Energy Efficient Topology for Unicast and Broadcast Routings

Wen-Zhan Song * Yu Wang * Xiang-Yang Li * Ophir Frieder *

Abstract— **We propose a novel topology control strategy for each wireless node to locally select communication neighbors and adjust transmission range, hence all nodes together self-form an energy efficient topology for unicast and broadcast routings in ad hoc environment. We first propose an improved strategy to construct degree-bounded planar spanner called $S\Theta GG$, by using relative positions only. $S\Theta GG$ keeps the same power spanning ratio with $SYaoGG$[30], while further reducing the node degree from k to $k-1$ when same parameter k is used. In addition, it is Θ-separated: the directions between any two neighbors of a node are separated by at least an angle θ. We then propose the first localized algorithm to construct a planar spanner with bounded node degree and low weight. The algorithm can apply to any known degree-bounded planar spanner (including BPS, $OrdYaoGG$, $SYaoGG$, $S\Theta GG$) and make low-weighted while keeping all its previous property, except increasing the power spanning ratio from ρ to $2\rho+1$ theoretically. For unicast applications, given any two nodes u and v, there is a path connecting them with total energy cost at most a constant factor times cost of the shortest path between them in the original communication graph. For broadcast applications, the total energy consumption is minimal among all locally constructed constructed topologies. Moreover, by assuming that the node ID and its position can be represented in $O(\log n)$ bits for a wireless network of n nodes, the total messages of construction is at most $3n$, where each message is $O(\log n)$ bits.**

Keywords— **Wireless ad hoc networks, topology control, self-adaptive, bounded degree, planar, spanner, low weight, efficient localized algorithm, power assignment.**

I. Introduction

A wireless *ad hoc* network consists of an arbitrary distribution of radios in certain geographical area. Unlike cellular wireless networks, there is no centralized control in the network, and mobile devices can communicate via multi-hop wireless channels: a node can reach all nodes in its transmission range while two far-away nodes communicate through the relaying by intermediate nodes. Wireless *ad hoc* networks trigger many challenging research problems, as they intrinsically have some special characteristics and unavoidable limitations, compared with other wired or wireless network. An important requirement of these networks is that they should be self-organizing, i.e., transmission ranges and data paths are dynamically restructured with changing topology. Energy conservation and network performance are probably the most critical issues in *ad hoc* wireless networks, because wireless devices are usually powered by batteries only and have limited computing capability and memory.

The self-adaptive *topology control* technique is to let each wireless device *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a global structure that can support energy efficient routing and improve the overall network performance. By enabling each wireless node shrinking its transmission power (which is usually much smaller than the maximal transmission power) to sufficiently cover the farthest selected neighbor, topology control scheme can not only save energy and prolong network life, but also can improve network throughput through mitigating the MAC-level medium contention. Unlike traditional wired networks and cellular wireless networks, the wireless devices are often moving during the communication, which could change the network topology in some extent. Hence it is more challenging to design a topology control algorithm for *ad hoc* wireless networks: the topology should be locally and self-adaptively maintained, without affecting the whole network and with low communication cost.

A wireless *ad hoc* network is modelled by a set V of n wireless nodes distributed in a two-dimensional plane. Each node has same *maximum* transmission range. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes iff their Euclidean distance is at most one. In other words, we assume that two nodes can always receive the signal from each other directly if the Euclidean distance between them is no more than one unit. Hereafter, $UDG(V)$ is always assumed to be connected. We also assume that all wireless nodes have distinctive identities and each wireless node knows its position information. More specifically, it is enough for our protocol when each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of signal arrival* and the *strength of signal*. We adopt the most common power-attenuation model from literature: the power needed to support a link uv is assumed to be $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 depending on the wireless transmission environment.

In the paper, We first propose an improved strategy to construct degree-bounded planar spanner called $S\Theta GG$, by using relative positions only. $S\Theta GG$ keeps the same power spanning ratio $\rho = \frac{\sqrt{2}^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta}$ as $SYaoGG$[30], while further reducing the node degree from k to $k-1$ when same parameter $k \geq 9$ is used. In addition, it is Θ-separated: the directions between any two neighbors of a node are separated by at least an angle θ. We then propose the first localized algorithm to construct a planar spanner with bounded node degree and low weight, which facilitates both unicast and broadcast routings in an energy efficient way. If we apply the algorithm on $S\Theta GG$, the final network topology has the following attractive properties:

1. power efficient: given any two nodes u and v, there is a path connecting them in the structure with total power cost no more than $2\rho+1$ times of the power cost of any path connecting them in the original network.
2. bounded node degree: each node has to communicate with at most $k-1$ neighbors;

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3. planar: this enables several localized routing algorithms [13], [2], [18], [19];
4. Θ-separated neighbors: the directions between any two neighbors of any node are separated by at least an angle θ.
5. low-weighted: for broadcast applications, the power consumption is within a constant factor of optimum among all locally constructed structures.

When a node moves, the topology can be locally and dynamically self-maintained without affecting the whole network. Moreover, by assuming that the node ID and its position can be represented in $O(\log n)$ bits each for a wireless network of n nodes, we show that the structure can be initially constructed using at most $3n$ messages. Our theoretical results are corroborated in simulations.

The rest of the paper is organized as follows. In Section II, we review some priori arts in this area, and summarize the most preferred properties of topology control protocol for unicast and broadcast applications. In Section III, an improved algorithm is presented to build degree-bounded planar spanner with Θ-separated property and lower degree. We then propose, in Section IV, the first localized algorithm to construct planar spanner with bounded-degree and low weight, which can support energy efficient unicast and broadcast applications simultaneously. Finally, we conclude our paper in Section V.

