

# Power Efficient 3-Dimensional Topology Control for Ad Hoc and Sensor Networks

Yu Wang   Fan Li   Teresa A. Dahlberg

Dept. of Computer Science, University of North Carolina at Charlotte, USA

Email: {ywang32, fli, tdhlber}@uncc.edu

**Abstract**—Topology control in wireless ad hoc and sensor networks has been heavily studied recently. Different geometric topologies were proposed to be the underlying network topologies to achieve the sparseness of the communication networks or to guarantee the package delivery of specific routing methods. However, most of the proposed topology control algorithms were only applied to 2-dimensional (2D) networks where all nodes are distributed in a 2D plane. In practice, the ad hoc and sensor networks are often deployed in 3-dimensional (3D) space, such as notebooks in a multi-floor building and sensor nodes in a forest. This paper seeks to investigate power efficient topology control protocols for 3D ad hoc and sensor networks. In our new protocols, we extend several 2D geometric topologies to 3D case, and propose some new 3D Yao-based topologies. We also prove several properties (e.g., bounded degree and constant power stretch factor) for them in 3D space. The simulation confirms our theoretical proofs for these proposed 3D topologies.

## I. INTRODUCTION

Wireless ad hoc and sensor networks have been undergoing a revolution that promises a significant impact on society. Ad hoc network intrigues many challenging research problems because it intrinsically has many special characteristics and some unavoidable limitations compared with traditional fixed infrastructure networks. Energy conservation and network performance are probably the most critical issues in ad hoc and sensor networks because mobile devices are usually powered by batteries only and have limited computing capability. The *topology control* technique is to let each wireless device *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a structure that can support energy efficient routing and improve the overall network performance. There exist several topology control techniques such as localized geometrical structures, dynamic cluster techniques and power management protocols. In this paper, we only focus on geometrical structures based methods. Most proposed topology control algorithms based geometrical structures [1]–[4] are only applied to two dimensional networks where all nodes are distributed in a two dimensional plane. However, in practice, ad hoc and sensor networks are often deployed in three dimensional space, such as notebooks in a multi-floor building and sensor nodes in an *under-water sensor network* (UWSN). Three dimensional UWSN is used to detect and observe phenomena that can not be adequately observed by means of ocean bottom sensor nodes, i.e., to perform cooperative sampling of the 3D ocean environment. In this paper, we study how to efficiently construct topology for 3D ad hoc and sensor networks to maintain network connectivity,

conserve energy and enable energy efficient routing.

**Network Model:** A 3D ad hoc network consists of a set  $V$  of  $n$  wireless nodes distributed in a 3-dimensional plane  $\mathbb{R}^3$ . Each node has the same *maximum transmission range*  $R_m$ . These wireless nodes define a *unit ball graph* (UBG), or called *unit sphere graph*, in which there is an edge  $uv$  between two nodes  $u$  and  $v$  iff (if and only if) the Euclidean distance  $\|uv\|$  between  $u$  and  $v$  in  $\mathbb{R}^3$  is at most  $R_m$ . In other words, two nodes can always receive the signal from each other directly if the distance between them is no more than  $R_m$ . We also assume that all wireless nodes have distinctive identities and each node knows its position information either through a low-power GPS receiver or some other ways. By one-hop broadcasting, each node  $u$  can gather the location information of all nodes within its transmission range. As in the most common power-attenuation model, the power to support a link  $uv$  is assumed to be  $\|uv\|^\beta$ , where  $\beta$  is a real constant between 2 and 5 depending on the wireless transmission environment.

**Preferred Properties:** Topology control protocols are to maintain a structure that can improve the overall networking performance (such as lifetime). In the literature, the following desirable features of the structure are well-regarded and preferred in ad hoc and sensor networks: (1) A good network topology should be *energy efficient*, i.e., the total power consumption of the least energy cost path between any two nodes in final topology should not exceed a constant factor of the power consumption of the least energy cost path in original network. Given a path  $v_1v_2 \cdots v_h$  connecting two nodes  $v_1$  and  $v_h$ , the energy cost of this path is  $\sum_{j=1}^{h-1} \|v_jv_{j+1}\|^\beta$ . The path with the least energy cost is called the shortest path in a graph. A subgraph  $H$  is called a *power spanner* of a graph  $G$  if there is a positive real constant  $\rho$  such that for any two nodes, the power consumption of the shortest path in  $H$  is at most  $\rho$  times of the power consumption of the shortest path in  $G$ . The constant  $\rho$  is called the *power stretch factor*. A power spanner of the communication graph (e.g., UBG) is usually energy efficient for routing. (2) It is also desirable that node degree in the constructed topology is small and upper-bounded by a constant. A small node degree reduces the MAC-level contention and interference, and may help to mitigate the well-known hidden and exposed terminal problems. (3) Due to limited resources and high mobility of wireless nodes, it is preferred that the topology can be constructed locally, which means every node  $u$  can decide all edges incident on  $u$  based only on the information of nodes within a constant hops of  $u$ .

