

Traffic Load Distribution of Circular Sailing Routing in Dense Wireless Networks

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Abstract—Shortest path routing protocol intends to minimize the total delay between every pair of destination node and source node. However, it is also well-known that shortest path routing suffers from uneven distribution of traffic load, especially in dense wireless networks. Recently, several new routing protocols are proposed in order to balance traffic load among nodes in a network. One of them is *circular sailing routing*(CSR) [1], [2] which maps nodes on the surface of a sphere and select routes based on surface distances. CSR has been demonstrated with better load balance than shortest path routing via simulations. However, it is still open that what load distribution CSR can achieve. Therefore, in this paper, we theoretically analyze the traffic load distribution of CSR in a dense circular wireless network. Using the techniques developed by Hyytiä and Virtamo [3], we are able to derive the traffic load of any point inside the network. We then conduct extensive simulations to verify our theoretical results with grid and random networks.

Index Terms—Load balancing, load distribution, circular sailing routing, routing

I. INTRODUCTION

In a dense wireless network, a route traveled by a packet from the source to the destination usually consists of multiple hops where the intermediate nodes act as relays. Routing algorithms aim to find an optimal route for each pair of source and destination in a given network. To minimize the traveled distance and the total delay experienced by packets, routing algorithms typically choose the shortest path among all possible routes between a source and a destination. However, it has been shown that shortest path routing (SPR) may lead to uneven distribution of traffic load in a network [1]–[4].

Fig. 1 shows a simple simulation result of a well-known “crowded center effect” caused by the shortest path routing. Here, 309 nodes are distributed on a regular grid in a unit disk area, as shown in Fig. 8(a). We assume a scenario with uniform traffic demands where each node sends one packet to all other nodes. Fig. 1(a) illustrates the cumulative node traffic (*i.e.*, number of packets passing through) of SPR for each node. It is clear that nodes in the center area have the most traffic load. This is due to the fact that more shortest paths go through the center than through the periphery of the network. Crowded

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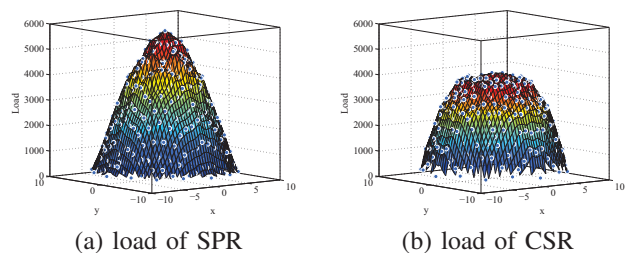


Fig. 1. (a) *Crowded center effect* of SPR: nodes in the center area have much heavier traffic load than nodes in the periphery; (b) CSR can effectively reduce crowded center effect by spreading the traffic on the sphere.

center effect can also be observed in transportation systems where the roads in center of a metropolitan region are usually the most congested. Such uneven traffic distribution may hurt the performance of the network in a long run. For example, the nodes in the center of the network may experience more congestion and run out of their batteries more quickly than nodes in the periphery. Therefore, load balancing in routing protocols becomes important.

Different load balancing routing protocols [5]–[7] have been proposed for wireless networks to address uneven load distribution within a network. Most of these load balancing routing protocols dynamically adjust the routes based on estimated real-time traffic loads. Unlike these load-aware routing protocols, there are also routing solutions that focus on balancing the load for the whole network without knowing the current load information. The key idea of such approaches is spreading the traffic across the wireless network via the elaborate design of the routing algorithm. Multi-path routing [4] by spreading the traffic across multiple paths between a source and a destination can result in a better load balancing throughout the network. However, Ganjali and Keshavarzian [8] showed unless using a very large number of paths, the load distribution of multi-path routing is almost the same as single path routing. Hyytiä and Virtamo [3] studied how to avoid the crowded center problem by analyzing the load distribution of different routing methods in a dense network. They not only provided a general framework to analyze the load distribution of a given set of paths and traffic demands, but also proposed a randomized choice between shortest path and routing on inner/outer radii to level the load.