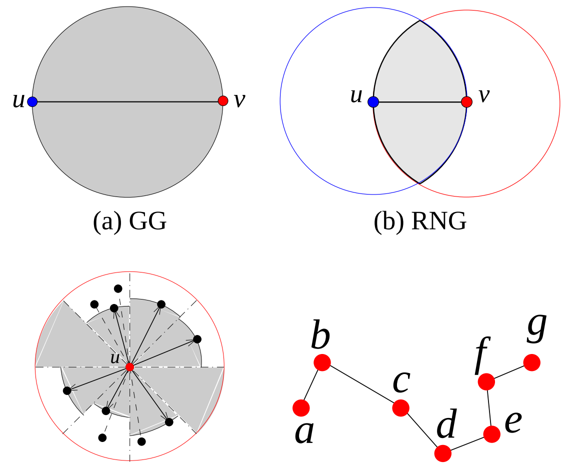
II. Priori Arts

A. Priori Arts on Unicast Topology

Several structures (such as relative neighborhood graph RNG, Gabriel graph GG, Yao structure, etc) have been proposed for topology control in wireless ad hoc networks. The *relative neighborhood graph*, denoted by $RNG(V)$ [31], see Figure 1(a), consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ do not contain any vertex w from the set V. The *Gabriel graph* [9] $GG(V)$ contains edge an uv if and only if $disk(u, v)$ contains no other points of V, where $disk(u, v)$ is the disk with edge uv as a diameter. See Figure 1(b) for illustration. Denote $GG(UDG)$ and $RNG(UDG)$ be the intersection of $GG(V)$ and $RNG(V)$ with $UDG(V)$ respectively. Both $GG(UDG)$ and $RNG(UDG)$ are connected, planar, and contain the Euclidean *minimum spanning tree* MST of V if UDG is connected. In [2], [13], relative neighborhood graph and Gabriel graph are used as underlying network topologies. However, RNG is not power efficient for unicasting, since its power stretch factor of RNG is $n-1$. Both RNG and GG are not degree-bounded.

For our later convenience, if it is clear that these structures are constructed on $UDG(V)$, we omit the (UDG) in the representation of all structures. For instance, we will use GG to denote Gabriel Graph instead of $GG(UDG)$.

The *Yao graph* [36] with an integer parameter $k > 6$, denoted by $\overrightarrow{YG}_k$, is defined as follows. At each node u, any k equally-separated rays originated at u define k cones. In each cone, choose the shortest edge $uv \in UDG(V)$ among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. See Figure 1(c). The resulting directed graph is called the *Yao graph*. Let YG_k be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k$. Some researchers used a similar construction named θ-graph [24], [14], the difference is that it chooses the edge which has the shortest projection on the axis of each cone instead of the shortest edge in each cone.



(a) GG (b) RNG

(c) YG (d) MST

Fig. 1. The definitions of RNG, GG, YG and MST. The shaded area is empty of nodes inside.

Bose *et al.* [3] proposed a centralized method with running time $O(n \log n)$ to build a degree-bounded planar spanner for a two-dimensional point set. It constructs a planar t-spanner with low-weight for a given nodes set V, for $t = (1 + \pi) \cdot C_{del} \simeq 10.02$, such that the node degree is bounded from above by 27. Hereafter, we use C_{del} to denote the spanning ratio of the Delaunay triangulation [8], [15], [14]. However, the distributed implementation of this centralized method takes $O(n^2)$ communications in the worst case for a set V of n nodes.

Recently, Wang and Li [34] proposed the first efficient localized algorithm to build a degree-bounded planar spanner BPS for wireless ad hoc networks. Though their method can achieve these three desirable features: planar, degree-bounded, and power efficient, the theoretical bound on the node degree of their structure is a large constant. For example, when $\alpha = \pi/6$, the theoretical bound on node degree is 25. In addition, the communication cost of their method can be very high, although it is $O(n)$ theoretically, because it needs to collect the 2-hop information for every wireless node. Even as mentioned in [34] the method by Calinescu [4] to collect 2-hop neighbors information takes $O(n)$ messages, however the hidden constant is large: it is several hundreds.

In [30], Song *et. al* proposed two methods to construct degree-bounded power spanner, by applying the ordered Yao structures on Gabriel graph. They achieved better performance with much lower communication cost, compared with the first known degree-bounded planar spanner [34] constructed in localized way. The second structure in [30] only costs $3n$ messages during the construction, and guarantees that there are at most one neighbor node in each of the $k = 9$ equal-sized cones. The trade-off is that the structure constructed in [30] is not length spanner while the structure constructed in [34] is.

In summary, for energy efficient unicast routing, the desired topology is preferred to have following features:

1. POWER SPANNER: For energy efficient unicast routing, the total power consumption of the shortest path (most power efficient path) between any two nodes in the constructed topology should not exceed a constant factor of the power consumption of the shortest path in the original graph. In other words, the topology is preferred to be a *power spanner*. Formally speaking, a subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. The constant ρ is called the *power stretch factor*.

2. DEGREE BOUNDED: It is also desirable that the node degree in the constructed topology is bounded from above by a small constant. A small node degree reduces the MAC-level contention and interference practically, also may help to mitigate the well known hidden and exposed terminal problems. Notice that, the small node degree cannot guarantee that the total link interferences are small, but we found that it does have small interferences compared with the structure with the minimum interferences. Another advantage of bounded degree is in Bluetooth based wireless ad hoc networks. According to Bluetooth specification, the *master* node degree need be bounded by 7 to maximize the efficiency. In addition, a structure with small degree will improve the overall network throughout [17].

3. PLANAR: A network topology is also preferred to be planar (no two edges cross each other in the graph) to enable some routing algorithms work correctly and efficiently, such as *Greedy Perimeter Stateless Routing* (GPSR) [13], *Greedy Face Routing* (GFG) [2], *Adaptive Face Routing* (AFR) [18], and *Greedy Other Adaptive Face Routing* (GOAFR) [19].

B. Priori Arts on Broadcast Topology

Any broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. Let $f_T(p)$ denote the transmission power of the node p required by T. For any leaf node p of T, $f_T(p) = 0$. For any internal node p of T,

$$f_T(p) = \max_{pq \in T} \|pq\|^\beta ,$$

in other words, the β-th power of the longest distance between p and its children in T. The total energy required by T is $\sum_{p\in V} f_T(p)$.

