm achieving the following three objectives: a constant bounded node degree, a constant bounded unicasting power stretch factor, and a constant bounded broadcasting (multicasting) power stretch factor.

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