Both Li *et al.* [1], [2] (*circular sailing routing*, CSR) and

Popa *et al.* [9] (*curveball routing*, CBR) proposed to map the network onto a sphere with stereographic projection to balance the traffic load. The major difference between CSR and CBR is using different mapping method. However, with an easy coordinate transformation, it can be proved that they are essentially equivalent. After nodes are mapped onto the surface of a sphere, packets can be routed on the virtual coordinates of nodes on the sphere. Instead of using the Euclidean distance, CSR or CBR uses the spherical distance between the virtual coordinates as the routing metric. Since there is no center on the spherical surface, the crowded center effect caused by SPR could be effectively reduced. This has been confirmed by Fig. 1(b) (load of CSR for the same grid network) and extensive simulation/testbed results from [1], [2] and [9], [10]. However, it is still open that what load distribution CSR or CBR can theoretically achieve in a dense network. Notice that such theoretical results are crucial to understand the advantage and limitation of performances of these methods. This paper aims to provide the first theoretical analysis of load distribution of CSR in a dense wireless network. We consider a dense wireless multi-hop network at the limit when the number of nodes is extremely large. Using the techniques developed by Hyytiä and Virtamo [3], we are able to derive the traffic load of any point inside the network. We also conduct extensive simulations to verify our theoretical results with grid and random networks.

The rest of the paper is organized as follows. Section II reviews the basic idea of circular sailing routing and its projection method. Section III provides our detailed analysis on the load distribution of CSR in a dense wireless network. Section IV presents some simulation results to verify our theoretical analysis. Finally, Section V concludes the paper.

II. CIRCULAR SAILING ROUTING (CSR)

The basic idea of circular sailing routing (CSR) [1], [2] is mapping all wireless nodes in a 2D network onto a 3D sphere via reversed stereographic projection. Since the surface of the sphere is symmetric, if nodes only communicate on the surface and traffic demand is uniformly distributed in the network, there will be no crowded center effect on the spherical surface. One-to-one mapping between nodes in a plane and nodes on a sphere has been well-studied in projective geometry [11]. A *stereographic projection* can map an infinite plane onto a sphere and vice versa. Fig. 2 illustrates the mapping method used by CSR, which is a stereographic projection. For a wireless network, the area in which the wireless nodes lie corresponds to a finite region \mathbb{P} in the plane. We can place a sphere \mathbb{S} centered at the center $S(0, 0, 0)$ of the network. The radius R of \mathbb{S} is an adjustable parameter for CSR. Any point $m(x, y, 0)$ in \mathbb{P} maps to its projection $m'(x', y', z')$ on the sphere \mathbb{S} , which is the intersection of \mathbb{S} and the line through m and the north pole $N(0, 0, 2R)$. Notice that stereographic projection preserves circles perfectly. That is, a circle on the sphere maps to a circle in the plane and vice versa

After mapping the nodes on the sphere, CSR routes packets on the spherical shortest paths on the sphere instead of

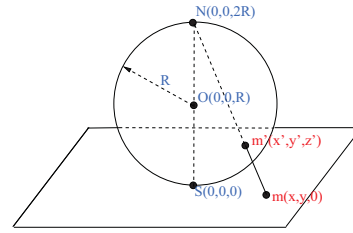


Fig. 2. Stereographic projection between a 3D sphere and a 2D plane: one-to-one mapping between a node m' on a sphere \mathbb{S} and a node m in a plane.

the Euclidean shortest paths in the 2D plane. Note that the mapping preserves topological neighborhoods, it only changes the cost of the links. For any existing link mn between two nodes m and n in the network, CSR uses the shortest distance on the sphere between their projected nodes m' and n' as the cost of link mn . CSR then chooses the route with smallest total spherical distance. CSR is easy to be implemented by the simple modification of shortest path routing (SPR) and has negligible additional computational overhead. The only introduced overhead is that each node needs to compute its neighbors' spherical coordinates. In addition, Li *et al.* [1], [2] also showed that CSR has a bounded stretch factor, *i.e.*, the distance traveled by the packets using CSR is no more than a small constant factor of the distance of the shortest path from the source to the destination.

Popa *et al.* [9] also proposed a similar routing technique, called *curveball routing* (CBR), which maps the 2D network plane is placed on the equator of the 3D sphere) and then route the packets based on spherical distances on the sphere. Notice that CBR and CSR are essentially equivalent. Therefore, the load analysis presented in this paper can be easily adopted to CBR too.