## II. PRIORI ARTS ON GEOMETRICAL TOPOLOGIES

Several geometrical structures have been proposed for topology control in 2D networks. The underlying communication graph is modeled by a *unit disk graph* (UDG) in which there is an edge between two nodes iff their distance is at most one.

Li *et al.* [5] proposed a localized topology called *local minimum spanning tree* (LMST) to estimate the *minimum spanning tree* (MST) and keep the network connected. In LMST, each node builds its local minimum spanning tree in one-hop neighborhood independently and only keeps one-hop on-tree nodes as neighbors. The *relative neighborhood graph* (RNG), consists of all edges  $uv \in \text{UDG}$  such that the intersection of two circles centered at  $u$  and  $v$  with radius  $\|uv\|$  does not contain any node from the set  $V$ . The *Gabriel graph* (GG) contains edge  $uv \in \text{UDG}$  iff  $\text{disk}(u, v)$  contains no other node of  $V$ , where  $\text{disk}(u, v)$  is the disk with edge  $uv$  as a diameter. Both GG and RNG are connected, planar, and contain the Euclidean MST of  $V$  if UDG is connected. RNG and GG have been used as underlying routing topologies for several proposed routing algorithms [6], [7]. However, Li *et al.* [2], showed that the power stretch factor of RNG is  $n - 1$  while the power stretch factor of GG is 1.

The *Yao graph* with an integer parameter  $k > 6$ , denoted by  $\vec{Y}G_k$ , is defined as follows: at each node  $u$ , any  $k$  equally-separated rays originating at  $u$  define  $k$  cones. In each cone, choose the shortest edge  $uv \in \text{UDG}$  among all edges emanated from  $u$ , if there is any, and add a directed link  $u\vec{v}$ . Let  $Y G_k$  be the undirected graph by ignoring the direction of each link in  $\vec{Y}G_k$ . The node out-degree of Yao graph is bounded by constant  $k$ . In [2], Li *et al.* proposed to construct the wireless network topology based on Yao graph and proved that Yao graph is a power spanner of UDG. Their proof of the spanner property is based on the induction of link length. The key result of their proof can be summarized by the following lemma:

*Lemma 1:* The power stretch factor of the Yao-based graph is at most  $\frac{1}{1 - (2 \sin \frac{\alpha}{2})^\beta}$ , if for every link  $uv$  who is not in the final graph, there exists a shorter link  $uw$  in the graph and  $\angle vuw < \delta$ , where  $\delta$  is a constant smaller than  $\pi/3$ .

Li *et al.* [3] further proposed to use another sparse topology, *Yao and Reversed Yao* (also called *Sparsified Yao graph*), which has constant-bounded node in/out-degree and constant-bounded power stretch factor [8]. The basic idea is to apply reversed Yao structure on  $\vec{Y}G_k$  to bound the in-degree.

Wattenhofer and Li *et al.* [1], [4] introduced a distributed *cone-based topology control* (CBTC) protocol based on directional information to guarantee the global connectivity of the network. They proved the spanner properties and bounded node degree of their topology if the top angle of the cone  $\alpha \leq \pi/2$ . While the CBTC protocol was designed specifically for topology control deployed on a 2-dimensional surface, it has been extended to the 3-dimensional space by [9]. The main idea is that each node increases its transmission power until there is no 3D-cone of degree  $\alpha$  without any nodes. They proved that running the algorithm with  $\alpha = \frac{2\pi}{3k}$  is an upper bound for preserving  $k$ -connectivity. However, there is

no proof of spanner property and node degree bound for their proposed topology. This paper is the only paper we found in literature that addresses 3D topology control.