Minimum-energy broadcast routing in a simple ad hoc networking environment has been addressed in [5], [6], [16], [35]. The minimum-energy broadcast routing problem is different from the conventional link-based minimum spanning tree problem. It is known [5] that the minimum-energy broadcast routing problem cannot be solved in polynomial time. Three greedy heuristics were proposed in [35] for the minimum-energy broadcast routing problem: MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). The MST heuristic first applies the Prim's algorithm to obtain a MST, and then orient it as an arborescence rooted at the source node. The SPT heuristic applies the Dijkstra's algorithm to obtain a SPT rooted at the source node. The BIP heuristic is the node version of Dijkstra's algorithm for SPT. It maintains, throughout its execution, a single arborescence rooted at the source node. Another slight variation of BIP was discussed in detail in [32].

Wan *et al.* [32], [33] showed that the approximation ratios of MST and BIP are between 6 and 12 and between $\frac{13}{3}$ and 12 respectively; on the other hand, the approximation ratios of SPT and BAIP are at least $\frac{n}{2}$ and $\frac{4n}{\ln n} - o(1)$ respectively, where n is the number of nodes.

Unfortunately, none of the above structures can be formed and updated locally. RNG has been widely used for broadcasting in wireless ad hoc networks [28], since it can be constructed locally. However, the ratio of the weight of RNG over the weight of MST could be $O(n)$ for n points set [22]. An example was given in [20] to show that the total energy used by broadcasting on RNG could be about $O(n^\beta)$ times of the minimum-energy used by an optimum method. Li [20] showed that, there is *no* deterministic localized algorithm to find a structure that approximates the total energy consumption of broadcasting within a constant factor of the optimum. Furthermore, in the worst case, the energy cost for broadcasting on any locally constructed and connected structure is at least $O(n^{\beta-1})$ times optimum.

On the other hand, given any low-weighted structure H, i.e., $\omega(H) \leq O(1) \cdot \omega(MST)$, we have

Lemma 1: [20] $\omega_\beta(H) \leq O(n^{\beta-1}) \cdot \omega_\beta(MST)$, where H is any low-weighted structure.

Here $\omega(G)$ is the total length of the links in G, i.e., $\omega(G) = \sum_{uv\in G} \|uv\|$, and $\omega_\beta(G)$ is the total power consumption of links in G, i.e., $\omega_\beta(G) = \sum_{uv\in G} \|uv\|^\beta$. Consequently, low-weighted structure is optimal for broadcasting among any connected structures built in a localized manner. In other words, to support energy efficient broadcast applications, the underlying topology is preferred to be low-weighted. Recently, several localized algorithms [20], [23] have been proposed to construct low-weighted structure, which indeed approximates the energy efficiency of MST as the network density increasing. However, none of them is power efficient for unicast routing.

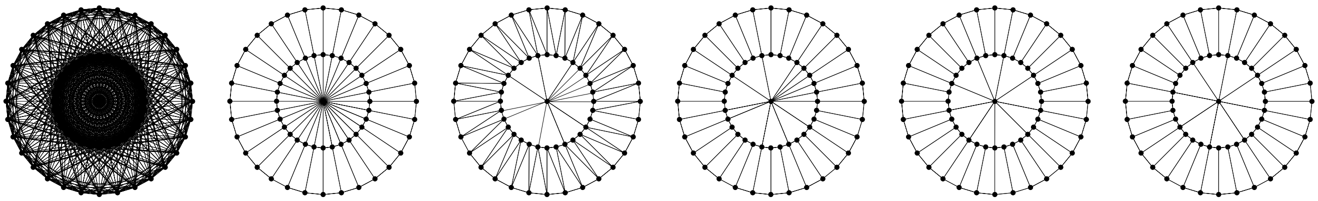
In summary, to enable energy efficient broadcasting, the locally constructed topology is preferred to be *low-weighted*, in addition to being power spanner, degree bounded, and planar.

4. LOW WEIGHTED: the total link length of final topology is within a constant factor of the total link length of MST.

III. AN IMPROVED BOUNDED-DEGREE PLANAR SPANNER

In this section, we propose a novel method to build degree-bounded planar power spanner, in addition, any two neighbors of each node are separated by an angle θ. Hereafter, we call it the Θ-*separation* property.

First, we review the property of Gabriel graph GG. It is not difficult to prove that structure GG is connected if UDG is connected. In addition, since we remove a link uv only if there are two links uw and wv with w inside $disk(u,v)$, it is easy to show that the power stretch factor of structure GG is exactly 1 [22]. In other words, the minimum power consumption path for any two nodes u and v in UDG is still kept in GG. Remember that here we assume the power needed to support a link uv is $\|uv\|^\beta$, for $\beta \in [2,5]$. Notice that GG is not degree bounded as mentioned



(a) UDG (b) RNG,GG (c) BPS (d) $OrdYaoGG$ (e) $SYaoGG$ (f) $S\Theta GG$

Fig. 2. Several planar power spanners on the UDG shown in (a). Here $k = 9$ during constructing $SYaoGG$ and $S\Theta GG$.

in literature. For example, when all $n - 1$ nodes are uniformly distributed on two circles with the nth node u as center, the node degree of u is $\frac{n-1}{2}$, as shown in Figure 2(b).

The following result is a folklore.

Theorem 2: [21] GG is a planar power spanner, whose power stretch factor is 1.