III. TRAFFIC LOAD ANALYSIS OF CSR

The ability of load balancing of CSR and CBR has been demonstrated via simulations or testbed experiments by [1], [2] and [9], [10], respectively. However, until now there is no any theoretical result on load distribution of CSR or CBR. In this section, we will give the detailed load analysis of CSR in a dense multi-hop wireless network by using a technique by Hyytiä and Virtamo [3].

A. Model and Assumptions

We consider a dense wireless multi-hop network at the limit when the number of nodes is extremely large. Therefore, a typical routing path in such a network consists of a large number of hops. We assume that the spatial scales of a typical distance between source and destination nodes and the mean distance between neighboring nodes are strongly separated. Under such assumption the routes are modeled as smooth and continuous geometric curves. For example, shortest path routing transmits a packet over a line segment between source and destination, while CSR transmits over a circular segment. Recall that CSR sends packets over spherical shortest paths on

the surface of the sphere, which are big circles of the sphere. Since stereographic projection preserves circles, all paths of CSR are also on circles in the 2D plane.

We assume that the network is located in a 2D disk region \mathcal{D} with radius one and centered at $S(0, 0)$, i.e., $\mathcal{D} = \{\mathbf{r} \in \mathbf{R}^2 : |\mathbf{r}| < 1\}$. The rate of traffic of packets from a differential area element dA about \mathbf{r}_1 to a differential area element dA about \mathbf{r}_2 is $\lambda(\mathbf{r}_1, \mathbf{r}_2) \times dA^2$, where $\lambda(\mathbf{r}_1, \mathbf{r}_2)$ is the traffic demand density (in units of $1/s/m^4$) and dA is the area of the element. Then the total packet generation rate (in units $1/s$) over the network is $\Lambda = \pi^2 \lambda(\mathbf{r}_1, \mathbf{r}_2)$. In our analysis, we assume uniform traffic demands in the network, i.e., $\lambda(\mathbf{r}_1, \mathbf{r}_2) = \frac{\Lambda}{\pi^2}$. Then the packet arrival rate (or called traffic load or scalar flux) for a given location \mathbf{r} can be defined as $\Phi(\mathbf{r})$. For a routing method \mathcal{A} , its traffic load distribution is denoted as $\Phi_{\mathcal{A}}(\mathbf{r})$ for all $\mathbf{r} \in \mathcal{D}$.

For CSR, we assume that the projection sphere \mathbb{S} is centered at $O(0, 0, R)$ with radius R . We assume that $R \geq 1/2$. As shown in Fig. 3, if $R \geq 1/2$, the projections of all nodes are located only on the southern hemisphere; while $R < 1/2$, the projections of some nodes will surpass the equator to the northern hemisphere. When $R < 1/2$, CSR may cause the following problems for our analysis: (1) the spherical shortest path between projections of two nodes near the boundary of the network may go via the area around north pole, where no projection nodes of any nodes inside the network. In other words, either CSR needs to transmit packets via nodes outside the network (the unit disk) or CSR will choose longer paths within the unit disk. (2) For two nodes whose projections are on the equator, there are infinite number of spherical shortest paths on the surface of the sphere. In addition, simulation results in [1], [2] show that better performance can be achieved when $R \geq 1/2$ than $R < 1/2$. Therefore, in this paper, we only analyze the load distribution of CSR when $R \geq 1/2$.

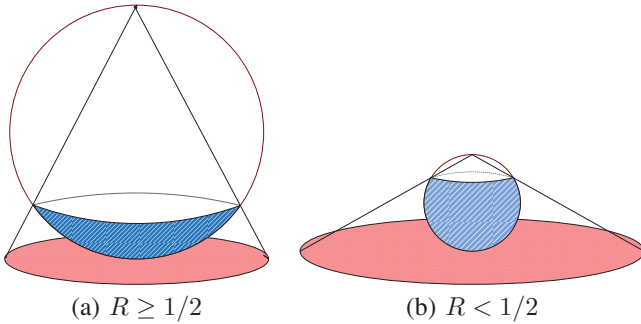


Fig. 3. Stereographic projection in CSR over a sphere \mathbb{S} with radius R .