To the best of our knowledge, our paper is the first one to study power efficient 3D topology control protocols for ad hoc and sensor networks. Besides topology control, there are also some other research in 3D ad hoc and sensor networks, such as coverage [10] and location [11] problems.

## III. 3D TOPOLOGY CONTROL PROTOCOLS

The first thought about 3D topology control is whether there exists an embedding method mapping the 3D networks into a 2D plane so that all 2D geometric topology control protocols can still be applied. Unfortunately, there is no such mapping method which still keeps the scale of the edge length. Consider a regular tetrahedron where the distances between any two nodes from the four endpoints are the same. However, in a 2D plane, there is no graph formed by four nodes where the distances between any two nodes are the same. Therefore, simply mapping 3D networks to 2D ones does not work when we want to achieve power efficiency.

Notice that local minimum spanning tree still works for 3D networks. It can guarantee the connectivity of the network and is easy to be constructed by using local information. However, as in 2D case, the stretch factor of LMST can be arbitrarily large in 3D, i.e., unicast routing may need to travel a much longer distance than the shortest path in UBG.

### A. 3D RNG and 3D GG

It is very natural to extend the related neighborhood graph and Gabriel graph to 3D. The definitions of 3D RNG and 3D GG are as follows: an edge  $uv \in \text{RNG}_{3D}$  iff the intersection of two balls centered at  $u$  and  $v$  with radius  $\|uv\|$  does not contain any node from the set  $V$ ; an edge  $uv \in \text{GG}_{3D}$  iff the ball with edge  $uv$  as a diameter contains no other node of  $V$  (see Figure 1).  $\text{RNG}_{3D}$  and  $\text{GG}_{3D}$  contains MST, which indicates that they are connected if the UBG is connected.  $\text{RNG}_{3D} \subseteq \text{GG}_{3D}$  since the shaded area in the definition of  $\text{GG}_{3D}$  is included in the definition of  $\text{RNG}_{3D}$ . By using the same example and proof in 2D [2], we can prove that  $\text{RNG}_{3D}$  is not a power spanner, while  $\text{GG}_{3D}$  is a power spanner with the power stretch factor of one. In other words, all edges in the least energy path in UBG are kept in  $\text{GG}_{3D}$ .

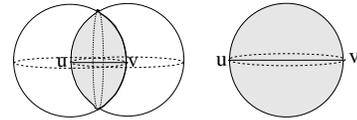


Fig. 1. 3D-RNG (left) and 3D-GG (right). Shaded area is empty of nodes.

### B. 3D Yao-based Structures

Since both  $\text{RNG}_{3D}$  and  $\text{GG}_{3D}$  can not bound node degree, we are also interested in extending Yao graph to 3D. However, it is hard to define the partition boundary of Yao structure of a node in 3D. Notice that a disk in 2D can be easily divided into  $k$  equal cones which do not intersect with each other, but

in 3D case, it is impossible to divide a ball into  $k$  equal 3D cones without intersections among each other. Here, we give two sets of methods to partition the transmission range:

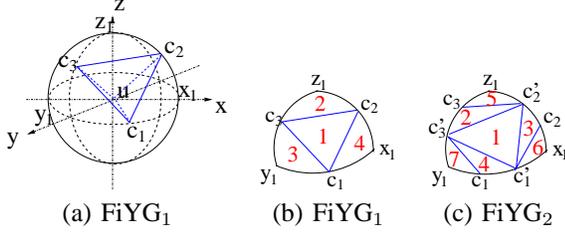


Fig. 2. Definitions of 3D Yao Graph with fixed partitions.

