Energy-efficient topology control for three-dimensional sensor networks

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Abstract: Topology control in sensor networks has been heavily studied recently. Different geometric topologies were proposed to be the underlying network topologies to achieve the sparseness of the communication networks or to guarantee the package delivery of specific routing methods. However, most of the proposed topology control algorithms were only applied to Two-Dimensional (2D) networks where all sensor nodes are distributed in a 2D plane. In practice, the sensor networks are often deployed in 3D space, such as sensor nodes in a forest. This paper seeks to investigate efficient topology control protocols for 3D sensor networks. In our new protocols, we extend several 2D geometric topologies to 3D case, and propose some new 3D Yao-based topologies for sensor networks. We also prove several properties (e.g. bounded degree and constant power stretch factor) for them in 3D space. The simulation results confirm our theoretical proofs for these proposed 3D topologies.

Keywords: topology control; energy efficient; Three-Dimensional; 3D; distributed algorithm; wireless; sensor networks.

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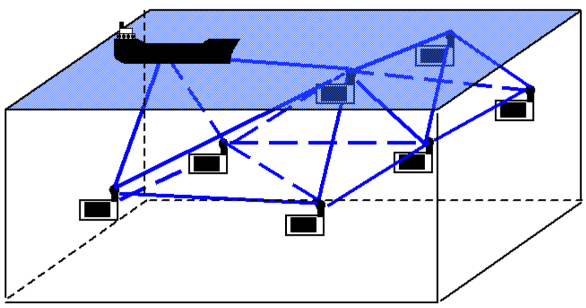
1 Introduction

Sensor networks have been undergoing a revolution that promises a significant impact on society. Unlike traditional fixed infrastructure networks, there is no centralised control over sensor networks, which consist of an arbitrary distribution of sensors in a certain geographical area. Sensor network intrigues many challenging research problems because it intrinsically has many special characteristics and some unavoidable limitations compared with traditional fixed infrastructure networks. An important requirement of a sensor network is that it should be self-organising, i.e. transmission ranges and data paths are dynamically restructured with changing topologies. Energy conservation and network performance are probably the most critical issues in sensor networks because sensor nodes are usually powered by batteries only and have limited computing capability and memory.

The *topology control* technique is to let each sensor node *locally* adjust its transmission range and select certain neighbours for communication, while maintaining a structure that can support energy-efficient routing and improve the overall network performance. Unlike traditional wired networks, sensor nodes are often moving during the communication, which could change the network topology. Hence it is more challenging to design a topology control algorithm for sensor networks: the topology should be locally and self-adaptively maintained without affecting the whole network, and the communication cost during maintaining should not be too high. In the past several years, topology control algorithms (Ramanathan and Hain, 2000; Li et al., 2001a, b; Wattenhofer et al., 2001; Grünewald et al., 2002; Li et al., 2002; Rajaraman, 2002) have drawn significant amount of research interests. The primary topology control algorithms for sensor networks aim to maintain network connectivity, optimise network throughput with power-efficient routing, conserve energy and increase the fault tolerance. There exist several topology control techniques such as localised geometrical structures (Li et al., 2001 a, b; Wattenhofer et al., 2001; Li et al., 2002), dynamic cluster techniques (Liang and Haas, 2000; Alzoubi et al., 2003; Bao and Garcia-Luna-Aceves, 2003; Nikaein and Bonnet, 2004) and power management protocols (Cerpa and Estrin, 2002; Chen et al., 2002; Schurgers et al., 2002). In this paper, we focus on geometrical structures-based methods.

Figure 1 A 3D Under-Water Sensor Network (UWSN) (see online version for colours)



Most proposed geometrical structures-based topology control algorithms are only applied to Two-Dimensional (2D) networks where all nodes are distributed in a 2D plane. However, in practice, sensor networks are often deployed in 3D space, such as sensor nodes in a multi-floor building or an Under-Water Sensor Network (UWSN) as shown in Figure 1. UWSN (Akyildiz et al., 2005) can find applications in oceanographic data collection, pollution monitoring, offshore exploration, disaster prevention, assisted navigation and tactical surveillance applications. A 3D UWSN is used to detect and observe phenomena that cannot be adequately observed by means of ocean bottom sensor nodes, i.e. to perform cooperative sampling of the 3D ocean environment. In 3D UWSN, sensor nodes float at different depths to enable the exploration of natural undersea resources and gathering of scientific data in collaborative monitoring missions. Underwater sensor nodes must possess self-configuration capabilities, i.e. they must be able to coordinate their operation by exchanging configuration, location and movement information, and to relay monitored data to an onshore station. In this paper, we will focus on how to efficiently construct topology for 3D sensor networks to maintain network connectivity, conserve energy and enable energy-efficient routing.

1.1 Network model

A 3D sensor network consists of a set V of n sensor nodes distributed in a 3D plane $\mathbb{R}^3$. Each node has the same *maximum transmission range R*. These sensor nodes define a *Unit Ball Graph* (UBG) (Hansen and Schmutz, 2005), or also called *unit sphere graph*, in which there is an edge uv between two nodes u and v if and only if the Euclidean distance $\|uv\|$ between u and v in $\mathbb{R}^3$ is at most R. In other words, we assume that two nodes can always receive the signal from each other directly if the distance between them is no more than R. Hereafter, *UBG* is always assumed to be connected. We also assume that all sensor nodes have distinctive identities and each node knows its position information either through a low-power GPS receiver or some other ways. By one-hop broadcasting, each node u can gather the location information of all nodes within its transmission range. As in the most common power-attenuation model, the power to support a link uv is assumed to be $\|uv\|^\beta$, where β is a real constant between 2 and 5, depending on the wireless transmission environment.

1.2 Preferred properties

Topology control protocols are to maintain a structure that can improve the overall networking performance (such as lifetime). In the literature, the following desirable features of the structure are well-regarded and preferred in sensor networks.

In sensor networks, two far-apart nodes can communicate with each other through the relay of intermediate nodes. Hence, each node only needs to set small transmission range which reduces the signal interference and saves power for transmissions. To guarantee the advantage, a good network topology should be *energy efficient*, that is to say, the total power consumption of the shortest path (most power-efficient path) between any two nodes in final topology should not exceed a constant factor of the power consumption of the shortest path in original network. Given a path $v_1 v_2 \ldots v_h$ connecting two nodes v_1 and v_h, the energy cost of this path is $\sum_{j=1}^{h-1} \|v_j v_{j+1}\|^\beta$. The path with the least energy cost is called the shortest path in a graph. A subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. The constant ρ is called the *power stretch factor*. A power spanner of the communication graph (e.g. UBG) is usually energy efficient for routing.

It is also desirable that node degree in the constructed topology is small and upper-bounded by a constant. A small node degree reduces the MAC-level contention and interference, and also may help to mitigate the well-known hidden and exposed terminal problems. In addition, a structure with small degree will improve the overall network throughout (Kleinrock and Silvester, 1978).