B. General Results on Load Distribution of Circularly Symmetric Paths in Unit Disk from Hyytiä and Virtamo

Hyytiä and Virtamo [3] presented a general framework for analyzing the traffic load resulting from a given set of paths and traffic demands by using integration of angular flux over angles. They further derived an explicit formula for the load distribution of a general family of circularly symmetric paths

by making the use of the symmetry. The paths of this general family can be always defined by a *basic set of paths*.

The basic set of paths is given by the set of curves $y = y(x, a)$, where $y(x, a)$ is an even function of x , i.e., $y(x, a) = y(-x, a)$ or each curve is symmetric on both sides of y -axis. In addition, the mirror image of a curve $y(x, a)$ with respect to the x -axis, $-y(x, a)$, also belongs to the basic set. For example, Fig. 4 shows basic set of paths of the shortest paths and the circular paths [3]. Here, circular paths consist of such sections of circumference of circles that cut the unit disk at the opposite points as shown in Fig. 4(b). It is clear that routing with circular paths (hereafter called *circular path routing* CPR) move some portion of the traffic away from the centre of the network. It is also notable that CPR is a special case of CSR when $R = 1/2$, where the boundary of the network is projected to the equator of the sphere.

For a set of basic paths, we can choose the curve parameter a so that $y(0, a) = a$, $a \in [-1, 1]$. Hyytiä and Virtamo [3] also assumed that for $a \geq 0$, it holds that $0 \leq y(x, a) \leq y(0, a)$ for all x , i.e., the maximum height of a curve is always at $y(0, a)$. In addition, $y(x, 0) = 0$, that is, the path corresponding to value $a = 0$ is the horizontal diagonal of the disk. Last, most importantly, the curves in the basic set fill the unit disk completely so that each interior point of the disk belongs to *one and only one* path in the basic set. Notice that both SPR and CPR can have basic set of paths that satisfy all these assumption and requirement as shown in Fig. 4.

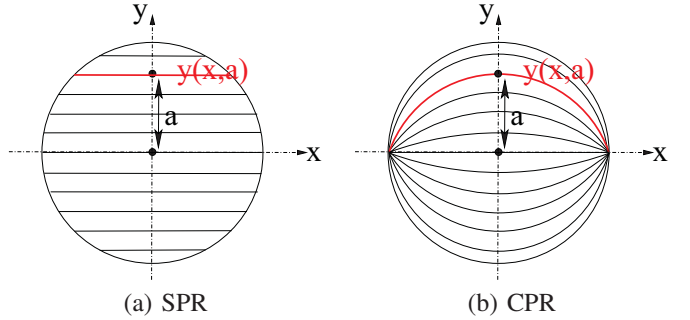


Fig. 4. Basic paths for (a) shortest path routing and (b) circular path routing.

After defining the basic set of paths, the full set of paths of a routing method can be obtained by rotations of the whole set around the origin $(0, 0)$ by an angle in the range $[0, \pi]$. In the full set of paths, there is a unique path through any given point in any given direction. Due to the symmetry of these paths, we now can focus on the load at any point on the circle with radius r from the origin, denoted by $\Phi(r)$. By fully using the symmetry of the basic set of paths, Hyytiä and Virtamo [3] have successfully derived the following formula for the load $\Phi(r)$ as a function of the radius for a general family of paths.

$$\Phi(r) = \frac{2\Lambda}{\pi^2} \frac{1-r^2}{r} \int_0^r \frac{A'(a)}{\cos\theta(r, a)} da, \quad (1)$$

where

$$A'(a) = \int_{-X(1, a)}^{X(1, a)} y_a(x, a) dx, \quad (2)$$

and

$$\cos\theta(r, a) = \frac{X(r, a) + Y(r, a)y_x(X(r, a), a)}{r\sqrt{1 + y_x(X(r, a), a)^2}}. \quad (3)$$

A few notations used in these equations are as follows. $y_x(x, a)$ and $y_a(x, a)$ are partial derivatives defined as

$$y_x(x, a) = \frac{\partial y(x, a)}{\partial x} = \frac{\partial}{\partial x}y(x, a), \quad (4)$$

$$y_a(x, a) = \frac{\partial y(x, a)}{\partial a} = \frac{\partial}{\partial a}y(x, a). \quad (5)$$

$X(r, a)$ and $Y(r, a)$ are the positive x -coordinate and y -coordinate of the intersection point of the a -path $y(x, a)$ and the circle with radius r , respectively. $\theta(r, a)$ is the angle of incidence of curve $y(x, a)$ and r -circle.