1) *Fixed Partition*: In fixed partition, cones from one node do not intersect with each other and the partition method is the same for all nodes. We will give two methods to divide the transmission range of a node into certain number of cones. In the first method, for each node  $u$ , it first divides its transmission region into 8 pieces by three orthogonal planes (i.e.,  $xy$ -plane,  $yz$ -plane and  $zx$ -plane), where each piece is a  $1/8$  ball. Then it uses three more planes as shown in Figure 2(a) to cut each piece into four cones. Figure 2(b) shows how to select these three planes where nodes  $c_1, c_2$  and  $c_3$  are the middle points of arc  $x_1y_1, y_1z_1$  and  $z_1x_1$  respectively. Therefore, the total number of cones at node  $u$  is  $8 \times 4 = 32$ . Notice that these cones have different shapes and do not intersect with each other. For each cone, node  $u$  chooses the shortest edge  $uv \in UBG$  among all edges emanated from  $u$ , if there is any, and adds a directed link  $\vec{uv}$ . Ties are broken arbitrarily or by ID. The resulting directed graph is denoted by  $FiYG_1$ . It is obvious that  $FiYG_1$  has a bounded out-degree of 32. However, it is easy to show that the angle  $\angle c_1uc_2 = \pi/3$ . Thus, the spanning ratio got from Lemma 1  $\frac{1}{1-(2\sin(\frac{\delta}{2}))^\beta}$  could be infinite when  $\delta = \pi/3$ . To fix this problem, we propose to use smaller cones for partition. The new partition method is shown in Figure 2(c), where for each piece of the  $1/8$  ball, we cut it into 7 cones. Here,  $c_1$  and  $c'_1, c_2$  and  $c'_2, c_3$  and  $c'_3$  trisect the arc  $x_1y_1, x_1z_1$ , and  $z_1y_1$  respectively. Using position information, we know the angle  $\angle c'_1uc'_3 = \pi/6$ . This can guarantee the power spanning ratio is bounded by  $\frac{1}{1-(2\sin(\frac{\pi}{12}))^\beta}$ . We use  $FiYG_2$  to denote the resulting structure. The out-degree of  $FiYG_2$  is bounded by  $8 \times 7 = 56$ .

2) *Flexible Partition*: Our second set of methods uses flexible partitions where different nodes may get different partitions. Three methods (Algorithm 1, 2, and 3) are proposed. In all these methods, we use cones with a top angle  $\theta$  to partition the transmission ball, and where to define the cones depends on the locations of neighbors. Here  $\theta$  is an adjustable parameter that could be any angle smaller than  $\pi/3$  for Algorithm 2 or  $2\pi/3$  for Algorithm 1 and 3. Notice that these cones are in the same size/shape and can intersect with each other. In Algorithm 1, for a node  $u$ , each link  $uv$  defines a 3D cone  $C_{uv}$  with angle  $\theta$ , and  $u$  adds the shortest link  $uv$  in each cone. In Algorithm 2, instead of defining a cone for each link  $uv$ ,  $u$  only defines a cone for *unprocessed* link.

Here, initially all links  $uv$  are *unprocessed*. And when link  $uv$  is in processing, it defines  $C_{uv}$  and adds the shortest link  $uv$ , then it marks all links in  $C_{uv}$  as *processed*. Algorithm 3 first orders all links  $uv_i$  in term of link length. Then it processes link  $uv_i$  from the shortest link and follows an ascending order. When it processes  $uv_i$ , it defines  $C_{uv_i}$ , adds the link  $uv_i$ , and marks all other links in  $C_{uv_i}$  as *processed*. We denote the final structures of these three algorithms as  $FIYG_1, FIYG_2$ , and  $FIYG_3$  respectively. Figure 3 illustrates the difference among these three algorithms.

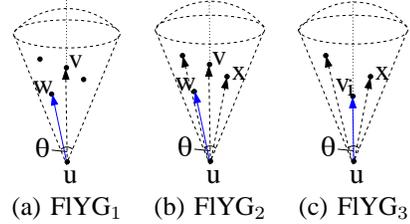


Fig. 3. Definitions of 3D Yao Graph with flexible partitions.

---

#### Algorithm 1 Construct 3D Yao Structure $FIYG_1$ for Node $u$

---

- 1:  $u$  collects the positions of its neighbors  $N_1(u)$  in UBG.
  - 2: **for all**  $v \in N_1(u)$  **do**
  - 3: As shown in Figure 3(a), in the cone  $C_{uv}$  defined by axis  $uv$  and top angle  $\theta$ , node  $u$  only selects the shortest edge  $uw \in UBG$  among all edges emanated from  $u$ , if there is any. Ties are broken by ID.
- 