Due to limited resources and high mobility of the sensor nodes, it is preferred that the underlying network topology can be constructed and maintained in a localised manner. Here a distributed algorithm constructing a graph G is a *localised* algorithm if every node u can exactly decide all edges incident on u based only on the information of all nodes within a constant hops of u.

The remainder of this paper is organised as follows. First, Section 2 and Section 3 provide overviews of the prior literature related to topology control and 3D networks. Then, Section 4 describes our new 3D topology control protocols based on 3D geometric structures. Later, some simulation results are shown in Section 5. Finally, the brief conclusion of our research work is highlighted in Section 6.

2 Priori arts on geometrical topologies

Several geometrical structures have been proposed for topology control in 2D sensor networks. The underlying communication graph is modelled by a *Unit Disk Graph* (UDG) in which there is an edge between two nodes if and only if their distance is at most one.

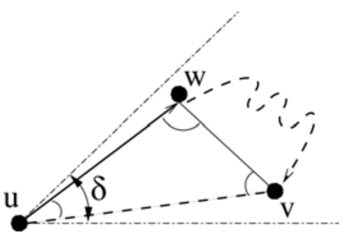
The *Minimum Spanning Tree*, denoted by MST, is the tree belonging to E which connects all nodes with minimised total edge length. MST is obviously one of the sparsest possible connected subgraph, but its length and power stretch factor can be as large as $n-1$ and it cannot be built locally. Therefore, Li et al. (2003a) proposed a localised topology called *Local Minimum Spanning Tree* (LMST) to estimate MST and keep the network connected. In LMST, each node builds its local minimum spanning tree in one-hop neighbourhood independently and only keeps one-hop on-tree nodes as neighbours. They proved that LMST is connected, and has a bounded degree of 6. Then, Li et al. (2004) further proposed *k-Localised Minimum Spanning Tree* (LMST_k) which can be locally constructed using k-hop information. LMST_k is connected and planar, the total edge length of the LMST_k is within a constant factor of that of the MST when $k \geq 2$.

The *relative neighbourhood graph* (Toussaint, 1980), denoted as RNG, consists of all edges $uv \in \text{UDG}$ such that the intersection of two circles centred at u and v with radius $\|uv\|$ does not contain any node from the set V. The *Gabriel Graph* (GG) (Gabriel and Sokal, 1969) contains edge $uv \in \text{UDG}$ if and only if *disk*(u, v) contains no other node of V, where *disk*(u,v) is the disk with edge uv as a diameter. Both GG and RNG are connected, planar and contain the Euclidean MST of V if UDG is connected. In Jennings and Okino (2002) and Seddigh et al. (2001), RNG is used as the underlying network topology for broadcasting and information dissemination. In Bose et al. (2001) and Karp and Kung (2000), RNG and GG are used as underlying routing topologies for their routing algorithms. However, Bose et al. (2002) proved that the length stretch factors of these two graphs are $\Theta(n)$ and $\Theta\left(\sqrt{n}\right)$ respectively. Moreover, Li et al. (2001b) showed that the power stretch factor of RNG is $n-1$ while the power stretch factor of GG is 1.

The *Yao graph* (Yao, 1982) with an integer parameter $k > 6$, denoted by $\overrightarrow{\text{YG}_k}$, is defined as follows. At each node u, any k equally separated rays originating at u define k cones. In each cone, choose the shortest edge $uv \in \text{UDG}$ among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is called the *Yao graph*. Let YG_k be the undirected graph by ignoring the direction of each link in $\overrightarrow{\text{YG}_k}$. Lukovszki (1999) and Keil and Gutwin (1992) used a similar construction named θ-graph. The difference is that it chooses the edge which has the shortest projection on the axis of each cone instead of the shortest edge in each cone. The node out-degree of *Yao graph* is bounded by constant k. Li et al. (2001b) proposed to construct the wireless network topology based on *Yao graph* and proved that Yao graph is a power spanner. Their proof of the spanner property is based on the induction of link length. As shown in Figure 2, for each removed link $uv \notin \text{YG}_k$, they proved the power consumed by a shorter link uw and a path from w to v is within constant times of the power consumed by link uv. The key result of their proof can be summarised by the following lemma:

Lemma 1: The power stretch factor of the Yao-based graph is at most $1/(1-(2\sin(\delta/2))^{\beta})$, if for every link uv which is not in the final graph, there exists a shorter link uw in the graph and $\angle vuw < \delta$, where δ is a constant smaller than $\pi/3$.

Figure 2 Proof of spanner property of Yao graph by Li et al. (2001b).



However, all of these Yao-based graphs are not guaranteed to be planar. Wang and Li (2003) proposed the first efficient localised algorithm to build a degree-bounded planar spanner for ad hoc networks. Song et al. (2004) further proposed two more communication-efficient methods to construct small degree-bounded planar power-efficient structures. Li et al. (2002) considered to use another sparse topology, *Yao and Reverse Yao* (called *Sparsified Yao graph* or *Yao Yao graph* in other papers, and denoted by $\overrightarrow{YY}_k(V)$), which has constant bounded node in-degree and out-degree. The basic idea is to apply reversed Yao structure on $\overrightarrow{YG}_k$ to bound the in-degree. It is proved by Jia et al. (2003) and Schindelhauer et al. (2007) that $\overrightarrow{YY}_k(V)$ has a constant bounded power stretch factor theoretically.

Wattenhofer et al. (2001) and Li et al. (2001a) introduced a distributed *Cone-Based Topology Control* (CBTC) protocol based on directional information. They demonstrated a simple distributed algorithm in which each node uses only local decisions about its transmission power to guarantee the global connectivity of the network. More precisely, in the first phase, by using only directional information, each node increases its transmission power until it detects a neighbour in every cone of α degree. The second phase improves the performance by eliminating non-efficient edges in the communication graph. The algorithm is simple and simultaneously reduces both transmission power and traffic interference. The algorithms tend to minimise the power consumption in each node, but there might have congestion and hot spots in some nodes, which in turn might drain battery power. Fault tolerance is also one of the central challenges in designing sensor networks. Bahramgiri et al. (2002) constructed a *k*-connected CBTC algorithm, which guarantees a network to be fault-tolerant. A graph is *k*-connected if and only if for each pair of nodes *u* and *v*, there exist *k* pairwise internally disjoint paths whose endpoints are *u* and *v*. It shows that running the algorithm with $\alpha = 2\pi/3k$ is sufficient for preserving *k*-connectivity. In addition, if *k* is even, this upper bound is tight, and if *k* is odd, this upper bound is very near to the optimal α. Li et al. (2003b) showed that resulting graph from cone-based CBTC approach is also a length spanner even with $k-1$ nodes faults. However, Bahramgiri et al.'s (2002) method does not bound the node degree. Then Li et al. (2003b) gave a careful enhancement of their protocol to bound the node degree. The node degree of enhanced CBTC is bounded by $4\pi/\alpha$. When $\alpha = 2\pi/3k$, the node degree is bounded by 6*k*.