By a limit consideration, an even simpler expression can be derived from Equation (1) for the flux at the center of the network:

$$\Phi(0) = \frac{\Lambda}{\pi} A'(0) = \frac{\Lambda}{\pi} \int_{-1}^1 \lim_{a \rightarrow 0} \frac{y(x, a)}{a} dx \quad (6)$$

By using these equations, Hyttiä and Virtamo [3] have obtained load distribution of SPR and CPR. For SPR, the load distribution is given by the following equations.

$$\Phi_{SPR}(r) = \frac{2(1-r)^2 \cdot \Lambda}{\pi^2} \int_0^\pi \sqrt{1-r^2 \cos^2 \phi} d\phi, \quad (7)$$

$$\Phi_{SPR}(0) = \frac{2}{\pi} \cdot \Lambda \approx 0.637 \cdot \Lambda. \quad (8)$$

For CPR, the equation for its basic set of circular paths is

$$y_{CPR}(x, a) = \sqrt{(1-x^2) + \left(\frac{1-a^2}{2a}\right)^2} - \frac{1-a^2}{2a}. \quad (9)$$

Thus, with Equation (1), the load distribution of CPR can be plotted. In addition, the maximum load is obtained at the center of the disk from Equation (6), which is

$$\Phi_{CPR}(0) = \frac{4}{3\pi} \cdot \Lambda \approx 0.424 \cdot \Lambda.$$

C. Load Distribution of CSR

Now we are ready to use the general result of Hyttiä and Virtamo [3] to analyze the load distribution of CSR. First, we define the basic set of paths for CSR. Fig. 5 show the possible set of paths for CSR when $R \geq 1/2$ and $R < 1/2$. Notice that the paths in Fig. 5(a) satisfy all requirements and assumptions for basic set of paths, however, the paths in Fig. 5(b) do not meet the requirement that a point can only belongs one path in the basic set. There are two points that belong to infinite of number of paths. Therefore, the technique by Hyttiä and Virtamo cannot be directly applied to the case when $R < 1/2$. Next, we will only focus on the case when $R \geq 1/2$.

We first derive the equation for the basic set of CSR. Notice that the curve $y(x, a)$ with height of a is a portion of a circle C' whose projection on the sphere is a big circle. Let O' and R' be the center and radius of circle C' , respectively, as shown in Fig. 6(a). Assume that the intersections of circle C' with y -axis are A and B . We know the coordinate of A is

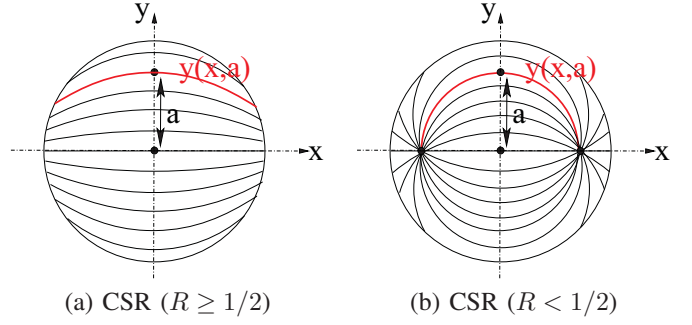


Fig. 5. Basic paths for circular sailing routing: (a) $R \geq 1/2$, (b) $R < 1/2$.

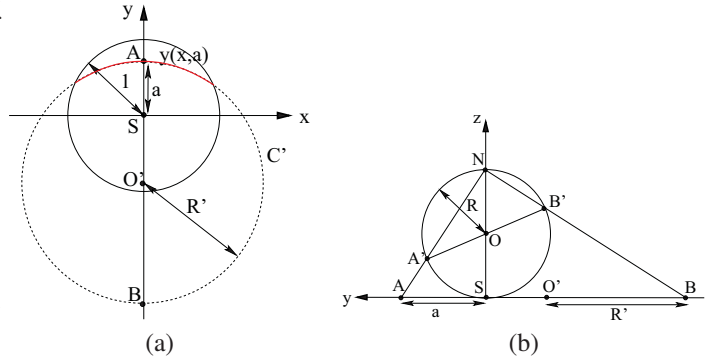


Fig. 6. Illustration of a -curve in the basic set of paths for CSR.