One natural way to construct a degree-bounded planar power spanner is to apply the Yao structure on Gabriel graph. In [22], Li *et al.* showed that the final structure by directly applying the Yao structure on GG is a planar power spanner, called $YaoGG$, however its in-degree can be as large as $O(n)$, as in the example shown in Figure 2(b). In [30], Song *et. al* proposed two new methods to bound node degree by applying the ordered Yao structures on Gabriel graph. They achieved better performance with much lower communication cost, compared with the first known degree-bounded planar spanner [34] constructed in localized way. The second structure in [30] guarantees that there is at most one neighbor node in each of the k equal-sized cones. To further reduce the signal interferences, the direction between any two neighbors of one node is preferred to be separated by at least angle θ, especially with the use of directional antennas. Hence, we propose a new structure based on $S\Theta$-graph. It has the Θ-separation property and does not demand the absolute cone partition, instead, only relative directions from two neighbors are needed. In addition, as will see later, the $S\Theta$-graph is stable and unique, if the point set V is given. While the final topology based on Yao graph varies as the direction of k-equal sized cones varies.

The difference between $S\Theta$ graph and Yao graph is that we use a concept called *θ-dominating regions* instead of *cones*. Once a node u keep a communication neighbor v, it will delete all the other neighbors in the θ-dominating region of node v.

Definition 1: θ-DOMINATING REGION: For each neighbor node v of a node u, the *dominating region* of v is the 2θ-cone emanated from u, with the edge uv as axis.

Figure 3 illustrates the θ-dominating region by a node v emanated from a node u. Another advantage of using the concept of θ-dominating region is that the theoretical node degree bound is reduced by 1 while keeping the same power spanning ratio as Yao graph when $k = 1 + 2\pi/\theta$.

In the following, we give a new method to construct a degree-$(k-1)$ ($k \geq 9$) planar power spanner, where any two neighbors of one node are guaranteed to be separated by an angle at least θ. Simulation late shows the average node power assignment is indeed smaller than $SYaoGG$ [30]. In the algorithm, one data structures will be used: $N(u)$ is the set of neighbors of each node u in the final topology, which is initialized as the set

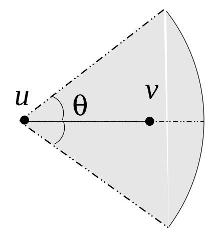


Fig. 3. Node v's θ-Dominating Region with respect to node u.

of neighbor nodes in GG. A node is marked WHITE if it is unprocessed and it is marked BLACK when it is processed.

Algorithm 1: CONSTRUCT $S\Theta GG$: DEGREE-$(k-1)$ PLANAR POWER SPANNER

1. First, each node self-constructs the Gabriel graph GG locally. Initially, all nodes mark themselves with WHITE color, i.e., *unprocessed*.

2. Once a WHITE node u has the smallest ID among all its WHITE neighbors in $N(u)$, it uses the following strategy to select neighbors:

(a) Node u first sorts all its BLACK neighbors (if available) in $N(u)$ in the distance-increasing order, then sorts all its WHITE neighbors (if available) in $N(u)$ similarly. The sorted results are then restored to $N(u)$, by first writing the sorted list of BLACK neighbors then appending the sorted list of WHITE neighbors.

(b) Node u scans the sorted list $N(u)$ from left to right. In each step, it keeps the current pointed neighbor w in the list, while deletes every *conflicted* node v in the remainder of the list. Here a node v is conflicted with w means that node v is in the θ-dominating region of node w. Here $\theta = 2\pi/k$ ($k \geq 9$) is an adjustable parameter.

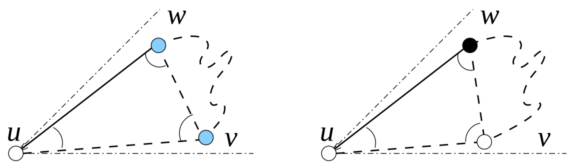
Node u then marks itself BLACK, i.e. *processed*, and notifies each deleted neighboring node v in $N(u)$ by a broadcasting message UPDATEN.

3. Once a node v receives the message UPDATEN from a neighbor u in $N(v)$, it checks whether itself is in the nodes set for deleting: if so, it deletes the sending node u from list $N(v)$, otherwise, marks u as BLACK in $N(v)$.

4. When all nodes are processed, all selected links $\{uv | v \in N(u), \forall v \in GG\}$ form the final network topology, denoted by $S\Theta GG$. Each node can shrink its transmission range as long as it sufficiently reaches its farthest neighbor in the final topology.

Lemma 3: Graph $S\Theta GG$ is connected if the underlying graph GG is connected. Furthermore, given any two nodes u and v, there exists a path $\{u, t_1, ..., t_r, v\}$ connecting them such that all edges have length less than $\sqrt{2}\|uv\|$.

Proof: We prove the connectivity by contradiction. Suppose a link uv is the shortest link in UDG whose connectivity is broken by Algorithm 1. W.L.O.G, assume the link uv is removed while processing node u, because of the existence of another node w.



(a) $\|uw\| < \|uv\|$ (b) $\|uw\| > \|uv\|$

Fig. 4. Two cases when link uv is removed while processing node u.

As shown in Figure 4, there are only two cases (ties are broken by ID) that the link uv can be removed by node u:
1. $\|uw\| < \|uv\|$. Notice that $\angle vuw \le \theta < \pi/4$, hence $\|wv\| < \|uv\|$. In other words, both link wv and uw are smaller than link uv. Since there are no paths $u \leftrightsquigarrow v$ according to the assumption, either the path $u \leftrightsquigarrow w$ or $v \leftrightsquigarrow w$ is broken. That is to say, either the connectivity of wv or uw is broken. Thus, uv is not the shortest link whose connectivity is broken, it is a contradiction to our assumption.
2. $\|uw\| > \|uv\|$. It happens only when node $w$is *processed* and node v is *unprocessed*. Similarly, $\angle vuw \le \theta < \pi/4 < \angle uwv$ (otherwise $\angle uvw > \pi/2$ violates the Gabriel Graph property), hence $\|wv\| < \|uv\|$. Since node w is a *processed* node and node u decides to keep link uw, the link uw will be kept in $S\Theta GG$. According to assumption that u and v are not connected in $S\Theta GG$, w and v are not connected either. That is to say, uv is not the shortest link whose connectivity is broken. It is a contradiction.