---

#### Algorithm 2 Construct 3D Yao Structure $FIYG_2$ for Node $u$

---

- 1:  $u$  collects the positions of its neighbors  $N_1(u)$  in UBG.
  - 2: Set PROCESSED( $v$ ) = 0 for each neighbor  $v \in N_1(u)$ .
  - 3: **for all**  $v \in N_1(u)$  and PROCESSED( $v$ ) = 0 **do**
  - 4: As shown in Figure 3(b), in the cone  $C_{uv}$  defined by node  $v$ , node  $u$  only selects the shortest edge  $uv \in UBG$  among all edges emanated from  $u$ , if there is any. Ties are broken by ID.
  - 5: Set PROCESSED( $x$ ) = 1 for all neighbor  $x \in C_{uv}$ .
- 

---

#### Algorithm 3 Construct 3D Yao Structure $FIYG_3$ for Node $u$

---

- 1:  $u$  collects the positions of its neighbors  $N_1(u)$  in UBG.
  - 2: Sort all neighbors  $v_i \in N_1(u)$  by its length such that  $\|uv_i\| \leq \|uv_{i+1}\|$ , where  $i = 1$  to  $m$ . Here  $m$  is the number of neighbors.
  - 3: Set PROCESSED( $v_i$ ) = 0 for all neighbor  $v_i \in N_1(u)$ .
  - 4: **for**  $i = 1$  to  $m$  **do**
  - 5: **if** PROCESSED( $v_i$ ) = 0 **then**
  - 6: As shown in Figure 3(c), in the cone  $C_{uv_i}$ , add edge  $uv_i$ .
  - 7: Set PROCESSED( $x$ ) = 1 for all neighbor  $x \in C_{uv_i}$ .
- 

Now we prove the properties of these three structures as the following two theorems.

*Theorem 2:*  $FIYG_1$  does not have a bounded node out-degree, while the node out-degrees of  $FIYG_2$  and  $FIYG_3$  are bounded by  $\frac{2}{1-\cos(\frac{\theta}{4})}$ .

*Proof:* For Algorithm 1, consider an instance shown in Figure 4(a). Let  $v_0, v_1, \dots$  and  $w_0, w_1, \dots$  are all node  $u$ 's neighbors, their lengths satisfy (1)  $\|w_i u\| > \|w_{i+1} u\|$  and  $\|w_i u\| < \|v_j u\|$  for any  $i, j$ , and (2)  $\angle v_i u v_{i+1} =$

$\angle w_i u w_{i+1} = \alpha$ . Since  $w_i u$  is the shortest link in cone  $C_{w_i}$ , it will be added in  $FIYG_1$  so that all  $w_i$  are neighbors of  $u$  in  $FIYG_1$ . If the angle  $\alpha$  is arbitrarily small, the number of nodes  $w_i$  could be large, i.e., the node out-degree at  $u$  could be extremely large.

For Algorithm 2 and Algorithm 3, after  $u$  processes  $C_{uv}$ , it marks all links inside  $C_{uv}$  as *processed* and those links will never be processed. And each processed cone adds at most one outgoing link in the final structure. Therefore, we only need to prove the number of processed cone is bounded by a constant, then the node out-degree will be bounded. Here, we claim that for any two processed cones, the angle  $\alpha$  between their axes satisfies  $\alpha \geq \theta/2$  as shown in Figure 4(b). Assume there exists any two processed cones  $C_{uv}$  and  $C_{uv'}$ , the angle between their axes  $\angle v u v' = \alpha < \theta/2$ . Then  $v'$  is inside  $C_{uv}$  and  $v$  is inside  $C_{uv'}$ . One of  $v$  and  $v'$  will be processed first, let us assume it is  $v$ . Then after adding the shortest link  $uv$  in Algorithm 2 or  $uv$  in Algorithm 3,  $u$  will mark all nodes inside  $C_{uv}$  as *processed* including  $v'$ . Thus,  $v'$  will never be processed which is a contradiction. It is easy to show that the number of processed cones is bounded by a constant. Since  $\alpha \geq \theta/2$ , the cones with  $uv$  and  $uv'$  as axes and with  $\theta/2$  as top angle can not intersect each other. Thus, the total number of processed cones is bounded by how many such  $\theta/2$  cones can be put into a unit ball so that they do not intersect with each other. By using a volume argument, this number is bounded by  $\frac{4\pi/3}{2\pi(1-\cos(\theta/4))/3} = \frac{2}{1-\cos(\frac{\theta}{4})}$ . See Figure 4(c). ■ Notice that if  $\theta = \pi/2$ , the degree bound is  $\frac{2}{1-\cos(\frac{\theta}{4})} \approx 26$  which is much smaller than those of  $FiYGs$ .

