Corrections on “Data Collection Capacity of Random-Deployed Wireless Sensor Networks”

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***Abstract*— Our paper [1] gave a data collection method under physical interference model, which can match the optimal data collection capacity in order. In our analysis of the capacity, we first derived a lower bound of interference distance L such that all simultaneous transmissions based on interference blocks with length L can be successfully received. Recently, via personal communications, Dr. Gruia Calinescu from Illinois Institute of Technology let us know that there is a bug in our derivation. To fix the problem, this correspondence provides an alternative derivation which can still guarantee the same result of our capacity analysis.**

I. Introduction

Our paper [1] studied the data collection capacity under physical interference model. We provided a data collection method based on scheduling transmissions over interference blocks with size L, which can match the optimal data collection capacity in order. In our analysis of the capacity, we first derived a lower bound of interference distance L such that all simultaneous transmissions as shown in Fig. 1 can be successfully received.

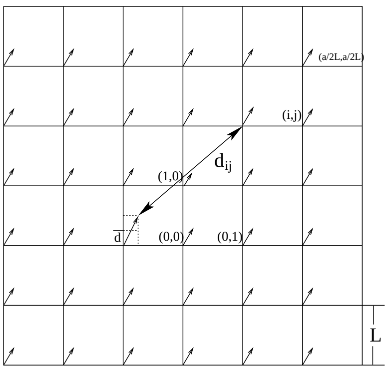

Fig. 1. Grid partition of interference blocks with size of $L \times L$.

Consider the SINR at the receiver in interference block $(0,0)$ (which is in the center of the field) since it has the minimum SINR among all receivers. Based on physical interference model, its SINR is

$$\frac{P \cdot r^{-\beta}}{B \cdot N_0 + \sum_{\text{all blocks } (i,j) \text{ except } (0,0)} P \cdot (d_{i,j})^{-\beta}}.$$

Here, $d_{i,j}$ is the distance from the sender in block (i,j) to the receiver in block $(0,0)$. Therefore, we need to derive L such that $SINR \geq \eta$, i.e.,

$$\sum_{\text{all blocks } (i,j) \text{ except } (0,0)} (d_{i,j})^{-\beta} \leq \frac{r^{-\beta}}{\eta} - \frac{BN_0}{P}.$$

In [1], we then proved that

$$\sum_{\text{all blocks } (i,j) \text{ except } (0,0)} (d_{i,j})^{-\beta} \leq \frac{2\pi}{3}(L-2d)^{-\beta}\left(\frac{1}{2^{\beta-1}}+2\right).$$

However, as pointed out by Gruia Calinescu, there are two errors in our derivation of the inequality above:

- $(\frac{1}{4}(\frac{1}{i^2}+\frac{1}{j^2}))^{\beta/2} \leq \frac{1}{2^\beta}(\frac{1}{i^\beta}+\frac{1}{j^\beta})$ is not true for $i>1$, $j>1$ and $\beta>2$.
- $\sum_{(i\geq1,j\geq1)} \frac{1}{2^\beta}(\frac{1}{i^\beta}+\frac{1}{j^\beta})$ is not equal to $\frac{1}{2^\beta}\sum_{i\geq1}\frac{1}{i^\beta}+\frac{1}{2^\beta}\sum_{j\geq1}\frac{1}{j^\beta}$, since the first term has much more items (around k^2 items where $k=\frac{a}{2L}$) than the second term has (around $2k$ items).

II. Corrections

Fortunately, Dr. Gruia Calinescu also referred us to [2], which already has a method to bound the interference under physical interference model. Next, we provide detailed derivation for our problem based on their method.

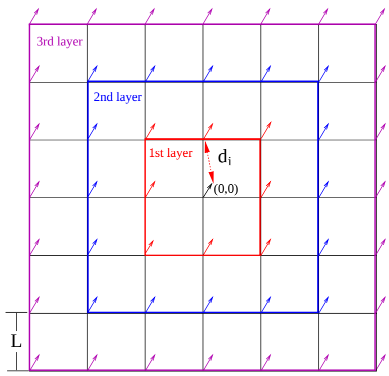

Fig. 2. Simultaneous transmissions are around the center by layers.