This finishes the proof of connectivity. Notice that the above proof implies that the shortest link uv in UDG is kept in the final topology. Clearly, the shortest link uv is in GG. Link uv cannot be removed in our algorithm due to the case illustrated by Figure 4 (a). Assume, for the sake of contradiction, that uv is removed due to the case (b) where $\|uw\| > \|uv\|$ and w is *processed* when processing u. Then $\|wv\| < \|uv\|$ is a contradiction to that uv is the shortest link in UDG.

We then show by induction that, given any link uv in UDG, there is a path connecting them using edges with length at most less than $\sqrt{2}\|uv\|$. Assume uv is removed when processing u and due to the existence of link uw. We build a path connecting u and v by concatenating $u \leftrightsquigarrow w$ and $w \leftrightsquigarrow v$, as shown in Figure 4. It is not difficult to see that the longest segment of the path is less than $\sqrt{2}\|uv\|$, which occurs in case (b). In this case, the link uw must be kept because both endpoints are *processed*, and $\|uw\| < \sqrt{2}\|uv\|$. This finishes the proof. ■

Theorem 4: The structure $S\Theta GG$ has node degree at most $k-1$ and is planar power spanner with neighbors Θ-separated. Its power stretch factor is at most $\rho = \frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$, where $k \ge 9$ is an adjustable parameter.

Proof: The proof would be similar with the proof of $SYaoGG$ in [30]. The only difference is that, we used the concept of dominating cones instead of Yao graph. Hence, while the power stretch factor keeps same theoretically, the degree bound is reduced from k to $k-1$. Obviously, the links in $S\Theta GG$ are Θ-separated, in other words, the direction of any two neighbors of a node is Θ-separated. ■

If $\theta = 2\pi/9$, Algorithm 1 constructs a degree-8 bounded planar power spanner, hence the degree bound is further decreased by 1 compared with $SYaoGG$ (where $k = 9$ is minimum) in [30]. In our new topology $S\Theta GG$, any two neighbors of any node are separated by at least an angle θ, which is called Θ-*separation* property. We believe this property is a great advantage if the directional antenna is adopted in wireless ad hoc networks. For each node, the signal interference caused by the communication with its neighbors is mitigated because their direction is wide separated. Figure 2 (e) and (f) shows the difference of $SYaoGG$ and $S\Theta GG$. Graph $S\Theta GG$ is more uniform distributed and the center node has lower node degree. Hence, the power assignment based on $S\Theta GG$ is also expected to be smaller.

More importantly, we will show later that, in terms of energy consumption, the broadcasting through its selected links is asymptotically optimal among all locally constructed structures.

IV. Localized Algorithm for Planar Spanner with Bounded Degree and Low Weight

To our best knowledge, so far, no localized topology control algorithm can achieve all the desirable properties summarized in Section II: *degree-bounded*, *planar*, *power spanner*, *low-weighted*. Hence, theoretically, none of them can support unicast and broadcast routing simultaneously in an energy efficient way. In real network applications, unicast and broadcast routings commonly co-exist in the same network at the same time, thus, a unified topology is demanded to support both applications. In the following, we propose the first strategy for all nodes together to locally maintain a network topology with all the four properties. The algorithm can apply to any known degree-bounded planar spanner (including BPS, $OrdYaoGG$, $SYaoGG$, $S\Theta GG$) and make low-weighted while keeping all its previous property, except increasing the power spanning ratio from ρ to $2\rho + 1$ theoretically.

Broadcast is also a very important operation in wireless ad hoc networks, as it provides an efficient way of communication that does not require global information and functions well in the case of changing topologies. For example, many unicast routing protocols [12], [25], [27], [26], [29] for wireless multi-hop networks use broadcast in the stage of route discovery. Similarly, several information dissemination protocols in wireless sensor networks use some form of broadcast/multicast for solicitation or collection of sensor information [10], [11], [37]. Since sensor networks mainly [1] use broadcast for communication, how to deliver messages to all the wireless devices in a scalable and power-efficient manner has drawn more and more attention. Not until recently have research efforts been made to devise power-efficient broadcast structures for wireless ad hoc networks.

Li [20] showed that, there is *no* deterministic localized algorithm to find a structure that approximates the total energy consumption of broadcasting within a constant factor of the optimum. Furthermore, in the worst case, the energy cost for broadcasting on any locally constructed and connected structure is at least $O(n^{\beta-1})$ times of the optimum. On the other hand, given any low-weighted structure H, the total power consumption of broadcasting based on it is no more than $O(n^{\beta-1})$ times of optimum as described in Lemma 1. Consequently, low-weighted structure is optimal for broadcasting among any connected structures built in localized manner. Li [20] also showed by example that it is *impossible* to construct a low-weighted structure using only one hop neighbor information. In the following, we give a communication efficient method to derive a sparse topology from $S\Theta GG$ whose total edge weight is within a constant factor of $\omega(MST)$, hence a routing protocol can be designed to support both applications simultaneously in this structure. Only the 2-hop information of $S\Theta GG$ graph [1] is used during the construction.

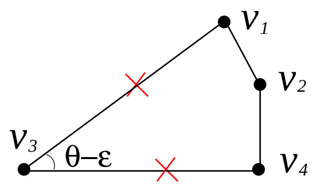
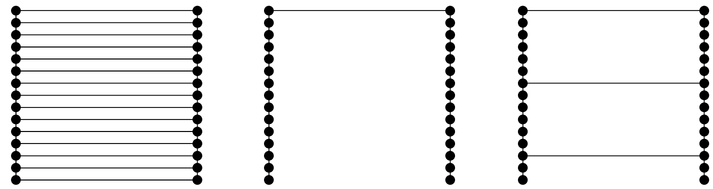


Fig. 5. The graph could be disconnected if applying the previous method to build low-weighted structure on $S\Theta GG$.