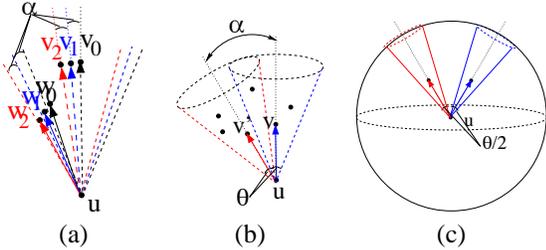


Fig. 4. Illustration of bounded degree for 3D Yao Graph  $FIYG$ .

**Theorem 3:**  $FIYG_1$ ,  $FIYG_2$  and  $FIYG_3$  are all power spanners with spanning ratios bounded by  $\frac{1}{1-(2\sin(\frac{\theta}{4}))^\beta}$ ,  $\frac{1}{1-(2\sin(\frac{\theta}{2}))^\beta}$  and  $\frac{1}{1-(2\sin(\frac{\theta}{4}))^\beta}$  respectively.

**Proof:** In Algorithm 1, for a link  $ux \notin FIYG_1$ , when it is processed, it defines a cone  $C_{ux}$ . Inside  $C_{ux}$ , there must exist a shorter link  $uw \in FIYG_1$ . The angle  $\angle x u w < \theta/2$ , since  $ux$  is the axis. By Lemma 1, the power spanning ratio of  $FIYG_1$  is  $\frac{1}{1-(2\sin(\frac{\theta}{4}))^\beta}$ . In Algorithm 2, for a link  $ux \notin FIYG_2$ , there must exist a shorter link  $uw \in FIYG_2$  in the same cone  $C_{ux}$  where  $ux$  is removed ( $x$  is marked as *processed* by  $u$ ). See Figure 3(b). The angle  $\angle x u w < \theta$ . By Lemma 1, the power spanning ratio of  $FIYG_2$  is  $\frac{1}{1-(2\sin(\frac{\theta}{2}))^\beta}$ . In Algorithm 3, for a link  $ux \notin FIYG_3$ , there must exist a shorter link  $uw \in FIYG_3$  who defined  $C_{ux}$  where  $ux$  is removed. See Figure 3(c). The angle  $\angle x u w < \theta/2$ , since  $uw$  is the axis. By Lemma 1, the power spanning ratio of  $FIYG_3$  is  $\frac{1}{1-(2\sin(\frac{\theta}{4}))^\beta}$ . ■

Notice that all proposed 3D topologies here only need 1-hop neighbor information to be constructed, i.e., all construction algorithms are localized algorithms. Thus, when nodes move, the updates of these topologies can be efficiently performed in a local area without any globe affects.

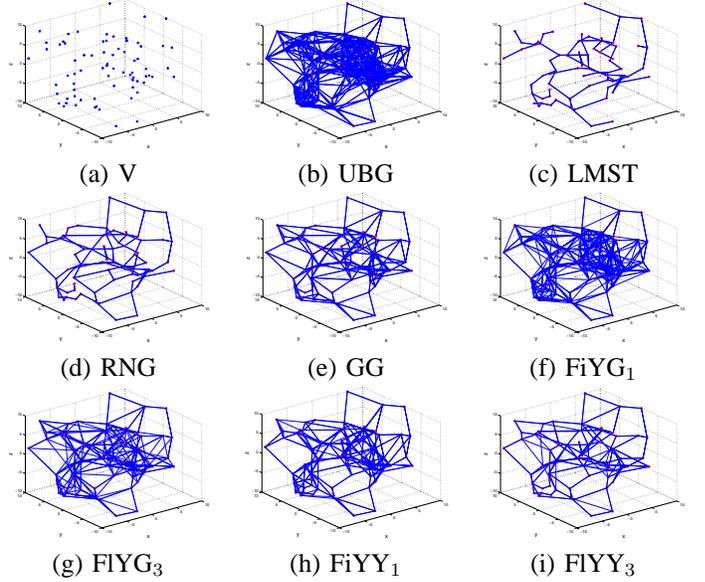


Fig. 5. 3D localized topologies (c-i) which are constructed from the same UBG (b). (a) shows the set  $V$  of 75 wireless nodes.