$(0, a, 0)$, thus $|AS| = a$. Based on the stereographic projection (as shown in Fig. 6(b)), we have

$$\frac{|AS|}{|NS|} = \frac{|NS|}{|SB|}.$$

Thus,

$$|SB| = \frac{|NS|^2}{|AS|} = \frac{4R^2}{a}.$$

Then, the coordinate of O' is $(0, \frac{a^2-4R^2}{2a}, 0)$ and $R' = \frac{a^2+4R^2}{2a}$. The equation for circle C' is as follows:

$$x^2 + \left(y - \frac{a^2 - 4R^2}{2a}\right)^2 = \left(\frac{a^2 + 4R^2}{2a}\right)^2.$$

Thus we have the equation for the basic set of CSR,

$$y_{CSR}(x, a) = \sqrt{\left(\frac{a^2 + 4R^2}{2a}\right)^2 - x^2} - \frac{4R^2 - a^2}{2a}. \quad (10)$$

Notice that when $R = 1/2$ this equation becomes Equation (9). This confirms that CSR becomes CPR when $R = 1/2$.

Given Equation (10) and Equations (1-3), we can obtain the load distribution of CSR $\Phi_{CSR}(r)$. Fig. 7 plots the load distribution of CSR when R 's value is from $1/2$ to ∞ . It is clear that the load $\Phi_{CSR}(r)$ decreases when the distance r from the center increases. When $R = 1/2$, CSR is basically CPR which has the least maximum load. When $R = \infty$, all nodes are distributed near the south pole of the sphere and CSR becomes SPR which has the largest maximum load.

Next, by using Equation (6), we can also explicitly obtain the accurate load $\Phi_{CSR}(0)$ at the center of the disk. Let $q =$

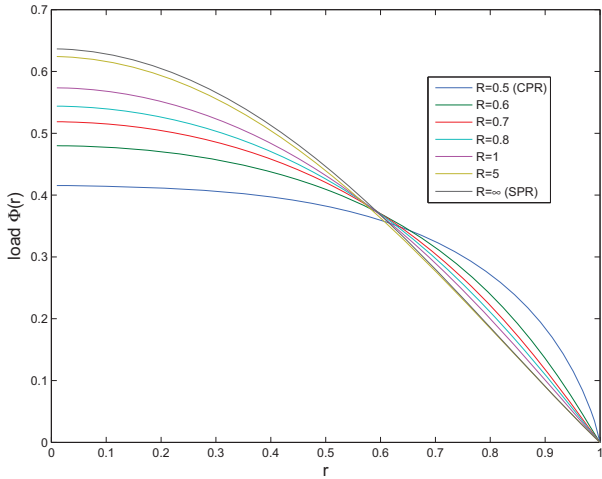


Fig. 7. Load distribution as the function of the distance r to the center for CSR with different R . Here we assume that $\Lambda = 1$.

$\sqrt{\left(\frac{a^2+4R^2}{2a}\right)^2 - x^2}$ and $p = \frac{4R^2-a^2}{2a}$. We have $y(x, a) = q - p = \frac{4R^2-x^2}{q+p}$. Now we calculate the limitation of $\frac{y(x, a)}{a}$ when a goes to 0, as follows,

$$\lim_{a \rightarrow 0} \frac{y(x, a)}{a} = \lim_{a \rightarrow 0} \frac{4R^2 - x^2}{aq + ap} \quad (11)$$

$$= \lim_{a \rightarrow 0} \frac{4R^2 - x^2}{\sqrt{\left(\frac{a^2+4R^2}{2}\right)^2 - a^2x^2} + \frac{4R^2-a^2}{2}} \quad (12)$$

$$= \frac{4R^2 - x^2}{2R^2 + 2R^2} \quad (13)$$

$$= 1 - \frac{x^2}{4R^2}. \quad (14)$$

Therefore, given by Equation (6), we have

$$\Phi_{CSR}(0) = \frac{\Lambda}{\pi} \int_{-1}^1 \lim_{a \rightarrow 0} \frac{y(x, a)}{a} dx \quad (15)$$