We then study how to transform the structure $S\Theta GG$ to a low-weighted one. One approach may directly use the technique presented in [20], [23]. The main technique used in [20], [23] is that for two links uv and xy of a graph G if xy is the longest link among quadrilateral $uvxy$, then xy is removed. They proved that the final structures are low-weighted if G is RNG' [20] or LMST$_2$ [23]. However, the technique in [20], [23] can not be applied here directly to bound the weight. Since, if we apply that one, the resulted topology could be disconnected as shown in Figure 5. The node ID of v_i is i, $\angle v_1v_3v_4 < \theta$ and $\|v_1v_3\| > \|v_3v_4\| > \max(\|v_1v_2\|, \|v_2v_4\|)$. While constructing $S\Theta GG$, v_1 will first select v_1v_2 and v_1v_3, then v_2 selects v_2v_1 and v_2v_4, and v_3 will select v_3v_1 and delete v_3v_4. Hence $v_3v_4 \notin S\Theta GG$. If applying the rule described in [20], [23], the link v_1v_3 will also be deleted because $\|v_1v_3\| > \max(\|v_1v_2\|, \|v_2v_4\|, \|v_3v_4\|)$. Then the graph will be disconnected. A modification would be that deleting xy only if $\|xy\| > \max(\|uv\|, \sqrt{2}\|ux\|, \sqrt{2}\|vy\|)$. This can work and generate a connected low-weighted planar structure from $S\Theta GG$, however, it is not a spanner. Figure 6(a) illustrates the original graph UDG and $S\Theta GG$. Two groups of $\frac{n}{2}$ nodes are distributed at two parallel lines, and the distance between the two lines is R. If we apply the similar method derived from [23] with the above modification, we indeed get a low-weighted structure, as shown in Figure 6(b) but the path between two bottom nodes could be very long and not power efficient for unicast routing. Hence, to keep the spanner property, we need keep some links between the left and right sides. Figure 6(c) illustrates our method presented late, which is based on local ordering.

In the following, we will describe a tricky algorithm to

[1]It is the 2-hop information of $S\Theta GG$ instead of UDG, Hence, during the construction, every node only need set its minimum transmission range in $S\Theta GG$.



(a) UDG and $S\Theta GG$ (b) Method derived from LMST and IMRG (c) Method with Ordering Deletion

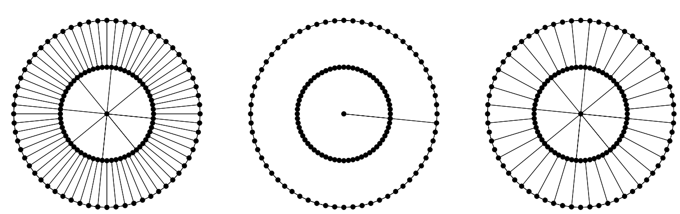
Fig. 6. Different Low Weight Strategies Comparison.

build a low-weighted structure from $S\Theta GG$ while keeping the bounded-degree planar power spanner property. The ID of an edge uv is defined as following: $ID(uv) = \{\|uv\|, \min(ID(u), ID(v)), \max(ID(u), ID(v))\}$.

Algorithm 2: CONSTRUCT $LS\Theta GG$: PLANAR SPANNER WITH BOUNDED DEGREE AND LOW WEIGHT

1. All nodes together construct the graph $S\Theta GG$ in a localized manner, as described in Algorithm 1. Initially, each node marks its incident edges in $S\Theta GG$ *unprocessed*.
2. Each node u locally broadcasts its incident edges in $S\Theta GG$ to its one-hop neighbors and listens to them at the same time. (Actually, this step can be omitted, if each node saved the edges information from its one hop neighbors while constructing $S\Theta GG$.)
Each node x can learn the existence of the set of 2-hop edges $E_2(x)$, which is defined as following: $E_2(x) = \{uv \in S\Theta GG | u \in N_{UDG}(x) \text{ or } v \in N_{UDG}(x)\}$. In other words, $E_2(x)$ represents the set of edges in $S\Theta GG$ with one endpoint in the transmission range of node x.
3. Once a node x learns its *unprocessed* incident edge xy has smallest ID among $E_2(x)$, it will delete edge xy if there exists an edge $uv \in E_2(x)$ (here both u and v are different from x and y), such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$; otherwise it simply marks edge xy *processed*. Here assume that $uvyx$ is the convex hull of u, v, x and y.
Repeat this step until all edges have been *processed*.
4. Let $LS\Theta GG$ be the final structure formed by all remaining edges in $S\Theta GG$.

Obviously, the construction is consistent for two endpoints of each edge: if an edge uv is kept by node u, then it is also kept by node v. Figure 7 illustrates the comparison of the weighted structure $LMST$, $IMRG$[23] and our $LS\Theta GG$ based on the original structure shown in Figure 7(a).



(a) $S\Theta GG$ (b) $LMST$, $IMRG$ (c) $LS\Theta GG$

Fig. 7. Low Weight Structures Comparison.

Theorem 5: The structure $LS\Theta GG$ is a degree-bounded planar spanner. It has constant power spanning ratio $2\beta + 1$, where

β is the power spanning ratio of $S\Theta GG$. The node degree is bounded by $k-1$ where $k>8$ is an adjustable parameter.

Proof: The degree-bounded and planar property is obviously derived from the $S\Theta GG$ graph, since we do not add any links in Algorithm 2.

To prove the spanner property is also kept, we only need show that the two endpoints of any deleted link $xy \in S\Theta GG$ is still connected in $LS\Theta GG$ with a constant spanning ratio path. We will prove it by induction on the length of deleted links from $S\Theta GG$.

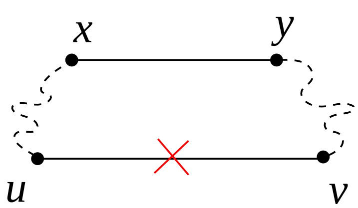


Fig. 8. The path between u and v is at most $(2\beta+1)\|uv\|$ in $LS\Theta GG$ if $uv \in S\Theta GG$.