#### IV. SIMULATIONS

We evaluate the performance of our 3D topology control protocols by conducting simulations with random networks. In our experiments, we randomly generate a set  $V$  of  $n$  wireless nodes and  $UBG$ , then test the connectivity of  $UBG$ . If it is connected, we construct different localized topologies on it, and measure the node degree and power efficiency of these topologies. We evaluate the following localized topologies:  $LMST$ ,  $RNG$ ,  $GG$ ,  $FiYG_1$ ,  $FIYG_3$ ,  $FiYY_1$  and  $FIYY_3$ . Here,  $FiYY_1$  and  $FIYY_3$  are the structures applied the corresponding 3D Yao structures on  $FiYG_1$  and  $FIYG_3$  respectively to bound node in-degree, same as the sparsified Yao graph [3] in 2D. Figure 5 shows a set of topologies generated for a UBG with 75 wireless nodes. In the experimental results presented here, we generate  $n$  random wireless nodes in a  $20 \times 20 \times 20$  cube; the parameter  $\theta = \pi/2$  for  $FIYG_3$  and  $FIYY_3$ ; the transmission range is set to 6; the power constant  $\beta = 2$ . We vary the number of nodes  $n$  in the network from 50 to 200, where 100 vertex sets are generated for each case. The average and the maximum are computed over all these 100 vertex sets.

All experimental results are plotted in Figure 6 and Figure 7. The average node degree of wireless networks should not be too large to avoid interference, collision, and overhead. It should also not be too small either: a low node degree usually implies that the network has a lower fault tolerance and tends to increase the overall network power consumption as longer paths may have to be taken. The upper figure in Figure 6 shows all localized topologies have lower degrees

compared with UBG and keep small degrees when the UBG becomes denser and denser with the number of node in the network increases.  $FiYY_1$  and  $FiYY_3$  have smaller degrees than  $FiYG_1$  and  $FiYG_3$ , while  $RNG$  and  $LMST$  are much sparser than all Yao-based structures. In addition,  $GG$  is in a similar level of  $FiYY_3$ . These can also be verified by Figure 5. The lower figure in Figure 6(b) shows that the maximum node out-degree of these localized 3D-topologies are small. The out-degrees of  $FiYG_1$  and  $FiYG_3$  are smaller than the theoretical bounds 32 and 26 respectively. Notice that  $RNG$  and  $GG$  do not have large degrees in this experiments, the reason is that the nodes are distributed randomly in the area. In real life, the network may not be distributed randomly, it is possible that  $RNG$  and  $GG$  have large degrees. Such examples and simulation results can be found in [3]. Besides connectivity, the most important design metric of wireless networks is perhaps the power efficiency, because it directly affects both the nodes and the network lifetime. Figure 7 shows all proposed structures have small power stretch factors even when the network is very dense except  $LMST$ . Notice that  $RNG$  has a little bit higher stretch factor than  $GG$  and Yao-based structures, however, its maximum power stretch factors is still smaller than 5. The reason again is that the nodes are distributed randomly in the area and the number of nodes is not too large. As we expected,  $GG$  has a power stretch factor of one and the power stretch factor of  $FiYG_3$  is smaller than the theoretical bound  $\frac{1}{1-(2\sin(\frac{\theta}{4}))^\beta} = \frac{1}{1-(2\sin(\frac{\pi}{8}))^2} \approx 2.4$ .

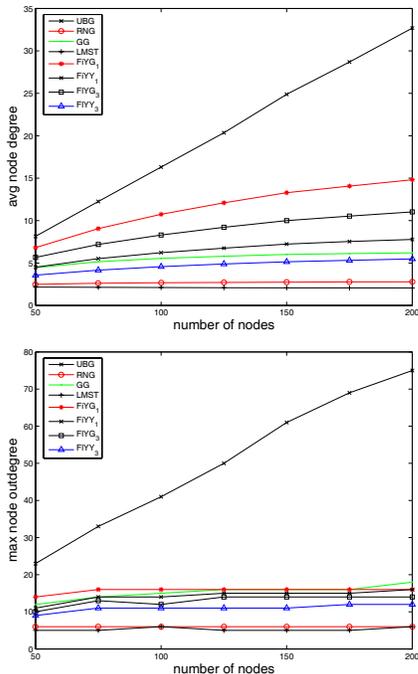


Fig. 6. Average node degree (upper) and maximum out-degree (lower)