While the CBTC protocol was designed specifically for topology control deployed on a 2D surface, it has been extended to the 3D space by Bahramgiri et al. (2002). The main idea is that each node increases its transmission power until there is no 3D cone of degree α without any nodes. They proved that running the algorithm with $\alpha = 2\pi/3k$ is an upper bound for preserving *k*-connectivity. However, there is no proof of spanner property and node degree bound for their proposed topology. Bahramgiri et al. (2002) is the only paper we found in literature that addresses 3D topology control. To the best of our knowledge, our paper is the first one to study power-efficient 3D topology control protocols for sensor networks.

3 Other related work

Besides geometrical structures, there are also various techniques proposed by researchers for topology control in sensor networks. For example, many dynamic clustering methods (Liang and Haas, 2000; Alzoubi et al., 2003; Bao and Garcia-Luna-Aceves, 2003; Nikaein and Bonnet, 2004) are proposed to build a virtual backbone for routing in sensor networks. Several power management methods are also proposed to address the power efficiency of the network topology. The most common power management technique (Cerpa and Estrin, 2002; Chen et al., 2002; Schurgers et al., 2002) is turning off the radio of the inactive sensor node to save power. However, sensor networks require nodes to forward packets for other nodes. Thus, there is a trade-off between energy conservation and network performance. The goal of power management is turning off as many nodes as possible without significantly diminishing the capacity or connectivity of the network. The strong assumption of all these power management protocols (Cerpa and Estrin, 2002; Chen et al., 2002; Schurgers et al., 2002) is that the density of sensor nodes is high enough so that putting some nodes to sleep will not hurt the connectivity.

Due to the various applications of sensor networks, the topology design is also various. For example, Pan et al. (2003) considered a two-tiered sensor network consisting of sensor clusters deployed around strategic locations and base-stations whose locations are relatively flexible. Within a sensor cluster, there are many small sensor nodes and at least one application node that receives data from sensor nodes and forwards towards a base-station. Pan et al. (2003) focused on the topology control process for application nodes and base-stations and proposed approaches to maximise the topological network lifetime, by arranging base-station location and inter-application-node relaying optimally. Surveys (Rajaraman, 2002; Li, 2003; Wang, 2008) and simulation results (Ruhrup et al., 2003; Srivastava et al., 2003) on topology control in ad hoc networks can be found in the literature.

Besides topology control, there are also some other research in 3D sensor networks. For example, Salhieh et al. (2001) proposed a directional source-aware routing protocol and showed the routing method can be applied to 3D sensor networks where sensors are located in a fixed 3D gird; Huang et al. (2004) and Alam and Haas (2006) studied coverage and connectivity problem in 3D networks; and Benlarbi-Delaï et al. (2003) proposed a 3D indoor location and position system for sensor networks.

4 3D topology control protocols

In this section, we study how to design energy-efficient topologies for 3D sensor networks. The first thought about

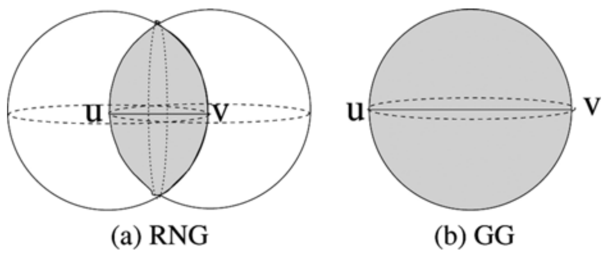
3D topology control is whether there exists an embedding method mapping a 3D network into a 2D plane so that all 2D geometric topology control protocols can still be applied. Unfortunately, there is no such mapping method which still keeps the scale of the edge length. Consider a regular tetrahedron in 3D where the distances between any two nodes from the four endpoints are the same. However, in a 2D plane, there is no graph formed by four nodes where the distances between any two nodes are the same. Therefore, simply mapping 3D networks to 2D ones does not work when we want to achieve power efficiency.

Notice that LMST still works for 3D networks. It can guarantee the connectivity of the network and is easy to be constructed by using local information. However, the similar problem as in 2D, the stretch factor of LMST can be arbitrarily large, i.e. unicast routing between a pair of nodes may need to travel a much longer distance than the shortest path in UBG.

4.1 3D RNG and 3D GG

It is very natural to extend the Relative Neighbourhood Graph and Gabriel graph to 3D. The definitions of 3D RNG and 3D GG are as follows: an edge $uv \in \mathrm{RNG}_{3D}$ if and only if the intersection of two balls centred at *u* and *v* with radius $\|uv\|$ does not contain any node from the set *V*; an edge $uv \in \mathrm{GG}_{3D}$ if and only if the ball with edge *uv* as a diameter contains no other node of *V*. Figure 3 illustrates the new definitions. RNG_{3D} and GG_{3D} contains MST, which indicates that they are connected if UBG is connected. From the definitions, $\mathrm{RNG}_{3D} \subseteq \mathrm{GG}_{3D}$, since the shaded area in GG_{3D} is included in the one in RNG_{3D}. By using Li et al.'s (2001b) example and proof in 2D case, we can prove that RNG_{3D} is not a power spanner, while GG_{3D} is a power spanner with the power stretch factor of 1. In other words, all edges in the least energy path in UBG are kept in GG_{3D}.

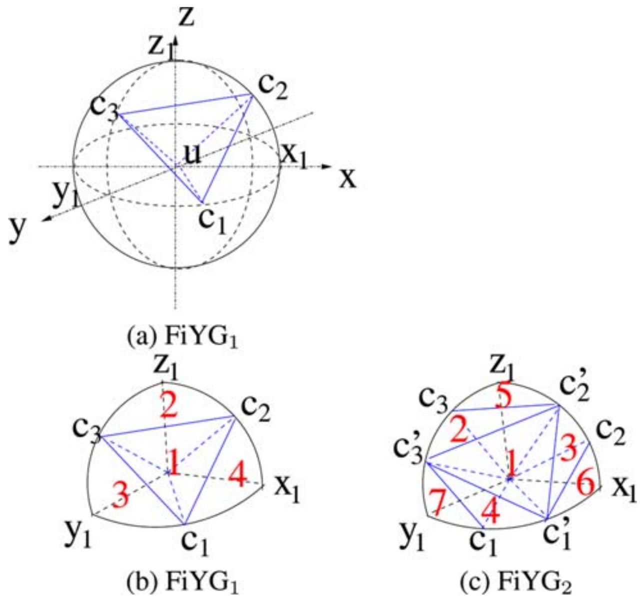
Figure 3 Definitions of 3D RNG and 3D GG. Shaded areas are empty of nodes



4.2 3D Yao-based Structures

Since both RNG_{3D} and GG_{3D} cannot bound node degree, we are also interested in extending Yao graph to 3D. However, it is difficult to define the partition boundary of Yao structure of a node in 3D. Notice that a disk in 2D can be easily divided into *k* equal cones which do not intersect with each other, but in 3D case, it is impossible to divide a ball into *k* equal cones without intersections among each other. Here, we give two sets of methods to partition the transmission range which is modelled as a ball: fixed partition and flexible partition.