Assume xy is the shortest link of $S\Theta GG$ which is deleted by Algorithm 2 because of the existence of link uv with smaller length. Obviously, path $x \leftrightsquigarrow y$ can be constructed through the concatenation of path $x \leftrightsquigarrow u$, link uv and path $v \leftrightsquigarrow y$, as shown in Figure 8. Since $\|xy\| > \max(\|ux\|, \|vy\|)$ and link xy is the shortest among deleted links in Algorithm 2, we have $p(x \leftrightsquigarrow u) < \beta\|ux\|$ and $p(v \leftrightsquigarrow y) < \beta\|vy\|$. Hence, $p(x \leftrightsquigarrow y) < \|uv\| + \beta\|ux\| + \beta\|vy\| < (2\beta+1)\|xy\|$.

Suppose all the i-th ($i \leq k-1$) deleted shortest links of $S\Theta GG$ have a path connecting their endpoints with spanning ratio $2\beta+1$. For the k-th deleted shortest link $xy \in S\Theta GG$, according to Algorithm 2, it must have been deleted because of the existence of a link uv: such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$ in a convex hull $uvyx$. Except that $p(x \leftrightsquigarrow u) < \beta\|ux\|$ and $p(v \leftrightsquigarrow y) < \beta\|vy\|$, similarly, we have

$$\begin{aligned}
p(x \leftrightsquigarrow y) &= \|uv\| + p(u \leftrightsquigarrow x) + p(v \leftrightsquigarrow y) \\
&< \|uv\| + (2\beta+1)\|ux\| + \beta\|vy\| \\
&< \|xy\| + (2\beta+1)\|xy\|/3 + (2\beta+1)\|xy\|/3 \\
&\leq (2\beta+1)\|xy\|
\end{aligned}$$

Here power spanning ratio $\beta \geq 1$ is obvious.

In summary, $LS\Theta GG$ is also a power spanner with spanning ratio $2\beta+1$. ∎

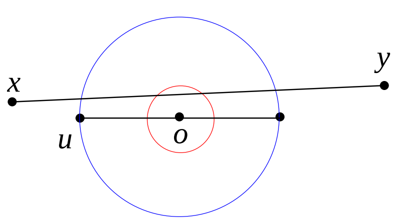


Fig. 9. The hypothetical case that an edge uv is not isolated. Here assume edge xy intersects its protecting disk.

We then show that graph $LS\Theta GG$ is low-weighted. To study the total weight of this structure, inspired by the method proposed in [20], we will show that the edges in $LS\Theta GG$ satisfy the *isolation property* [7], which is defined as follows. Let $c>0$ be a constant and E be a set of edges in d-dimensional space, and let $e \in E$ be an edge of length l. If it is possible to place a hyper-cylinder B of radius and height $c \cdot l$ each, such that the axis of B is a subedge of e and B does not intersect with any other edge, i.e., $B \cap (E - \{e\}) = \phi$, then edge e is said to be *isolated* [7]. If all the edges in E are isolated, then E is said to satisfy the *isolation property*. The following theorem is proved by Das *et al.* [7].

Theorem 6: [7] If a set of line segments E in d-dimensional space satisfies the isolation property, then $\omega(E) = O(1) \cdot \omega(SMT)$.

Here SMT is the Steiner minimum tree over the end points of E, and total edge weight of SMT is no more than that of the minimum spanning tree. It is also known [7] that, in the definition of the isolation property, we can replace the hyper-cylinder by a hypersphere, a hypercube etc., without affecting the correctness of the above theorem. We will use a disk and call it *protecting disk*. Specifically, the protecting disk of a segment uv is $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$, where o is the midpoint of segment uv. Obviously, we need all such disks do not intersect any edge except the one that defines it.

Theorem 7: The structure $LS\Theta GG$ is low-weighted.

Proof: We will prove by showing that all edges E in $LS\Theta GG$ satisfy the isolation property. For the sake of contradiction, assume that E does not satisfy the isolation property. Assume there is one edge uv that is not isolated. Thus, there is an edge, say xy, that intersects the protecting disk of uv. Figure 9 illustrates the hypothetical situation: a link xy intersects the protecting disk of link uv, i.e., $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$. First notice that, both x and y can not locate inside $disk(o, \frac{1}{2}\|uv\|)$, otherwise the property of Gabriel graph is violated.

We further divide the hypothetical situation into two cases:

1. The first case is that $\|xy\| < \|uv\|$.
We will show that the link uv itself must have been removed by our algorithm, by proving that both $\|ux\|$ and $\|vy\|$ are no more than $\frac{1}{3}\|uv\|$ in the hypothetical situation. To prove by inducing contradiction, w.l.o.g., we assume that $\|vy\| > \frac{1}{3}\|uv\|$.
Figure 10 illustrates our proof that follows. The link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w, and intersects the right half of $disk(v, \frac{1}{3}\|uv\|)$ with the point y'. Let t be a point on the top half of $disk(v, \frac{1}{3}\|uv\|)$ such that $\|ut\| = \|uv\|$. The segment ut intersects the $disk(o, \frac{1}{2}\|uv\|)$ with point t'. It is easy to verify that ut is the tangent line of protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$.
From the assumption $\|vy\| > \frac{1}{3}\|uv\|$, node y is out of the $disk(v, \frac{1}{3}\|uv\|)$. Hence, $\|xy\| > \|x'y'\|$. We continue to induce contradiction that $\|xy\| > \|uv\|$ by showing $\|x'y'\| > \|ut\| = \|uv\|$.
(a) Obviously, $\|x'w\| > \|ut'\|$, because the chord $x'w$ of $disk(o, \frac{1}{2}\|uv\|)$ is closer the center o than the chord ut' (because $x'w$ intersects the protecting disk while ut is the tangent line).
(b) Similarly, $\|wy'\| > \|tt'\|$, because $\|ww'\| > 2\|tt'\|$ (In $disk(v, \frac{1}{3}\|uv\|)$, the chord ww' is closer to center v than line tt', and segment tt' is half of the chord overlaping tt' since vt' is

perpendicular to ut' in $disk(o, \frac{1}{2}\|uv\|)$.) and $\|wy'\| > \frac{1}{2}\|ww'\|$ (In $disk(v, \frac{1}{3}\|uv\|)$, $\angle x'wv > \angle uwv = \frac{\pi}{2}$, hence $\|wy'\|$ is more than half of the chord ww'.).
Consequently, $\|xy\| > \|x'y'\| = \|x'w\| + \|wy'\| > \|ut'\| + \|tt'\| = \|ut\| = \|uv\|$, hence we get the contradiction. In other words, uv should have been deleted if $\|uv\| > \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is fake.