$$= \frac{\Lambda}{\pi} \int_{-1}^1 \left(1 - \frac{x^2}{4R^2}\right) dx \quad (16)$$

$$= \frac{\Lambda}{\pi} \left(2 - \frac{1}{6R^2}\right). \quad (17)$$

Clearly, when R increases, the load at the center increases too. When $R = 1/2$, $\Phi_{CSR}(0) = \frac{4}{3} \frac{\Lambda}{\pi} \approx 0.424 \cdot \Lambda$. When $R = \infty$, $\Phi_{CSR}(0) = 2 \frac{\Lambda}{\pi} \approx 0.637 \cdot \Lambda$. In both cases, the traffic load at the center matches the results for CPR or SPT obtained by Hyytiä and Virtamo [3] and the values shown in Fig. 7.

IV. SIMULATION

In this section, we conduct extensive simulations with both grid and random networks in our own developed simulator to verify our theoretical results. In both cases, wireless nodes are distributed in a unit circular area and a simple unit disk graph propagation model is used. In CSR, the south pole of the sphere \mathbb{S} is located at the center of the deployment area. The virtual coordinates of all projection nodes on the sphere are then generated. We are mainly interested in CSR's

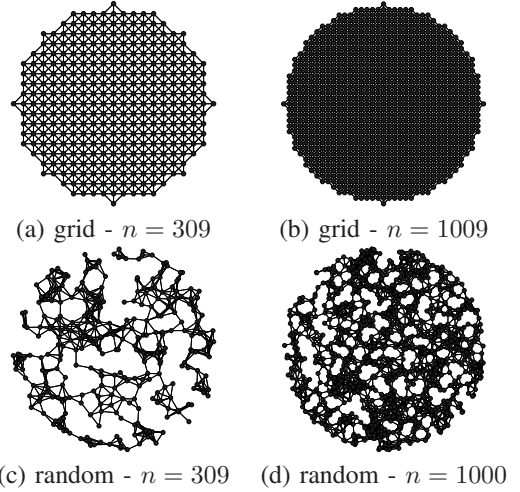


Fig. 8. Topology examples of grid and random networks.

load distribution in term of the distance to the center. It is clear that the size of the sphere affects the distribution of the mapped nodes on the sphere, thus affects the performance. In our simulations, we try different radii R of the sphere and different network sizes (denoted as n). For all simulations, we compare the load of SPR and CSR under the uniform communication scenario where every pair of nodes has unit message to communicate.

Grid Networks: We deploy n nodes on a grid inside the unit circular area, then set the transmission range to certain value so that each node can only reach its neighbors in the grid. Fig. 8(a) and (b) show two examples. Fig. 9 shows the load distribution of SPR and CSR under different scenarios. We first fix R at 0.5 and test the load distribution of SPR and CSR (n increases from 309 to 1009). The following conclusions can be drawn for all cases. It is clear that SPR suffers from the crowd center effect in which the maximum load occurs at the center of the network. CSR instead can eliminate the high load at the center area and its maximum load is much smaller than that of SPT. This confirms the load balancing capability of CSR. As shown in Fig. 9(b), we then fix n at 709 and test various values of R (from 0.5 to 2). With larger value of R , CSR has higher maximum load. This is consistent with our theoretical analysis. However, it is interesting to see that in all cases the maximum load of CSR does not occur at the center. This is mainly due to the structure of grid topology which is not a perfect circle around the boundary. In addition, the theoretical results we obtained in Section III are only true when the node number n goes to infinity.