Figure 4 Definitions of 3D Yao Graph with fixed partitions (see online version for colours)



4.2.1 Fixed partition

In fixed partition, cones from one node do not intersect with each other and the partition method is the same for all nodes. We will give two methods to divide the transmission range of a node into certain number of cones. In the first method, for each node *u*, it first divides its transmission region into eight pieces by three orthogonal planes (i.e. *xy*-plane, *yz*-plane and *zx*-plane), where each piece is a 1/8 ball. Then it uses three more planes as shown in Figure 4(a) to cut each piece into four cones. Figure 4(b) shows how to select these three planes where nodes c_1, c_2 and c_3 are the middle points of arc x_1y_1, y_1z_1 and z_1x_1, respectively. Therefore, the total number of cones at node *u* is $8 \times 4 = 32$. Notice that these cones are different and do not intersect with each other. For each cone, node *u* will choose the shortest edge $uv \in \mathrm{UBG}$ among all edges emanated from *u*, if there is any, and will add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is denoted by FiYG_1. It is obvious that FiYG_1 has a bounded out-degree of 32. However, it is easy to show that the largest angle inside a cone is the angle $\angle c_1uc_2 = \pi/3$. Thus, the spanning ratio got from Lemma 1, $1/(1-(2\sin(\delta/2))^{\beta})$, could be infinite when $\delta = \pi/3$.

To fix this problem, we propose to use smaller cones for partition. The new partition method is shown in Figure 4(c), where for each piece of the 1/8 ball, we cut it into 7 cones. Here, c_1 and c'_1, c_2 and c'_2, c_3 and c'_3 trisect the arc x_1y_1, x_1z_1, and z_1y_1, respectively. Using position information, we know the largest angle inside a cone is $\angle c'_1uc'_3 = \pi/6$. This can guarantee the power spanning ratio is bounded by $1/(1-(2\sin(\pi/12))^{\beta})$. We use FiYG_2 to denote the resulting structure. The out-degree of FiYG_2 is bounded by $8 \times 7 = 56$.

4.2.2 Flexible partition

Our second set of methods uses flexible partitions where different nodes may get different partitions. Three methods (Algorithms 1, 2 and 3) are proposed. In all these methods, we use identical cones with a top angle θ to partition the transmission ball, and where to define the cones depends on the locations of neighbours. Here θ is an adjustable parameter that could be any angle smaller than $\pi/3$ for Algorithm 2 or $2\pi/3$ for Algorithms 1 and 3. Notice that these cones are in the same size/shape and can intersect with each other. In Algorithm 1, for a node *u*, each link *uv* defines a 3D cone C_{uv} with angle θ, and *u* adds the shortest link *uw* in each cone. In Algorithm 2, instead of defining a cone for each link *uv*, *u* only defines a cone for *unprocessed* link. Here, initially all links *uv* are *unprocessed*. And when link *uv* is in processing, it defines C_{uv} and adds the shortest link *uw*, then it marks all links in C_{uv} as *processed*. Algorithm 3 first orders all links uv_i in terms of link length. Then it processes link uv_i from the shortest link and follows an ascending order. When it processes uv_i, it defines C_{uv_i}, adds the link uv_i, and marks all other links in C_{uv_i}, as *processed*. We denote the final structures of these three algorithms as FlYG_1, FlYG_2, and FlYG_3, respectively. Figure 5 illustrates the difference among these three algorithms.

Algorithm 1 Construct a 3D Yao Structure FlYG_1 for Node *u*

1: *u* collects the positions of its neighbours $N_1(u)$ in UBG.
2: **for all** $v \in N_1(u)$ **do**
3: Let C_{uv} be the cone using *uv* as the axis and θ as the top angle.
4: As shown in Figure 5(a), in C_{uv}, node *u* only selects the shortest edge $uw \in \text{UBG}$ among all edges emanated from *u*, if there is any. Ties are broken by ID.

Algorithm 2 Construct a 3D Yao Structure FlYG_2 for Node *u*

1: *u* collects the positions of its neighbours $N_1(u)$ in UBG.
2: Set PROCESSED(*v*) = 0 for all neighbours $v \in N_1(u)$.
3: **for all** $v \in N_1(u)$ and PROCESSED(*v*) = 0 **do**
4: Let C_{uv} be the cone using *uv* as the axis and θ as the top angle.
5: As shown in Figure 5(b), in C_{uv}, node *u* only selects the shortest edge $uw \in \text{UBG}$ among all edges emanated from *u*, if there is any. Ties are broken by ID.
6: Set PROCESSED(*x*) = 1 for all neighbours $x \in C_{uv}$.

Now we prove the properties of these three structures as the following two theorems.

Theorem 1: *FlYG_1 does not have a bounded node out-degree, while the node out-degrees of FlYG_2 and FlYG_3 are bounded by* $2/(1-\cos(\theta/4))$.

Proof. For Algorithm 1, consider an instance shown in Figure 6. Let $v_0, v_1, \ldots$ and $w_0, w_1, \ldots$ are all node *u*'s neighbours, their lengths satisfy (1) $\|w_i u\| > \|w_{i+1} u\|$ and $\|w_i u\| < \|v_j u\|$ for any *i, j* and (2) $\angle v_i u v_{i+1} = \angle w_i u w_{i+1} = \alpha$. Since $w_i u$ is the shortest link in cone C_{uv_i}, it will be added in FlYG_1 so that all w_i are neighbours of *u* in *FlYG_1*. If the angle α is arbitrarily small, the number of nodes w_i could be large, i.e. the node out-degree at *u* could be extremely large.

Algorithm 3 Construct 3D Yao Structure FlYG_3 for Node *u*

1: *u* collects the positions of its neighbours $N_1(u)$ in UBG.
2: Sort all neighbours $v_i \in N_1(u)$ by its length such that $\|uv_i\| \le \|uv_{i+1}\|$, where *i* = 1 to *m*. Here *m* is the number of neighbours.
3: Set PROCESSED(v_i) = 0 for all neighbours $v_i \in N_1(u)$.
4: **for** *i* = 1 to *m* **do**
5: **if** PROCESSED(v_i) = 0 **then**
6: As shown in Figure 5(c), let C_{uv_i} be the cone using uv_i as the axis and θ as the top angle. Add edge uv_i.
7: Set PROCESSED(*x*) = 1 for all neighbours $x \in C_{uv_i}$.