Once again we consider the SINR at the receiver in interference block $(0,0)$ (which is in the center of the field). We now label all simultaneous transmissions by layers from position $(0,0)$, as shown in Fig. 2. Based on physical interference model, its SINR is at least

$$\frac{P \cdot r^{-\beta}}{B \cdot N_0 + \sum_{\text{all layers } i\geq1} c_i P \cdot (d_i)^{-\beta}}.$$

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Here, d_i is the minimum distance from a transmitter on ith layer to the receiver in block $(0, 0)$ and c_i is the number of transmitters on ith layer. Therefore, we need to derive L such that $SINR \geq \eta$, i.e.,

$$\sum_{\text{all layers } i \geq 1} c_i (d_i)^{-\beta} \leq \frac{r^{-\beta}}{\eta} - \frac{BN_0}{P}.$$

Notice that $d_i = iL - 2d$ and $c_i = 8i$. For example, there are 8 transmitters at the first layer with distance at least $L - 2d$ and 16 transmitters at the second layer with distance at least $2L - 2d$, and so on. Thus,

$$\begin{aligned}
\sum_{i \geq 1} c_i (d_i)^{-\beta} &= \sum_{i \geq 1} 8i(iL - 2d)^{-\beta} \\
&\leq \sum_{i \geq 1} 8i(iL - 2id)^{-\beta} \\
&= 8(L - 2d)^{-\beta} \sum_{i \geq 1} i^{-(\beta-1)}.
\end{aligned}$$

Since $\beta > 2$, $\sum_{i \geq 1} i^{-(\beta-1)}$ converges to a constant, let it be denoted by ϕ. Then we only need

$$8\phi(L - 2d)^{-\beta} \leq \frac{r^{-\beta}}{\eta} - \frac{BN_0}{P},$$

to guarantee that the SINR at the receiver in the center is at least η. This can be satisfied by setting

$$L \geq \left(\frac{1}{8\phi} \cdot \left(\frac{r^{-\beta}}{\eta} - \frac{BN_0}{P}\right)\right)^{-\frac{1}{\beta}} + 2d.$$

Remember $r \leq (\frac{P}{B \cdot N_0 \cdot \eta})^{1/\beta}$, this makes sure we can find such suitable L. We can further select $L = (\frac{1}{8\phi} \cdot (\frac{r^{-\beta}}{\eta} - \frac{BN_0}{P}))^{-\frac{1}{\beta}} + 2d$. Since $r = \sqrt{5}d$,

$$\begin{aligned}
\frac{L}{d} &= \left(\frac{1}{8\phi} \cdot \left(\frac{(\sqrt{5}d)^{-\beta}}{\eta d^{-\beta}} - \frac{BN_0}{Pd^{-\beta}}\right)\right)^{-\frac{1}{\beta}} + 2 \\
&= \left(\frac{1}{8\phi} \cdot \left(\frac{5^{-\beta/2}}{\eta} - \frac{BN_0 d^{\beta}}{P}\right)\right)^{-\frac{1}{\beta}} + 2.
\end{aligned}$$

When $n \to \infty$, this ratio goes to a constant, denoted by α. This lead to the same analysis result which we need for our data collection method in [1] to achieve optimal capacity.

III. Conclusion

This correspondence provided a simple correction of our lower bound derivation for interference distance in [1] so that our data collection method and its capacity analysis stand the same. We would like to thank Dr. Gruia Calinescu for pointing out the problem and suggesting possible fix to us. In addition, we also want to acknowledge that similar interference-aware scheduling (taking turn among interference blocks) has been used in both [2] and [3] for unicast or broadcast.

References

[1] S. Chen, Y. Wang, X.-Y. Li, and X. Shi, "Data Collection Capacity of Random-Deployed Wireless Sensor Networks," in *Proceedings of the IEEE Global Telecommunications Conference (Globecom 2009)*, 2009.

[2] M. Franceschetti, O. Dousse, D. N.C. Tse, and P. Thiran, "Closing the gap in the capacity of random wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 4, pp. 1009-1018, 2007.

[3] G. Calinescu and S. Tongngam, "Interference-Aware Broadcast Scheduling in Wireless Networks," in *Proc. of the 4th International Conference on Mobile Ad-hoc and Sensor Networks (MSN)*, 2008.