Random Networks: We also test CSR with different random networks. In each case, we generate 30 random networks and take the average for all results. The results are plotted in Fig. 10. Overall, the conclusions are consistent with our theoretical results too. With more nodes, the load curves become much smoother as expected. When $n = 1000$, it is interesting to see that the load of CSR with $R = 2$ is already near the SPR's one. This confirm our observation that

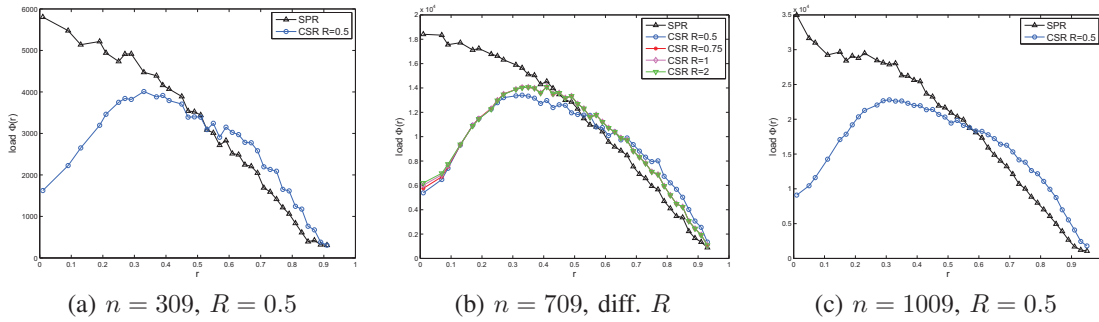


Fig. 9. Load distributions of SPR and CSR for grid networks.

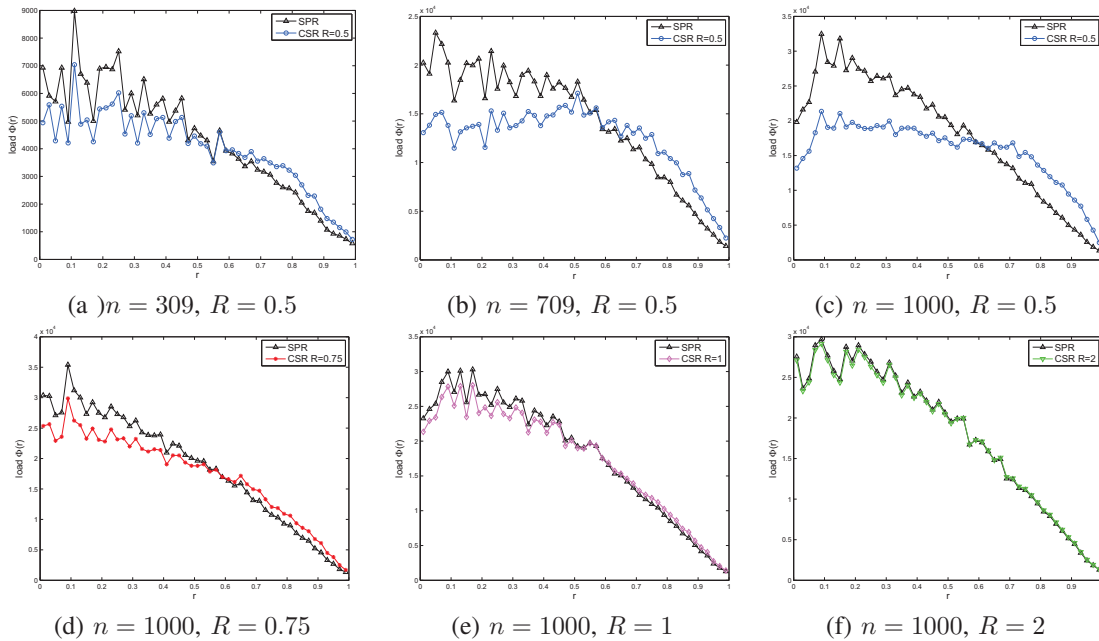


Fig. 10. Load distributions of SPR and CSR for random networks.

CSR becomes SPR when R goes to infinity. In addition, the maximum load of CSR indeed occurs near the center now.

V. CONCLUSION

In this paper, we have presented the first theoretical results on load distribution of CSR in a large-scale dense multihop network. Using the techniques developed by Hyytiä and Virtamo [3], we are able to derive the traffic load $\Phi(r)$ of CSR ($R \geq 1/2$) at any point with a distance r to the center of the network under uniform traffic demands. Our results show that when $R = 1/2$, CSR can achieve the best load balancing. Larger R will increase the maximum load at the center of the network. However, it is unclear what is the load distribution of CSR when $R < 1/2$. Simulation results in [9], [10] seem to indicate that a better load balancing may happen when R is slightly smaller than $1/2$. We leave further study of load distribution of CSR when $R < 1/2$ as our next step.

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