Figure 5 Definitions of 3D Yao graph with flexible partitions (see online version for colours)

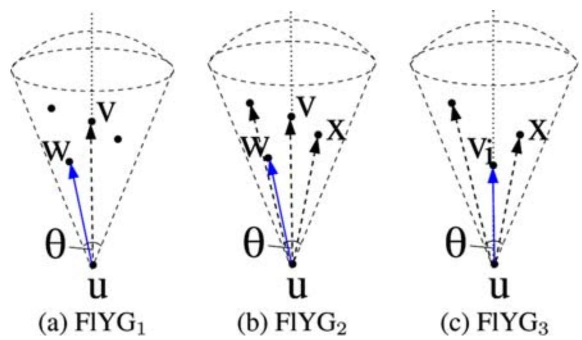
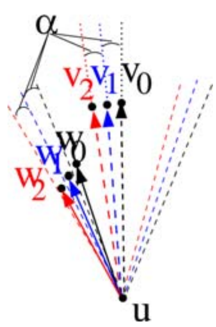


Figure 6 Illustration of unbounded degree for FlYG_1 (see online version for colours)



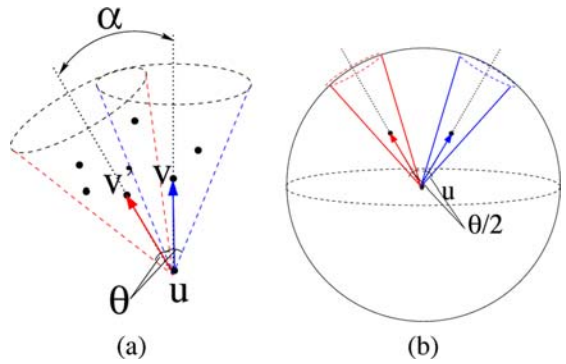
For Algorithms 2 and 3, after *u* processes C_{uv}, it marks all links inside C_{uv} as *processed* and those links will never be processed. And each processed cone adds at most one outgoing link in the final structure. Therefore, we only need to prove the number of processed cone is bounded by a constant, then the node out-degree will be bounded. Here, we claim that for any two processed cones, the angle α between their axes satisfies $\alpha \geq \theta/2$, as shown in Figure 7(a). Assume there exist any two processed cones C_{uv} and $C_{uv'}$, the angle between their axes $\angle vuv' = \alpha < \theta/2$. Then v' is inside C_{uv} and v is inside $C_{uv'}$. One of v and v' will be processed first, let us assume it is v. Then after adding the shortest link *uw* in Algorithm 2 or *uv* in Algorithm 3, *u* will mark all nodes inside C_{uv} as *processed* including v'. Thus, v' will never be processed, which is a contradiction. It is easy to show that the number of processed cones is bounded by a constant. Since $\alpha \geq \theta/2$, the cones with *uv* and *uv*′ as axes and with $\theta/2$ as top angle cannot intersect each other. Thus, the total number of processed cones is bounded by how many such $\theta/2$ cones can be put into a unit ball so that they do not intersect with each other. By using a volume argument, this number is bounded by

$$\frac{4\pi/3}{2\pi\left(1-\cos\left(\frac{\theta}{4}\right)\right)/3} = \frac{2}{1-\cos\left(\frac{\theta}{4}\right)}.$$

See Figure 7(b) for reference.

Notice that if $\theta = \pi/2$, the degree bound is $2/(1-\cos(\theta/4)) \approx 26$ which is much smaller than those of FiYGs, and if $\theta = \pi/3$, the degree bound is 58.

Figure 7 Illustration of bounded degree for $FlYG_2$ and $FlYG_3$ (see online version for colours)



Theorem 2: *$FlYG_1$, $FlYG_2$ and $FlYG_3$ are all power spanners with spanning ratios bounded by* $1/(1-(2\sin(\theta/4))^{\beta})$, $1/(1-(2\sin(\theta/2))^{\beta})$ and $1/(1-(2\sin(\theta/4))^{\beta})$, *respectively.*

Proof. In Algorithm 1, for a link $ux \notin \text{FlYG}_1$, when it is processed, it defines a cone C_{ux}. Inside C_{ux}, there must exist a shorter link $uw \in \text{FlYG}_1$. The angle $\angle xuw < \theta/2$, since *ux* is the axis. By Lemma 1, the power spanning ratio of $FlYG_1$ is $1/(1-(2\sin(\theta/4))^{\beta})$.

In Algorithm 2, for a link $ux \notin \text{FlYG}_2$, there must exist a shorter link $uw \in \text{FlYG}_2$ in the same cone C_{uv} where *ux* is removed (*x* is marked as *processed* by *u*); see Figure 5(b), the angle $\angle xuw < \theta$. By Lemma 1, the power spanning ratio of $FlYG_2$ is $1/(1-(2\sin(\theta/4))^{\beta})$.

In Algorithm 3, for a link $ux \notin \text{FlYG}_3$, there must exist a shorter link $uv \in \text{FlYG}_3$ which defined C_{uv} where *ux* is removed; see Figure 5(c), the angle $\angle xuv < \theta/2$ since *uv* is the axis. By Lemma 1, the power spanning ratio of $FlYG_3$ is $1/(1-(2\sin(\theta/4))^{\beta})$.

The properties summary for several proposed 3D geometric structures discussed above can be found in Table 1. Notice that all proposed 3D topologies here only need one-hop neighbour information to be constructed, i.e. all construction algorithms are localised algorithms. Thus, when nodes move, the updates of these topologies can be efficiently performed in a local area without any global effects.

Table 1 Property summary for proposed 3D topologies

3D structure	*Bounded out-degree*	*Power stretch factor*
RNG_{3D}	O(n)	O(n)
GG_{3D}	O(n)	1
$FiYG_1$	32	O(n)
$FiYG_2$	56	$\frac{1}{1-\left(2\sin\left(\frac{\pi}{12}\right)\right)^{\beta}}$
$FlYG_1$	O(n)	$\frac{1}{1-\left(2\sin\left(\frac{\theta}{4}\right)\right)^{\beta}}$
$FlYG_2$	$\frac{2}{1-\cos\left(\frac{\theta}{4}\right)}$	$\frac{1}{1-\left(2\sin\left(\frac{\theta}{2}\right)\right)^{\beta}}$
$FlYG_3$	$\frac{2}{1-\cos\left(\frac{\theta}{4}\right)}$	$\frac{1}{1-\left(2\sin\left(\frac{\theta}{4}\right)\right)^{\beta}}$

5 Simulations

We evaluate the performance of our 3D topology control protocols by conducting simulations in random networks. In our experiments, we randomly generate a set *V* of *n* wireless nodes and UBG, then test the connectivity of UBG. If it is connected, we construct MST and different localised topologies on it, and measure the node degree and power efficiency of these topologies. We evaluate the following localised topologies: LMST, RNG, GG, $FiYG_1$, $FlYG_3$, $FiYY_1$ and $FlYY_3$. Here, $FiYY_1$ and $FlYY_3$ are the structures applied to the corresponding 3D Yao structures on $FiYG_1$ and $FlYG_3$, respectively, to bound node in-degree, same as the 2D sparsified Yao graph (Li et al., 2002). Figure 8 shows a set of topologies generated for a UBG with 75 wireless nodes. In the experimental results presented here, we generate *n* random wireless nodes in a 20×20×20 cube; the parameter $\theta = \pi/2$ for $FlYG_3$ and $FlYY_3$; the maximum transmission range *R* is set to 6; the power constant β = 2. We vary the number of nodes n in the network from 50 to 200, where 100 vertex sets are generated for each case. The average and the maximum are computed over all these 100 vertex sets.