## V. CONCLUSION

Topology control for ad hoc and sensor networks has been heavily studied recently and different geometric topologies were proposed to achieve the sparseness and connectivity of

the network. However, most of them are only applied to 2D networks. In this paper, we extend two 2D geometric topologies ( $RNG$  and  $GG$ ) to 3D case and propose several new 3D Yao-based topologies for ad hoc and sensor networks. All our 3D topologies can be constructed locally and efficiently, and we prove the power efficiency of several proposed topologies. Our simulation results confirm the nice performance of these 3D topologies. This paper is just the first step of research for 3D topology control, there are still a number of challenging questions left. E.g., if position information is not accurate, how to achieve power efficiency of these geometric topologies.

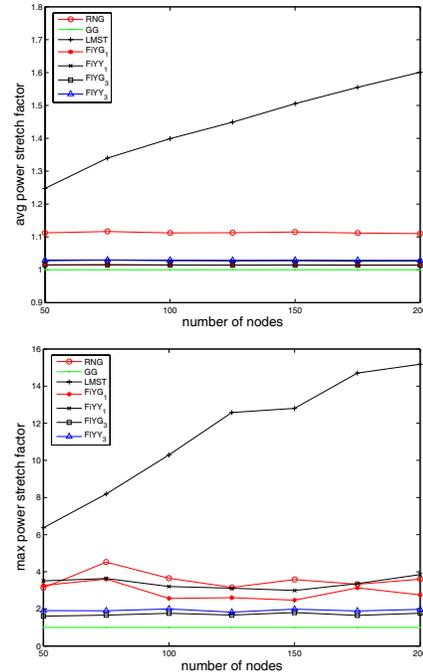


Fig. 7. Average (upper) and maximum (lower) power stretch factors.

## REFERENCES

- [1] L. Li, J. Y. Halpern, P. Bahl, Y.-M. Wang, and R. Wattenhofer, "Analysis of a cone-based distributed topology control algorithms for wireless multi-hop networks," in *ACM PODC*, 2001.
- [2] X.-Y. Li, P.-J. Wan, and Y. Wang, "Power efficient and sparse spanner for wireless ad hoc networks," in *IEEE ICCCN*, 2001.
- [3] X.-Y. Li, P.-J. Wan, Y. Wang, and O. Frieder, "Sparse power efficient topology for wireless networks," in *IEEE HICSS*, 2002.
- [4] R. Wattenhofer, L. Li, P. Bahl, and Y.-M. Wang, "Distributed topology control for wireless multihop ad-hoc networks," in *IEEE INFOCOM*, 2001.
- [5] N. Li, J. C. Hou, and L. Sha, "Design and analysis of a MST-based topology control algorithm," in *IEEE INFOCOM*, 2003.
- [6] P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia, "Routing with guaranteed delivery in ad hoc wireless networks," *ACM/Kluwer Wireless Networks*, vol. 7, no. 6, pp. 609-616, 2001.
- [7] B. Karp and H. Kung, "GPSR: Greedy perimeter stateless routing for wireless networks," in *ACM MobiCom*, 2000.
- [8] L. Jia, R. Rajaraman, and C. Scheideler, "On local algorithms for topology control and routing in ad hoc networks," in *ACM SPAA*, 2003.
- [9] M. Bahramgiri, M. T. Hajiaghayi, and V. S. Mirrokni, "Fault-tolerant and 3-dimensional distributed topology control algorithms in wireless multi-hop networks," in *IEEE ICCCN*, 2002.
- [10] C.-F. Huang, Y.-C. Tseng, and L.-C. Lo, "The coverage problem in three-dimensional wireless sensor networks," in *IEEE Globecom*, 2004.
- [11] A. Benlarbi-Delai, D. Simplot, J. Cartigny, J. C. Cousin, "Using 3D indoor microwave phase sensitive stereoscopic location system to reduce energy consumption in wireless ad-hoc networks," in *Smart Objects Conf.*, 2003.