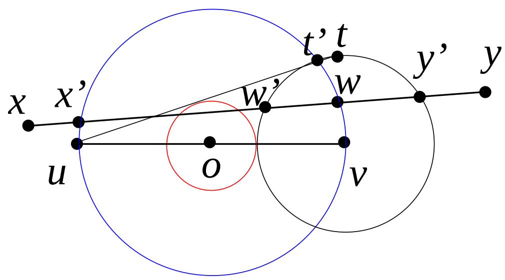


Fig. 10. $\|xy\| < \|uv\|$ is impossible.

2. The second case is that $\|xy\| > \|uv\|$.
We will show that xy will be deleted by Algorithm 2 by showing $\max(\|ux\|, \|vy\|) < \frac{1}{3}\|xy\|$ if xy intersects the protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$ of link uv. To prove by inducing contradiction, w.l.o.g., we assume that $\|vy\| > \frac{1}{3}\|xy\|$.
Figure 11 illustrates our proof that follows. ut is a tangent line of the protecting disk, the link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w. The segment $y'v$ is perpendicular to xv. Here point y' is on line xy.
Obviously, $\angle vxy < \angle vx'y$. And $\angle vx'y < \angle vut$ because the arc $\widehat{vw}$ is smaller than the arc $\widehat{vt}$. We have, $\|vy'\| = \|xy'\| \sin(\angle vxy) < \|xy'\| \sin(\angle vut) = \frac{\sqrt{35}}{18}\|xy'\| < \frac{1}{3}\|xy\|$. On the other hand, $\|vw\| < \|vt\| < \frac{1}{3}\|uv\| < \frac{1}{3}\|xy\|$. Hence, node y can not on the left side of y', instead only possible on the right side since $\|vy\| > \frac{1}{3}\|xy\|$. Then, we have $\angle xvy > \frac{\pi}{2}$, in other words, the Gabriel edge property of link xy is violated. Contradiction is induced here.
Consequently, xy should have been deleted if $\|uv\| < \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is also fake.

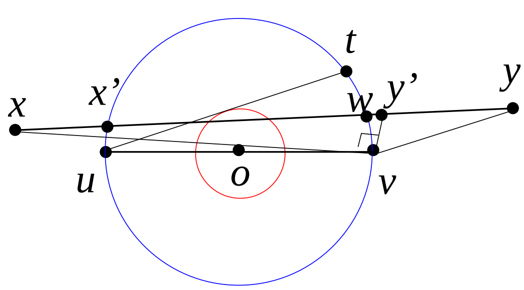


Fig. 11. $\|xy\| > \|uv\|$ is impossible.

In summary, each link $uv \in LS\Theta GG$ satisfies the isolation property, that is to say, $LS\Theta GG$ is low-weighted. This finishes the proof. ∎

The above proof also confirms our previous definition of partial 2-hop information for construction. Since we need remove the link xy only when $\sqrt{2}\max(\|xu\|, \|yv\|) \le \|xy\|$, each node x learns the existence of a shorter link uv by keeping its transmission range sufficient to cover all links in $S\Theta GG$ instead of using the maximum range. Hence the 2-hop information of $S\Theta GG$ is enough to ensure correctness.

We then analyze the communication cost of Algorithm 1 and 2. First, clearly, the first step of building GG in Algorithm 1 can be done using only n messages: each message contains the ID and geometry position of a node. Second, in the following steps of Algorithm 1, initially, the number of edges in Gabriel Graph is less than $3n$ since it is a planar graph. Clearly, there are at most $2n$ such removed edges since we keep at least $n-1$ edges from the connectivity of the final structure. Thus the total messages used to inform the deleted edges from GG is at most $2n$. Thirdly, in the marking process described in Algorithm 2, the communication cost of broadcasting its incident edges (or its neighbors) is in fact omitted in practice, since those neighbor information is already available during the construction of $S\Theta GG$. Hence, the marking process does not cost any communication. Notice that Algorithm 2 does not incur any communication cost when the 2-hop neighbor information is available. Then the following theorem directly follows.

Theorem 8: Assuming that both the ID and the geometry position can be represented by $\log n$ bits each, the total number of messages during constructing the structure $LS\Theta GG$ is at most $3n$, where each message has at most $O(\log n)$ bits.

V. Conclusion

Wireless *ad hoc* networks have been undergoing a revolution that promises to have a significant impact throughout society. Energy conservation and network performance are two critical issue in wireless ad hoc networks. This has drawn significant research interests from different approaches. Topology control scheme aims to conserve energy while improving network performance, hence it has low cost, compared with other approaches involved with hardware improvements.

We proposed a novel strategy for all wireless nodes to self-maintain an energy efficient network topology, called $LS\Theta GG$. It can support unicast and broadcast applications simultaneously in an energy efficient way. For unicast applications, the topology has following attractive properties: power spanner, bounded node degree, planar and Θ-separation. For broadcast applications, in terms of energy consumption, the topology is also within a constant factor of the power consumption of broadcast based on any locally constructed topology. All previous known localized topology control algorithms can only achieve part of those nice properties, especially, none of them can support both unicast and broadcast applications simultaneously in an energy efficient way. The power assignment based on our new structures shows low energy cost and small interference at each wireless node.

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