All experimental results are plotted in Figure 9. Here we only measure the node degrees, power and length stretch factors of the resulting topologies. The node degree of the wireless networks should not be too large to avoid interference, collision and overhead. The node degree should also not be too small either: a low node degree usually implies that the network has a lower fault tolerance and it also tends to increase the overall network power consumption as longer paths may have to be taken. Figure 9(a) shows all localised topologies have lower average degrees compared with UBG and keep small degrees when the UBG becomes denser and denser as the number of nodes in the network increases. $FiYY_1$ and $FlYY_3$ have smaller degrees than $FiYG_1$ and $FlYG_3$, while RNG and LMST are much sparser than all Yao-based structures. In addition, GG is in a similar level of $FlYY_3$. These can also be verified by Figure 8. Figure 9(c) shows that the maximum node out-degree of these localised 3D topologies are small. The out-degrees of $FiYG_1$ and $FlYG_3$ are smaller than the theoretical bounds 32 and 26, respectively. Figure 9(d) also gives the maximum node in-degree of these topologies. It is clear that $FiYY_1$ and $FlYY_3$ have smaller in-degrees than $FiYG_1$ and $FlYG_3$. Remember that $FiYY_1$ and $FlYY_3$ applied reversed 3D Yao structures on $FiYG_1$ and $FlYG_3$ to bound the node in-degree. The results also showed that RNG and GG do not have large degrees in this experiment, and the reason is that the nodes are distributed randomly in the area. In real life, the network may not be distributed randomly, so it is possible that RNG and GG have large degrees. Such examples and simulation results can be found in Li et al. (2002).

Besides connectivity, the most important design metric of wireless networks is perhaps the power efficiency, because it directly affects both the nodes and the network lifetime. Figures 9(e) and 9(f) show all proposed structures have small power stretch factors even when the network is very dense except LMST and MST. Notice that RNG has a little bit higher stretch factor than GG and Yao-based structures; however, its maximum power stretch factors is still smaller than 5. The reason again is that the nodes are distributed randomly in the area and the number of nodes is not too large. As we expected, GG has a power stretch factor of one and the power stretch factor of $FlYG_3$ is smaller than the theoretical bound $1/(1-(2\sin(\theta/4))^{\beta}) = 1/(1-(2\sin(\pi/8))^2) \approx 2.4$. Figures 9(g) and 9(h) show the results on length stretch factor, where the length stretch factor considers the total length of a path instead of the total power consumed by the path. The results have similar pattern with those on power stretch factor.

Figure 8 3D topologies generated from the UBG by different 3D topology control protocols (see online version for colours)

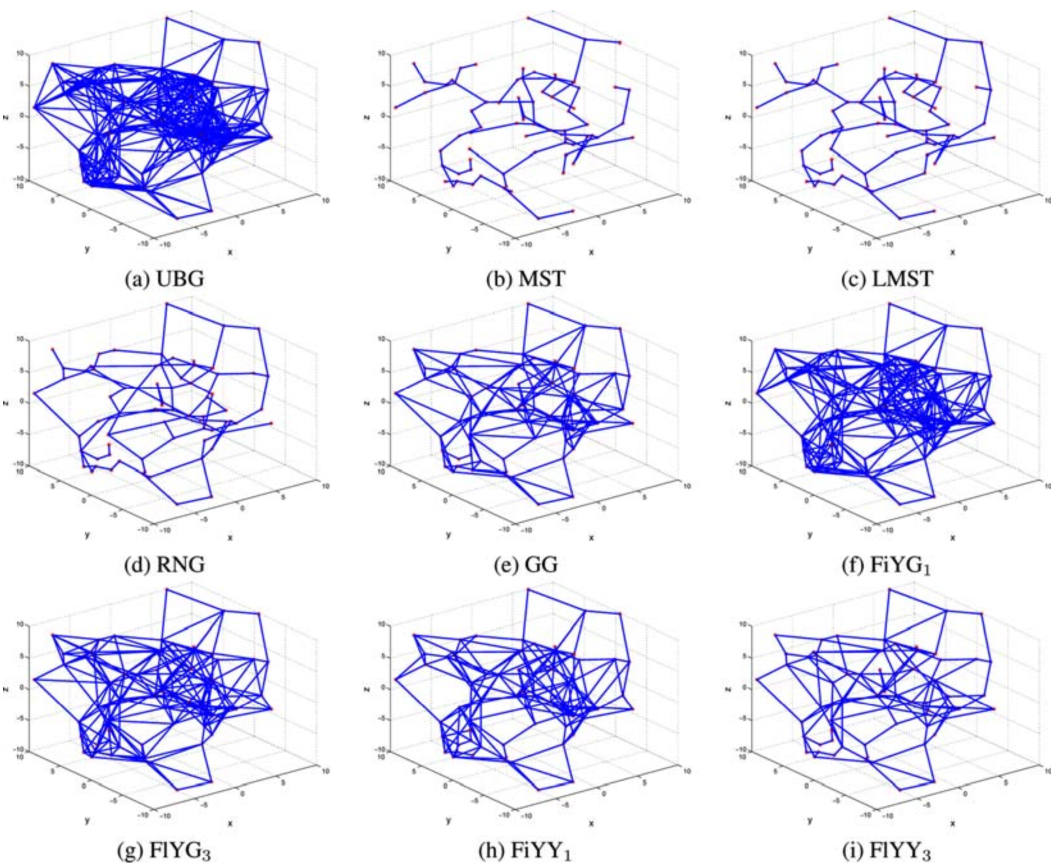
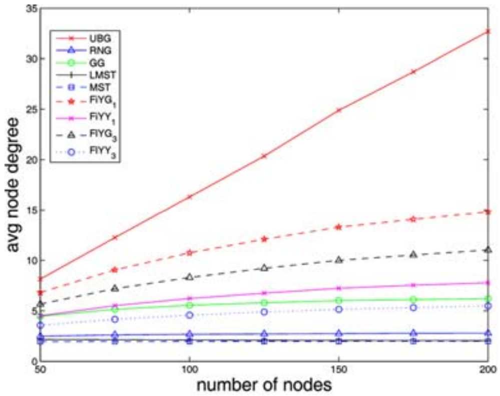
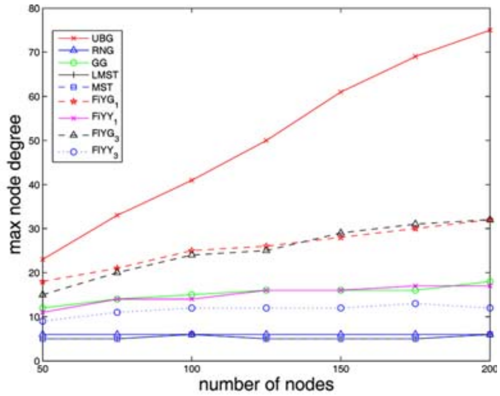


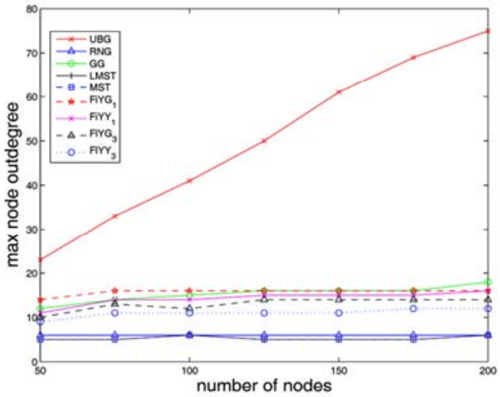
Figure 9 Results when number of sensor nodes increasing from 50 to 200 (see online version for colours)



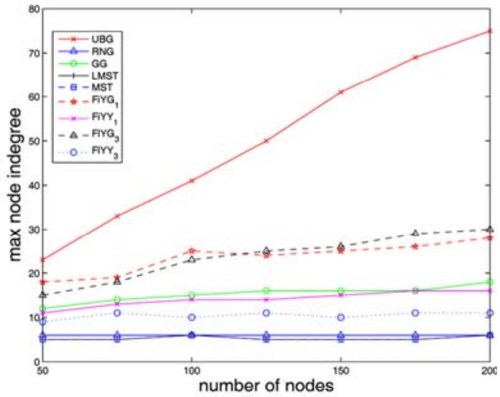
(a) avg-node-degree



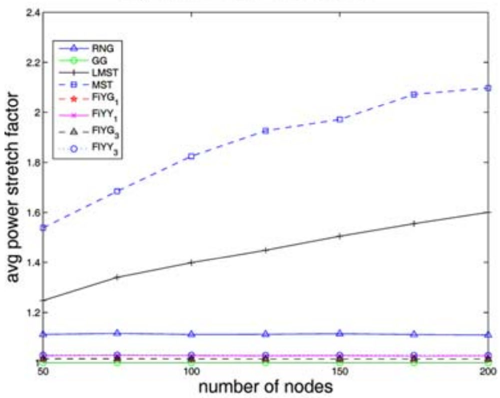
(b) max-node-degree



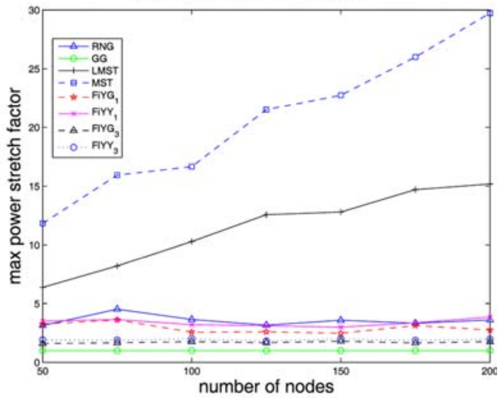
(c) max-node-out-degree



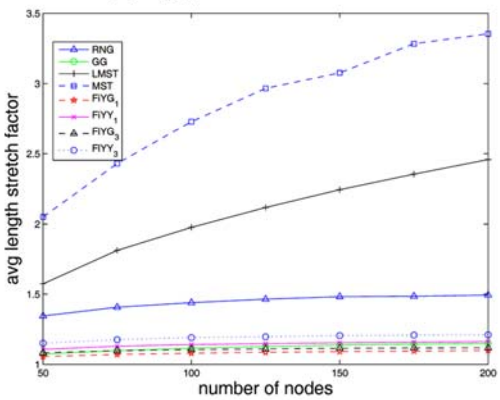
(d) max-node-in-degree



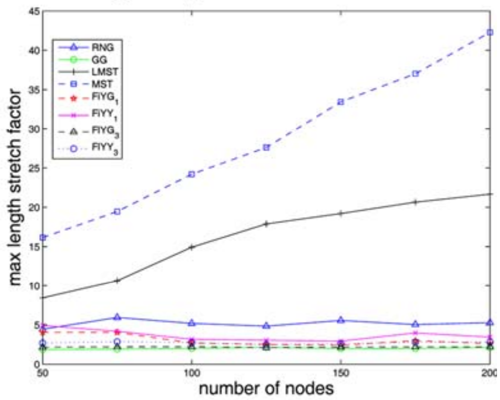
(e) avg-power-stretch-factor



(f) max-power-stretch-factor



(g) avg-length-stretch-factor



(h) max-length-stretch-factor

6 Conclusion

Topology control for sensor networks has been heavily studied recently and different geometric topologies were proposed to achieve the sparseness and connectivity of the communication network. However, most of them are only applied to 2D networks. In this paper, we extend two 2D geometric topologies (RNG and GG) to 3D case and propose several new 3D Yao-based topologies for sensor networks. All our 3D topologies can be constructed locally and efficiently, and we prove the power efficiency of several proposed topologies. Our simulation results confirm the nice performance of these 3D topologies. We are planning to implement the proposed methods in network simulators and evaluate their network performance. This paper is just the first step of research for 3D topology control; there are still a number of challenging questions left for future research. For example: If the position information is not accurate, how to achieve power efficiency of these geometric topologies? If we want to minimise the link interference among the links in the network, how to design the 3D network topology?

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References

Akyildiz, I.F., Pompili, D. and Melodia, T. (2005) 'Underwater acoustic sensor networks: research challenges', *Ad Hoc Networks*, Vol. 3, No. 3, pp.257–279.

Alam, S.M.N. and Haas, Z.J. (2006) 'Coverage and connectivity in three-dimensional networks', *Proceedings of the 12th ACM International Conference on Mobile Computing and Networking*, Los Angeles, CA.

Alzoubi, K., Li, X-Y., Wang, Y., Wan, P-J. and Frieder, O. (2003) 'Geometric spanners for wireless ad hoc networks', *IEEE Transactions on Parallel and Distributed Processing*, Vol. 14, No. 4, pp.408–421.

Bahramgiri, M., Hajiaghayi, M.T. and Mirrokni, V.S. (2002) 'Fault-tolerant and 3-dimensional distributed topology control algorithms in wireless multi-hop networks', *Proceedings of IEEE International Conference on Computer Communications and Networks*.

Bao, L. and Garcia-Luna-Aceves, J.J. (2003) 'Topology management in ad hoc networks', *Proceedings of the 4th ACM International Symposium on Mobile Ad Hoc Networking & Computing*, Annapolis, MD.

Benlarbi-Delaï, A., Simplot, D., Cartigny, J. and Cousin, J-C. (2003) 'Using 3D indoor microwave phase sensitive stereoscopic location system to reduce energy consumption in wireless ad-hoc networks', *Proceedings of Smart Objects Conference (sOc)*, Grenoble, France.

Bose, P., Devroye, L., Evans, W. and Kirkpatrick, D. (2002) 'On the spanning ratio of Gabriel graphs and beta-skeletons', *Proceedings of the Latin American Theoretical Infocomatics*.

Bose, P., Morin, P., Stojmenovic, I. and Urrutia, J. (2001) 'Routing with guaranteed delivery in ad hoc wireless networks', *ACM/Kluwer Wireless Networks*, Vol. 7, No. 6, pp.609–616.

Cerpa, A. and Estrin, D. (2002) 'ASCENT: adaptive self-configuring sensor network topologies', *SIGCOMM Computer Communication Review*, Vol. 32, No. 1, p.62.

Chen, B., Jamieson, K., Balakrishnan, H. and Morris, R. (2002) 'SPAN: an energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks', *Wireless Network*, Vol. 8, No. 5, pp.481–494.

Gabriel, K. and Sokal, R. (1969) 'A new statistical approach to geographic variation analysis', *Systematic Zoology*, Vol. 18, pp.259–278.

Grünewald, M., Lukovszki, T., Schindelhauer, C. and Volbert, K. (2002) 'Distributed maintenance of resource efficient wireless network topologies', *Proceedings of the 8th European Conference on Parallel Computing (Euro-Par'02)*.

Hansen, J.C. and Schmutz, E. (2005) 'Comparison of two CDS algorithms on random unit ball graphs', *Proceedings of the 7th Workshop on Algorithm Engineering and Experiments/the 2nd Workshop on Analytic Algorithms and Combinatorics*.

Huang, C-F., Tseng, Y-C. and Lo, L-C. (2004) 'The coverage problem in three-dimensional wireless sensor networks', *Proceedings of IEEE Globecom*.

Jennings, E. and Okino, C. (2002) 'Topology control for efficient information dissemination in ad-hoc networks', *Proceedings of International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS)*.

Jia, L., Rajaraman, R. and Scheideler, C. (2003) 'On local algorithms for topology control and routing in ad hoc networks', *Proceedings of the 15th Annual ACM Symposium on Parallel Algorithms and Architectures*.

Karp, B. and Kung, H. (2000) 'GPSR: greedy perimeter stateless routing for wireless networks', *Proceedings of the ACM International Conference on Mobile Computing and Networking (MobiCom)*.

Keil, J.M. and Gutwin, C.A. (1992) 'Classes of graphs which approximate the complete Euclidean graph', *Discrete Computational Geometry*, Vol. 7, No. 1, pp.13–28.

Kleinrock, L. and Silvester, J. (1978) 'Optimum transmission radii for packet radio networks or why six is a magic number', *Proceedings of the IEEE National Telecommunications Conference*.

Li, L., Halpern, J.Y., Bahl, P., Wang, Y-M. and Wattenhofer, R. (2001a) 'Analysis of a cone-based distributed topology control algorithms for wireless multi-hop networks', *Proceedings of ACM Symposium on Principle of Distributed Computing*.

Li, N., Hou, J.C. and Sha, L. (2003a) 'Design and analysis of a MST-based topology control algorithm', *Proceedings of IEEE INFOCOM*.

Li, X-Y. (2003) 'Topology control in wireless ad hoc networks', in Basagni, S., Conti, M., Giordano, S. and Stojmenovic, I. (Eds): *Ad Hoc Networking*, IEEE Press, pp.175–203.

Li, X-Y., Wan, P-J. and Wang, Y. (2001b) 'Power efficient and sparse spanner for wireless ad hoc networks', *Proceedings of IEEE International Conference on Computer Communications and Networks*.

Li, X-Y., Wan, P-J., Wang, Y. and Frieder, O. (2002) 'Sparse power efficient topology for wireless networks', *Proceedings of IEEE Hawaii International Conference on System Sciences (HICSS)*.

Li, X-Y., Wan, P-J., Wang, Y. and Yi, C-W. (2003b) 'Fault tolerant deployment and topology control for wireless ad hoc networks', *Proceedings of the 4th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*.

Li, X-Y., Wang, Y. and Song, W-Z. (2004) 'Applications of k-local MST for topology control and broadcasting in wireless ad hoc networks', *IEEE Transactions on Parallel and Distributed Systems*, Vol. 15, No. 12, pp.1057–1069.

Liang, B. and Haas, Z.J. (2000) 'Virtual backbone generation and maintenance in ad hoc network mobility management', *Proceedings of the 19th Annual Joint Conference of the IEEE Computer and Communications Societies INFOCOM*.

Lukovszki, T. (1999) *New Results on Geometric Spanners and Their Applications*, PhD thesis, University of Paderborn.

Nikaein, N. and Bonnet, C. (2004) 'Topology management for improving routing and network performances in mobile ad hoc networks', *Mobile Networks and Applications*, Vol. 9, No. 6, pp.583–594.

Pan, J., Hou, Y.T., Cai, L., Shi, Y. and Shen, S.X. (2003) 'Topology control for wireless sensor networks', *Proceedings of the 9th ACM International Conference on Mobile Computing and Networking*.

Rajaraman, R. (2002) 'Topology control and routing in ad hoc networks: a survey', *SIGACT News*, Vol. 33, pp.60–73.

Ramanathan, R. and Hain, R. (2000) 'Topology control of multihop wireless networks using transmit power adjustment', *Proceedings of IEEE INFOCOM*.

Ruhrup, S., Schindelhauer, C., Volbert, K. and Grunewald, M. (2003) 'Performance of distributed algorithms for topology control in wireless networks', *Proceedings of the International Parallel and Distributed Processing Symposium*.

Salhieh, A., Weinmann, J., Kochhal, M. and Schwiebert, L. (2001) 'Power efficient topologies for wireless sensor networks', *Proceedings of the International Conference on Parallel Processing*.

Schindelhauer, C., Volbert, K. and Ziegler, M. (2007) 'Geometric spanners with applications in wireless networks', *Computational Geometry: Theory and Applications*, Vol. 36, No. 3, pp.197–214.

Schurgers, C., Tsiatsis, V. and Srivastava, M.B. (2002) 'STEM: topology management for energy efficient sensor networks', *Proceedings of IEEE Aerospace Conference*.

Seddigh, M., Gonzalez, J.S. and Stojmenovic, I. (2001) 'RNG and internal node based broadcasting algorithms in wireless one-to-one networks', *ACM Mobile Computing and Communications Review*, Vol. 5, No. 2, pp.394–397.

Song, W-Z., Wang, Y., Li, X-Y. and Frieder, O. (2004) 'Localized algorithms for energy efficient topology in wireless ad hoc networks', *Proceedings of 5th ACM International Symposium on Mobile Ad Hoc Networking and Computing*.

Srivastava, G., Boustead, P. and Chicharo, J. (2003) 'A comparison of topology control algorithms for ad-hoc networks', *Proceedings of Australian Telecommunications Networks Applications Conference (ATNAC 2003)*.

Toussaint, G.T. (1980) 'The relative neighborhood graph of a finite planar set', *Pattern Recognition*, Vol. 12, No. 4, pp.261–268.

Wang, Y. (2008) 'Topology control for wireless sensor networks', in Li, Y., Thai, M. and Wu, W. (Eds): *Wireless Sensor Networks and Applications*, Springer.

Wang, Y. and Li, X-Y. (2003) 'Localized construction of bounded degree planar spanner for wireless networks', *Proceedings of ACM DIALM-POMC Joint Workshop on Foundations of Mobile Computing*.

Wattenhofer, R., Li, L., Bahl, P. and Wang, Y-M. (2001) 'Distributed topology control for wireless multihop ad-hoc networks', *Proceedings of IEEE INFOCOM*.

Yao, A.C-C. (1982) 'On constructing minimum spanning trees in k-dimensional spaces and related problems', *SIAM Journal on Computing*, Vol. 11, pp.721–736.